Development of software to improve uncertainty estimations on velocity of stars

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Abstract

This report has the goal of displaying the results obtained on the PEEC (Projecto de Estágio Extra-Curricular) internship at the Centro de Astrofísica da Universidade do Porto whose purpose was to develop a programming tool to improve the measurement on the uncertainties of velocities of stars with the ultimate goal of setting constrains in the dark matter content present in ultra-faint dwarf galaxies. To analyze the results, spectra from the MUSE telescope of stars in the Leo T ultra-faint dwarf galaxy were used. A comparison between the results obtained from the developed software and data from the spectra fitting framework *spexxy* was made and the average results were in agreement based on the average velocity difference between the two tools, which equaled 0 ± 1 km/s. Moreover, it was also shown that the developed tool returns velocities with lower uncertainties compared to *spexxy*. This result was also more evident for high SNR spectra. Using the developed tool, it was obtained an average line-of-sight velocity of $v_{\odot} = 39.16^{+1.65}_{-1.82}\,kms^{-1}$, with an relative error of 3%, a velocity dispersion of value $\sigma_v = 6.81^{+2.11}_{-1.60}\,kms^{-1}$ with a relative error of 9%, the dynamical mass of the system with value $M_{dyn} = 5.2 \pm 3.1 \times 10^6 M_{\odot}$ and a mass-to-light ratio of $M_{dyn}/L = 89 \pm 58 \frac{M_{\odot}}{L_{\odot}}$, in agreement with values on the scientific literature on the Leo T ultra-faint dwarf.

1 Introduction

There is overwhelming evidence that almost all the matter which composes the universe we live in is of unknown origin[1]. Some contribution comes from what is called "dark matter", a yet unknown type of matter that has this name due to the fact it does not absorb, reflect nor emit electromagnetic radiation and we can only know it's there by its gravitational influence. Astrophysical observations have shown that it makes up ~25% of the mass of the observable universe and combined with the ~75% contribution from what is called "dark energy", they make up ~95% of all the matter in the universe[2]. That means all baryonic matter - i.e things made up of electrons, protons and neutrons - only accounts to a small fraction of the composition of our universe: the Earth, the Sun, all the other hundreds of billions of stars both in the Milky Way and across hundreds of billions of galaxies in the entire universe only amounts to 5% of all there is.

Multiple candidates for the composition of dark matter have been proposed, ranging from baryonic MACHOs (Massive compact halo object)[3] to new, still undiscovered, non-baryonic particles such as WIMPs (Weakly interacting massive particles)[4], and active research into these and other possible dark matter particles is currently being made. One possibility to constrain dark matter is to probe into systems which appear to contain large concentrations of it - stellar objects whose mass observed from the light they give off is far smaller than the mass measured by their gravitational influence. In other words, they contain some kind of matter which doesn't give off light but still makes itself noticed by its gravity. And stellar objects called ultra-faint dwarfs, first discovered in 2005[5], are one of such systems.

Ultra-faint dwarfs are the smallest and least luminous galaxies in the universe. They represent the extreme limit of the process of star formation, providing hints into the formation of the early galaxies in the universe. Furthermore, these galaxies are of special interest due to the fact they contain the highest mass-to-light ratios measured in any galaxy, providing significant clues to the behavior of dark matter on small scales[6]. To better understand the dark matter content present on these stellar objects, it is of extreme importance to have good measurement of the intrinsic velocity dispersion of the stars within them. That's the case since the velocity dispersion, which ultimately reflects the statistical dispersion of the velocity of stars around the mean velocity, is used to find the dynamical mass of the system.

Bayesian based methods like Markov Chain Monte Carlo (MCMC) are often used in astronomy research as well as in many other scientific disciplines due to their robustness and applicability to a wide range of problems. With the purpose of trying to decrease the uncertainty on the velocities measured, this project utilized this statistical method in a Python programming algorithm which, given as input spectra of stars, returns their average stellar velocity as well as their velocity dispersion. More information about MCMC is given in section 2.2 .

To check the validity of the results obtained, data from the MUSE (Multi-unit spectroscopic explorer) telescope of the ultra-faint dwarf galaxy Leo T were used. More information about Leo T, its general properties and some of the previous studies about it will be discussed on section 3 . This section also presents a comparison between results obtained from the developed software with the *spexxy* fitting framework and also makes a comparison with values present on the astronomy literature on the Leo T system. An overview of the dynamical mass and mass-to-light ratio calculated for the Leo T system using the data is then discussed on section 4 .

2 Methods

2.1 Maximum likelihood estimation

The initial approach adopted was to approximate the absorption lines from the star's spectra as Gaussian curves with the goal of obtaining the star's velocity. To do so, a Python library called *lmfit*[1] was used. This library provides a set of tools to help build fitting models for non-linear least-squares problems, applying such models to observed data.

The Python library was then used to fit Gaussian lines on top of the spectrum's most prominent absorption lines, namely Hα and Hβ (on hotter stars Ca II absorption lines were also used), to obtain the radial velocity of the star using the Doppler effect. A Gaussian function is given by

$$M(\lambda; A, \sigma, \lambda_{true}) = \frac{A}{\sqrt{2\pi}\sigma} \exp\left(\frac{(\lambda - \lambda_{true})^2}{2\sigma^2}\right) \tag{1}$$

where $M(\lambda)$ is the best fit for a given absorption line in the spectrum, A is the amplitude, λ_{true} is the theoretical wavelength value for a given absorption line and σ is the standard deviation (i.e the "width" of the gaussian curve).

Given the spectral model $M(\lambda)$, the likelihood function is given by

$$\mathcal{L}(S(\lambda)|\theta) = \prod_i \frac{1}{\sqrt{2\pi}\sigma_i} \exp\left(-\frac{(S(\lambda_i) - M(\lambda_i;\theta))^2}{2\sigma_i{}^2}\right) \tag{2}$$

where θ represents the parameters for the model.

Therefore, the parameter θ must be such that L is maximum. That means

$$\frac{d\mathcal{L}}{d\theta} = 0 \tag{3}$$

It is worth pointing out that σ_i on equation (2) relates to the uncertainty on each measurement, where σ on equation (1) refers to the standard deviation on the gaussian curve fitting the observed spectra. Once calculating this derivative for each particular wavelength (and its particular value of A and σ), it is possible to show[7] that the best estimate for the theoretical mean for this likelihood function is the *weighted mean*

$$\overline{X}_w = \frac{\sum_i^N x_i/\sigma_i^2}{\sum_i^N 1/\sigma_i^2} \tag{4}$$

where x_i are the measurements made and σ_i is the uncertainty associated with each measurement. This value is also known as standard error (which here I called u) and is given by

$$u^2 = \frac{1}{\sum_i^N \frac{1}{\sigma_i^2}} \tag{5}$$

[1]For more information, visit https://lmfit.github.io/lmfit-py/intro.html

In the case where the uncertainties σ_i of the samples are all equal (i.e $\sigma_i = \sigma$), we have that

$$u = \frac{\sigma}{\sqrt{N}} \tag{6}$$

where N is the number of measurements used for the calculation of the mean and σ is the standard error on the sample. Thus, u is the uncertainty on the weighted mean $\overline{X}_w$.

The function which best fit the data is centered at a wavelength λ_{obs} (point where $|M(\lambda)|$ has maximum value). Because the theoretical wavelength λ_{true} is given (as a parameter), it is possible to calculate the velocity of the star using the Doppler effect

$$v_\odot = c\left(\frac{\lambda_{obs}}{\lambda_{true}} - 1\right) \tag{7}$$

with

$$\sigma_v = \sigma_\lambda \frac{c}{\lambda_{true}} \tag{8}$$

where σ_v is the uncertainty on the velocity, σ_λ is the uncertainty on the wavelength and c is the speed of light.

Hence, it was possible to implement the initial version of the Python program: given a star's spectrum and a set of wavelengths, it returns the average velocity and uncertainty of the star. Figure 1 shows an example of a Gaussian fit.

A special focus was given to finding the best gaussian fit centered at a particular wavelength λ_{true}. That was achieved by fitting multiple curves around λ_{true} and selecting the curve with the lowest uncertainty. The curves varied only in the range of data points it utilized for the fit. The parameters returned are then used to calculate the average velocity of the star, also being used later on as initial parameters for the Markov chain Monte Carlo statistical method.

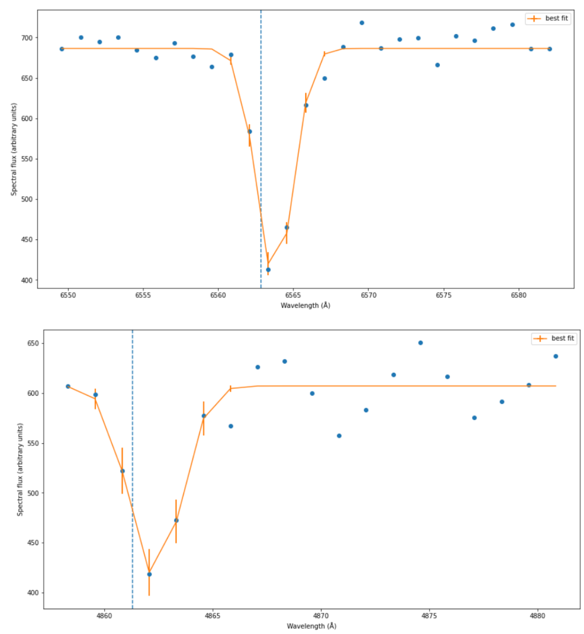

Figure 1: Gaussian fit for the Hα and Hβ absorption line of the spectrum of source ID 4031jd2458163p6789f000.

2.2 Markov chain Monte Carlo

Markov chain Monte Carlo techniques have existed for more than 50 years and one of the reason for their increase in use over the last decade has to do with the huge increase in computational power over the same period[8]. MCMC is a set of methods for sampling a *probability distribution function* (PDF) using a Markov chain whose equilibrium distribution is the distribution we want[8]. What distinguishes MCMC is the fact it allows the possibility of systematically including prior information on data. That means limiting where in the state space our "random walk" can go, based on the information we have *a priori* on the system. Bayes' Theorem states that

$$p(H|D,I) = \frac{p(D|H,I)p(H|I)}{p(D|I)} \tag{9}$$

where H is denoted as the hypothesis, D is the data and I is the prior information we have on the system. Therefore, $p(D|H,I)$ can be treated as the probability of observing the data D if the hypothesis H is true and it is known as the *likelihood*. The value $p(H,I)$ is called the *prior* and it specifies the prior knowledge on the hypothesis H being true. The quantity $p(H|D,I)$ is the updated belief about the system in case the hypothesis is true given the data and it is called the *posterior*. Finally, $p(D,I)$ is simply a constant to normalize $p(H|D,I)$, i.e to make the integral $\int p(H|D,I)dH = 1$ and it is known as the *evidence.* In other words,

$$posterior = \frac{likelihood \times prior}{evidence} \tag{10}$$

Going back to the implementation: from a initial value for the velocity of a star and its uncertainty, it was then possible to use these values as initial parameters for the MCMC analysis. The likelihood function for the MCMC method is the same as the one used on *lmfit* given by equation (2). The spectral model is given by

$$M(\lambda; \Delta_v, \sigma, A, c|\lambda_T) = c + \frac{A}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(\lambda + \Delta\lambda(\Delta_v) - \lambda_T)^2}{2\sigma^2}\right) \tag{11}$$

where Δ_v is the velocity given by the Doppler effect (which is derived by the change in wavelength $\Delta\lambda$), σ is the uncertainty of the velocity, A is the amplitude of the Gaussian fit and c is the parameter which dislocate the Gaussian function vertically to precisely fit the data. These values are the parameters used for the MCMC analysis and are represented by θ. All of the parameters listed before except for the speed of light c are obtained via *lmfit* for a given theoretical wavelength λ_T.

For the implementation in Python, the library *emcee*[2] was used. *emcee* is a pure-Python open-source implementation of Goodman & Weare's Affine Invariant Markov chain Monte Carlo Ensemble sampler[9].

It was assumed that the line-of-sight velocities analyzed were between -100 km/s and 100 km/s, and that information is the *prior* we'll use for the MCMC method. Moreover, for each wavelength it was calculated the star's velocity and its associated uncertainty using the Doppler shift. Since multiple wavelengths are used, it is necessary to take all of them into account for the *true* average velocity v_{avg} and average uncertainty $\sigma_{v,avg}$ given by equations (4) and (5).

The values regarding the average velocity and average uncertainty are the basis for the estimation of the velocity dispersion - in fact, they are the initial parameters for the calculation (which also utilizes MCMC).

The final code[3] ultimately has only one Python module, a file called *spectra.py*, containing the following public methods:

- *plot()*: returns a visual representation of the spectra of the star, with an option to display specific wavelength ranges;
- *fit_mcmc():* returns the average velocity of a star given its spectrum and multiple wavelengths (H alpha and H beta are already use by default). The method uses Markov Chain Monte Carlo to go over the entire n-th dimensional state space(where n is the number of parameters of the system), returning a series of plots which highlight the correlation between each parameter used and also their values.

[2]For more information about *emcee*, visit https://emcee.readthedocs.io/en/stable/.
[3]The code is available at https://github.com/w3slley/PEEC-star-velocities.

3 Observational data

For a proof that the program developed returns reasonable results, one would expect its results to be in agreement with obtained on observational data from routinely used tools utilized in astronomical research with the exact purpose of deriving line-of-sight velocities from spectra of stars. One of such tools is *spexxy*[4], a framework used for the analysis of astronomical spectra. spexxy, however, is complex and has a huge variety of models. Because of that, it becomes important to have more control over the process involved in obtaining the velocity of stars and, subsequently, the uncertainties associated with them. Building a software tool from scratch is one solution to try to minimize the uncertainties obtained and that was exactly the motivation for this project.

The data utilized was from the ultra-faint dwarf galaxy Leo-T, which was gathered by the MUSE (Multi-unit spectroscopic explorer) telescope. The Leo T system was discovered in 2007 using the Sloan Digital Sky Survey (SDSS) by M. J. Irwin et al. (2007)[10]. The ultra-faint dwarf system has been subject of study in the past[10][11][12] and has the following properties:

Properties	**Values**	**Reference**
Distance	~420 kpc	[10]
$v_{\odot}$ (kms^{-1})	38.1 ± 2.0	[13, 14]
σ_v(kms^{-1})	7.5 ± 1.6	[13, 14]
$R_{1/2}$(pc)	118 ± 11	[6]
M_v	-8.0 ± 0.5	[13, 14]
L ($10^4 L_{\odot}$)	5.9 ± 1.8	[15]

Table 1: Properties of Leo T

3.1 Velocity estimate for the stars

In total, 55 spectra of stars in the Leo T ultra-faint dwarf were available, but only 45 of those were used to calculate the line-of-sight velocity of the galaxy as well as its velocity dispersion. The tool was unable to return results for the remaining 10 spectra due to their low signal to noise ratio. Figure 3 shows the difference in line-of-sight velocity between the developed software and *spexxy* as a function of signal to noise ratio.

Figure 2 highlights the distribution of the data using a histogram.

[4]Available at https://github.com/thusser/spexxy.

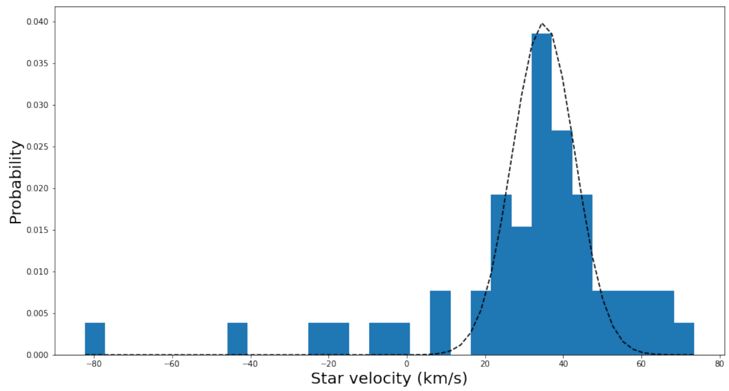

Figure 2: Distribution of velocities returned from the developed software.

Using equations (4) and (5), it is possible to calculate the average velocity difference and its equivalent average uncertainty

$$\Delta V_{avg} = \sum_i \frac{\Delta v_i}{\sigma^2_{\Delta v,i}} \tag{12}$$

$$\sigma^2_{\Delta v,avg} = \frac{1}{\sum_i \frac{1}{\sigma^2_{\Delta v,i}}} \tag{13}$$

where Δv_i is the velocity difference between the tool and *spexxy* for each spectra

$$\Delta v_i = V_{i(tool)} - V_{i(spexxy)} \tag{14}$$

$\sigma_{\Delta v,i}$ is the uncertainty on the previous value and is given by

$$\sigma_{\Delta v,i} = max(\sigma_{i(tool)}, \sigma_{i(spexxy)}) \tag{15}$$

ΔV_{avg} is the average velocity difference and $\sigma_{\Delta v,avg}$ is average velocity difference uncertainty. The weighted average of all the velocity differences was calculated to be $\Delta V_{avg} = 0 \pm 1$ km/s, ultimately showing that on average the results obtained agree with the ones from *spexxy*.

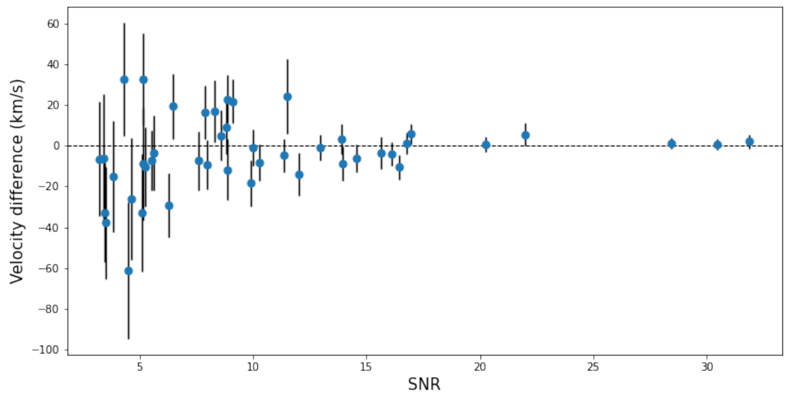

Figure 3: Velocity difference between developed software and *spexxy* as a function of signal to noise ratio (SNR).

It is worth highlighting the comparison between the uncertainties yielded by spexxy and the ones calculated in the developed tool. For high SNR spectra, the uncertainty values for both tools are not radically different. This indicates that spexxy's velocity uncertainties are indeed plausible. Figure 4 displays the velocity uncertainty difference between the two tools, showing visually that, as mentioned, the two uncertainties are not very different for high SNR spectra. Overall, the average velocity uncertainty difference was calculated to be -1 ± 1km/s, using the sample mean and an uncertainty given by equation (6).

Using equation (14) and the result calculated before, it is possible to show that the velocity uncertainties returned by the developed software are on average smaller than the ones from spexxy, and that is certainly the case for high SNR spectra as Figure 4 shows. This is a positive result since one of the main goals of the project was to decrease the uncertainties on the velocities to better characterize the dynamical mass of stellar systems.

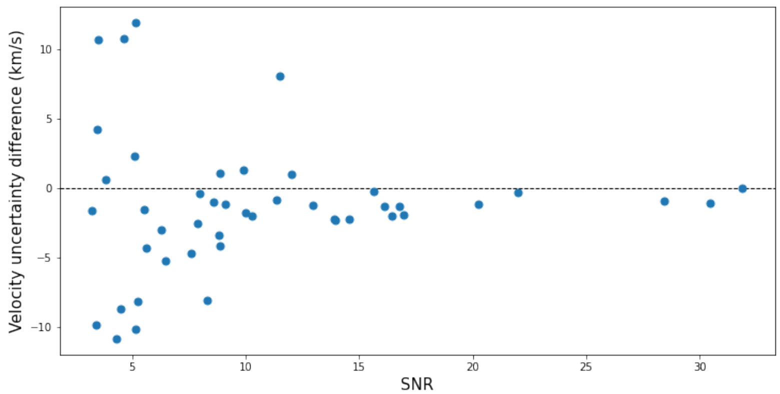

Figure 4: Velocity uncertainty differences between the developed tool and spexxy as a function of signal to noise ratio (SNR).

3.2 Velocity dispersion

The total likelihood is thus defined as:

$$\mathcal{L}(S(\lambda)|\theta) = \prod_i \frac{1}{\sqrt{2\pi}\sigma_i} \exp\left(-\frac{(S(\lambda_i) - M(\lambda_i))^2}{2\sigma_i{}^2}\right) f + \Upsilon(1-f) \tag{16}$$

where Υ is the constant likelihood function for the background of stars which are not in the system being studied and the parameter *f*, which is also highlighted as *frac* in Figure 6, has to do with the percentage of stars that are within the ultra-faint dwarf system. The gaussian function on equation (16) represents the distribution of stars in the system we are trying to study and it is usually an assumption made based on the dynamics of the system (e.g whether it rotates or not, etc).

The average velocity and its equivalent uncertainty obtained from the previous step were then used as initial parameters to obtain the velocity dispersion of the ultra-faint dwarf. Figure 5 shows the path taken by *walkers* throughout the system's state space using the data after the period of burn-in. Figure 6 displays the results on each parameter and the correlation between them. Both graphs were generated by a Python library called *corner.*

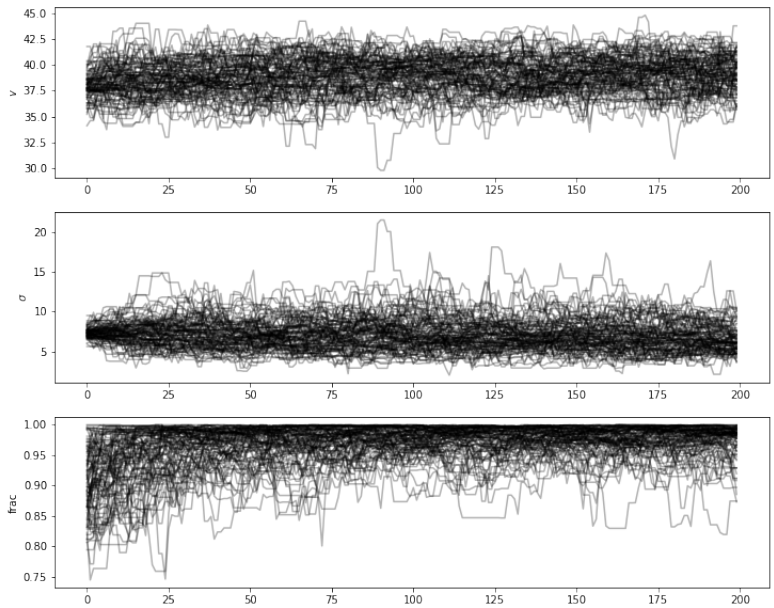

Figure 5: Corner plot with MCMC walker iterations after burn-in.

The *frac* parameter indicates that 98% of the stars are within Leo T. Furthermore, it was found an average line-of-sight velocity of $v_{\odot} = 39.16^{+1.65}_{-1.82}\ kms^{-1}$ and a velocity dispersion of $\sigma_v = 6.81^{+2.11}_{-1.60}\ kms^{-1}$. Comparing these results with values on the scientific literature, the value for the line-of-sight velocity obtained has 3% relative error and the velocity dispersion has 9% relative error (compared to values on Simon & Geha 2007[13]).

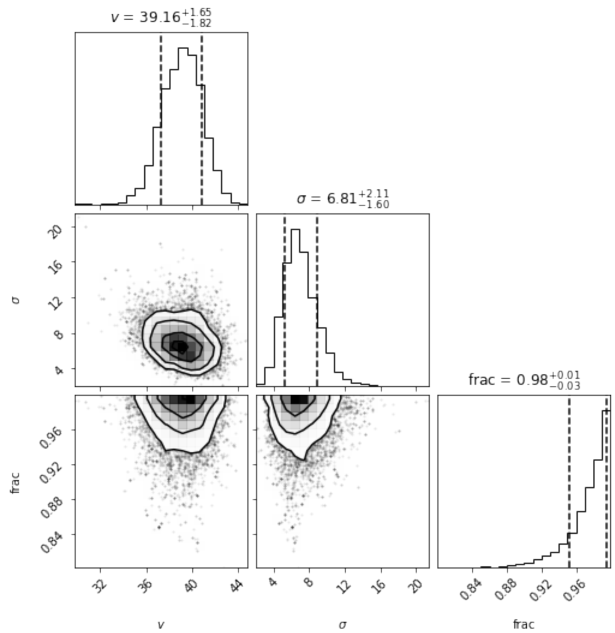

Figure 6: Corner plot with the results and correlation between parameters.

Table 2 summarizes the results obtained with the developed software on the Leo T ultra-faint dwarf system.

Parameter	Value
$v_{\odot}$ (kms^{-1})	$39.16^{+1.65}_{-1.82}$
σ_v (kms^{-1})	$6.81^{+2.11}_{-1.60}$
$M_{dyn}(10^6 M_{\odot})$	5.2 ± 3.1
$M_{dyn}/L(\frac{M_{\odot}}{L_{\odot}})$	89 ± 58

Table 2: Direct parameters (line-of-sight velocity $v_{\odot}$ and velocity dispersion σ_v) calculated by the developed tool and indirect results (dynamical mass M_{dyn} and mass-to-light ratio M_{dyn}/L) derived from them.

4 Analysis

Once having the velocity dispersion of the Leo T ultra-faint dwarf, now it is possible to estimate the dynamical mass of the system. Assuming that the Leo T stellar system is in dynamical equilibrium and that there is no interference due to binary system on the velocity dispersion, it's possible to estimate the dynamical mass using[16]

$$M_{dyn} = \frac{4\sigma_v^2 R_{1/2}}{G} \tag{17}$$

where σ_v is the velocity dispersion, $R_{1/2}$ is the half-light radius in parsec and G is the gravitational constant. Leo T has a half-light radius of 118 ± 11 pc[6] and a known luminosity of $L = 5.9 \pm 1.8 \times 10^4 L_\odot$[15]. Thus, the total dynamical mass has a value of $M_{dyn} = 5.2 \pm 3.1 \times 10^6 M_\odot$. Using a stellar mass-to-light ratio of 2[10], this yields a stellar mass of ~$10^5 M_\odot$. Utilizing the dynamical mass calculated previously, the observed mass-to-light ratio is therefore $M_{dyn}/L = 89 \pm 58 \frac{M_\odot}{L_\odot}$. These results are also highlighted in Table 2 . The high difference between the stellar mass and the dynamical mass combined with the high mass-to-light ratio indicates a strong presence of dark matter in the Leo T ultra-faint dwarf system. Moreover, the high uncertainties on M_{dyn} and in the mass-to-light ratio are due to uncertainties in the velocities of the stars themselves. By decreasing the uncertainties on the velocities even more one would be able to obtain more and more precise values for M_{dyn} and M_{dyn}/L . The results are in agreement with the values on the scientific literature, being placed in between the lower bound values by E. V. Ryan-Weber et al. (2007)[11] of $M_{dyn} \gtrapprox 3.3 \times 10^6 M_\odot$ and $M_{dyn}/L \gtrapprox 56 \frac{M_\odot}{L_\odot}$ and the upper bound values by Simon & Geha (2007)[13] of $M_{dyn} = 8.2 \pm 3.6 M_\odot$ and $M_{dyn}/L = 138 \pm 71 \frac{M_\odot}{L_\odot}$.

5 Conclusion

The present report discussed the implementation of a programming tool whose purpose was to improve the uncertainty estimations on the velocity of stars on ultra-faint stellar systems as a way to better constrain their dark matter content. Utilizing data from the ultra-faint dwarf Leo T gathered from the MUSE telescope, it was possible to 1) reproduce results similar to those from the *spexxy* spectra fitting framework, providing evidence to the validity of *spexxy's* velocity uncertainty estimations and 2) implement from scratch an algorithm using Bayesian analysis and Markov chain Monte Carlo which returned line-of-sight velocities with lower uncertainties on average compared to *spexxy*'s, with an average uncertainty difference on the velocities of -1 ± 1km/s - ultimately improving the dynamical mass and mass-to-light ratio estimations for the Leo T system and fulfilling one of the main goals of the project.

With the developed tool, the following results were obtained: a line-of-sight velocity $v_\odot = 39.16^{+1.65}_{-1.82}\ kms^{-1}$ with a relative error of 3%, a velocity dispersion $\sigma_v = 6.81^{+2.11}_{-1.60}\ kms^{-1}$ with a relative error of 9%, ultimately obtaining a dynamical mass $M_{dyn} = 5.2 \pm 3.1 \times 10^6 M_\odot$ and a mass-to-light ratio of $M_{dyn}/L = 89 \pm 58 \frac{M_\odot}{L_\odot}$, results which are in agreement with values obtained on previous studies made on the Leo T system. To further test the software implemented, a good sequel to this research project could be to apply the tool to other ultra-faint dwarf systems known to have a high value for the mass-to-light ratio, or even to compare the results acquired with another commonly used spectra fitting software.

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A Leo-T data

Below is the data used for the analysis on the present report.

ID	SNR	Velocity (kms⁻¹)	Uncertainty (kms⁻¹)	spexxy_Vel (kms⁻¹)	spexxy_Unc (kms⁻¹)
3188	7.9539	24.7145	11.5701	33.8895	11.9765
3234	3.4146	61.4271	21.594	67.4415	31.4754
3307	5.2458	28.6164	11.2393	39.0983	19.4139
3356	7.5811	27.5251	9.8029	34.8108	14.4898
3357	10.01	46.1016	7.0786	46.9275	8.8949
3632	20.2626	50.886	2.6104	50.2279	3.7808
3642	3.4612	35.8258	24.4927	68.4658	20.2693
3701	13.9221	36.6971	5.1542	33.384	7.4365
3720	12.0049	32.2429	10.4801	46.478	9.4738
3875	9.927	36.5708	11.0915	55.0062	9.7915
3961	14.583	43.9249	4.6596	50.0615	6.8826
4028	8.8108	52.2809	9.9496	43.2024	13.3452
4031	30.4425	44.4215	1.6587	43.7474	2.7694
4035	8.5737	54.7655	11.1777	49.6643	12.1866
4165	13.953	27.0638	6.0265	35.7928	8.3659
4227	9.0861	52.328	9.7739	30.4136	10.9493
4260	16.9479	32.3799	3.1416	26.6429	5.0967
4302	5.5513	40.9586	13.0611	48.0661	14.6331
4337	7.8723	37.1898	10.7561	20.7449	13.3145
5717	16.1284	35.3852	4.6052	39.4328	5.9283
5840	5.1681	36.2972	12.5175	3.6214	22.7121
5986	16.445	37.8106	4.0208	48.2578	6.0255
6053	8.8783	66.8736	12.0271	43.9409	11.0021
6156	28.4322	44.7336	1.5852	43.5216	2.5578
6412	22.0104	45.5013	5.0968	39.9675	5.448
6451	5.1372	41.2537	27.6456	50.2747	15.7056
6496	6.2917	59.0726	12.5872	88.4329	15.62
6596	15.6349	34.3205	7.6073	38.0941	7.8633
6693	4.6412	-2.7012	29.7462	23.4023	19.0112
6801	12.9653	53.5246	4.9936	54.4325	6.2556
7062	4.285	20.8152	17.1059	-11.8388	27.9986
7077	31.8816	40.9259	3.4763	38.8725	3.4728
7086	5.0846	-15.4553	28.9684	17.537	26.657
7103	5.6457	7.3146	14.2029	10.7408	18.522
7226	10.286	28.2218	6.7646	36.3753	8.7605
7422	3.526	9.9971	27.5449	47.8095	16.8881
7478	6.4933	33.5693	10.7607	14.1719	16.0557
7522	16.7573	35.5923	3.9621	34.3508	5.2819
7563	4.4719	-22.4536	24.9736	38.7314	33.7119
7602	11.3514	36.9217	7.1912	41.622	8.0389
7606	11.513	66.5628	18.1138	42.2894	10.0795
7658	8.8519	21.8938	10.7258	33.7592	14.9081
7768	8.3019	37.6614	7.2373	20.6054	15.3274
7780	3.8323	-43.5091	27.4206	-28.3802	26.8235
7979	3.2445	75.9827	26.3507	82.5626	28.0056