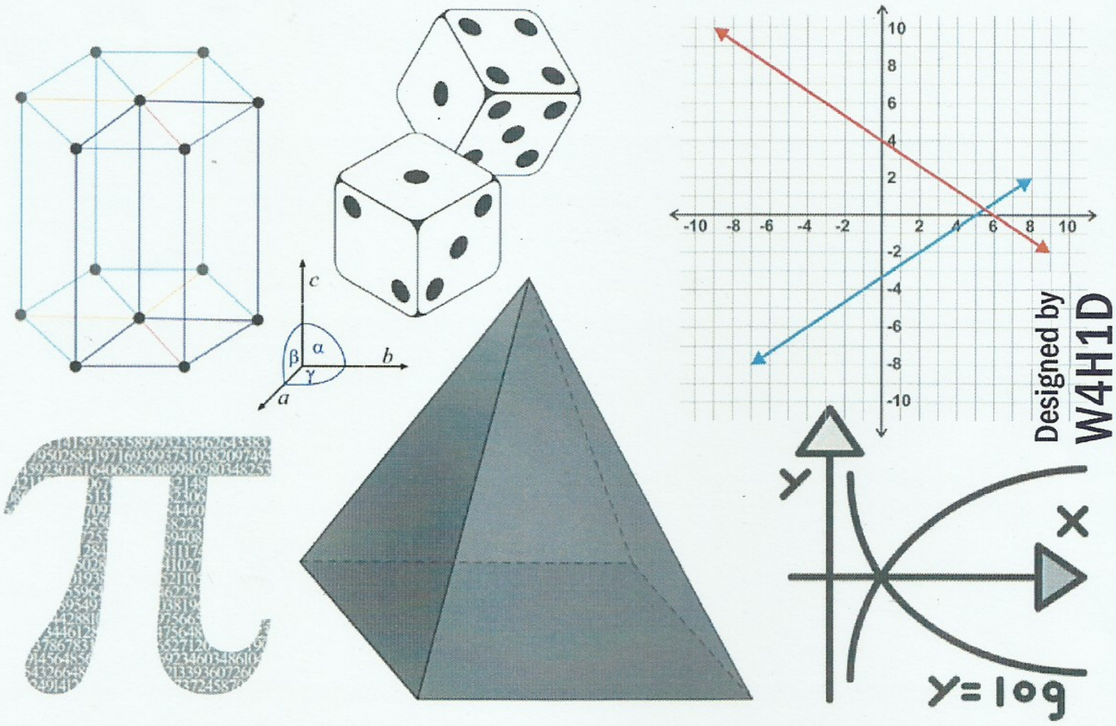
BANGLADESH SCHOOL & COLLEGE, SAHAM

Sultanate of Oman

Higher Mathematics
2nd Paper

Practical Notebook

Class: Eleven - Twelve



Designed by W4H1D

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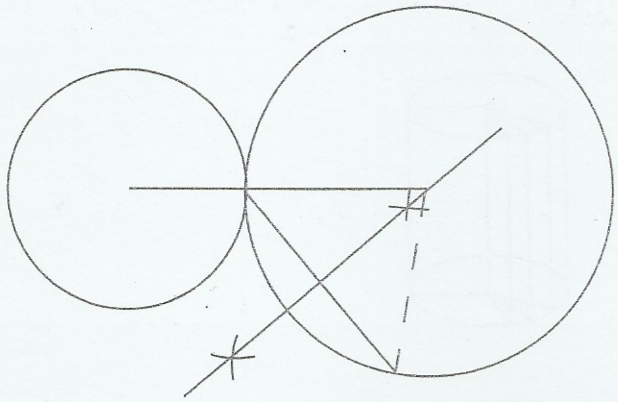
HIGHER MATHEMATICS



PRACTICAL NOTE BOOK

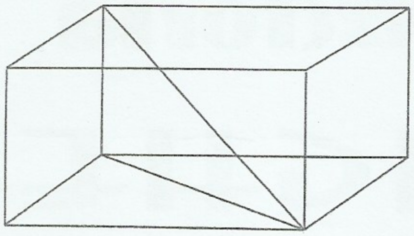
Name Najmul Huda

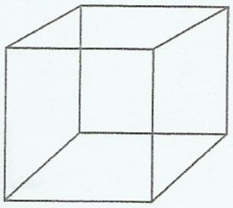
School/ College Bangladesh School & College, Saham

Class XII Sec Science

Subject Higher Mathematics 2nd Paper Roll No.

Reg. No. Year 2020-2021





১। আয়তিক ঘন বা আয়তাকার ঘনবস্তু

(ক) আয়তকার ঘণবস্তুর সমগ্রতলের ক্ষেত্রেফল $= 2\,(ab + bc + ca)$ বর্গ একক

(খ) আয়তন $= abc$ ঘন একক

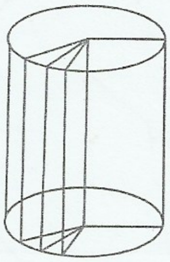
(গ) কর্ণ $= \sqrt{a^2+b^2+c^2}$

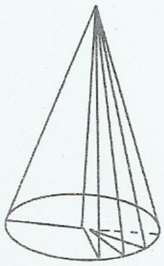
২। ঘনকের ক্ষেত্রে,

(ক) সমগ্রতলের ক্ষেত্রফল $= 6a^2$ বর্গ একক

(খ) আয়তন $= a^3$ ঘন একক

(গ) কর্ণ $= a\sqrt{3}$ একক।





৩। সমবৃত্ত ভূমিক সিলিন্ডার

(ক) বক্রতলের ক্ষেত্রেফল $= 2\pi rh$ বর্গ একক।

(খ) সমগ্রতলের ক্ষেত্রফল $= 2\pi r\,(r + h)$ বর্গ একক।

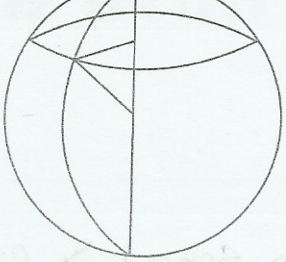
(গ) আয়তন $= \pi r^2 h$ ঘন একক।

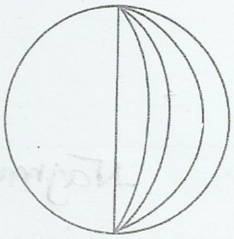
৪। সমবৃত্তভূমিক কোণক

(ক) বক্রতলের ক্ষেত্রেফল $= \pi rl$ বর্গ একক।

(খ) সমগ্রতলের ক্ষেত্রফল $= \pi r\,(r+l)$ বর্গ একক।

(গ) আয়তন $= \frac{1}{3}\pi r^2 h$ ঘন একক।



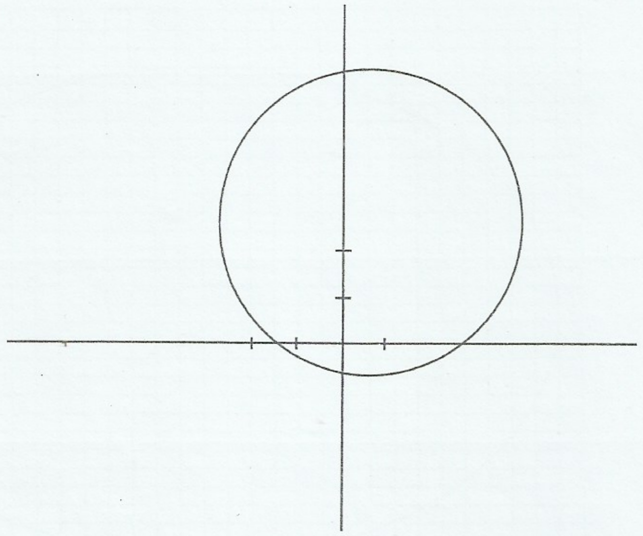


৫। গোলক

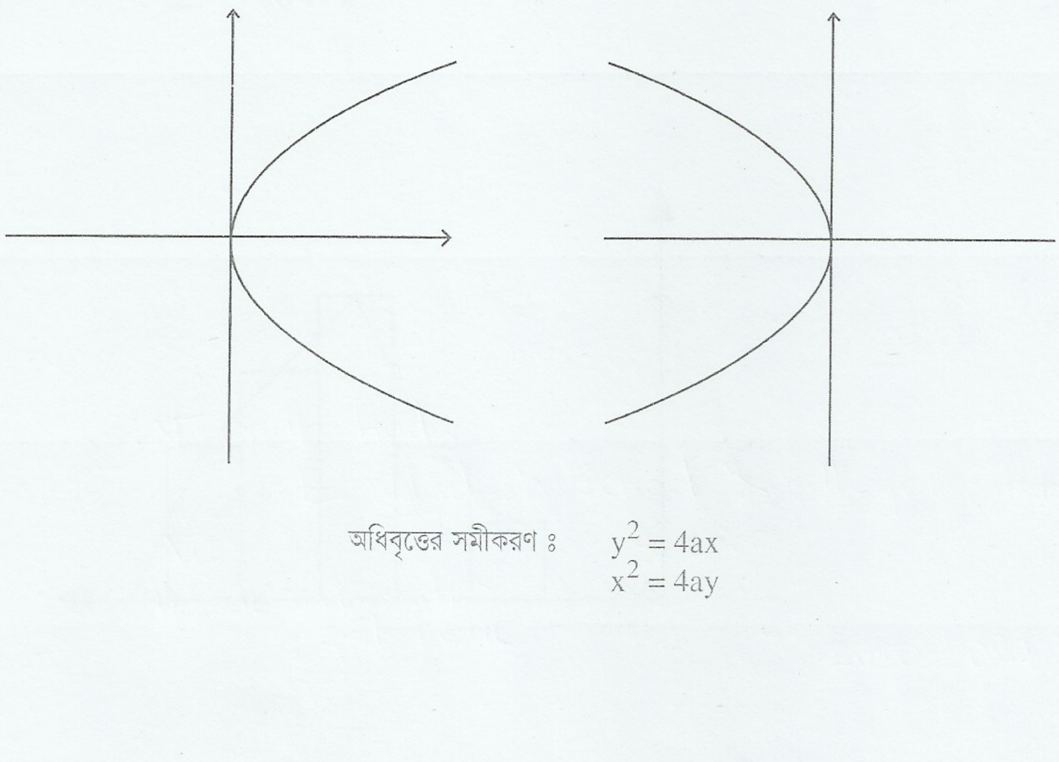
(ক) গোলকের তলের ক্ষেত্রফল $= 4\pi r^2$ বর্গ একক

(খ) আয়তন $= \frac{4}{3}\pi r^3$

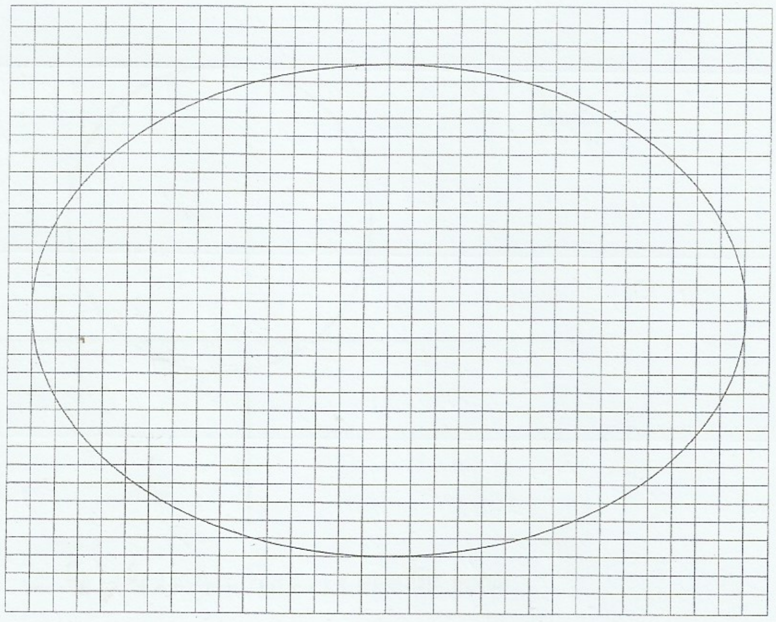
(গ) h উচ্চতায় তলচ্ছেদে উৎপন্ন বৃত্তের ব্যাসার্ধ $= \sqrt{r^2-h^2}$



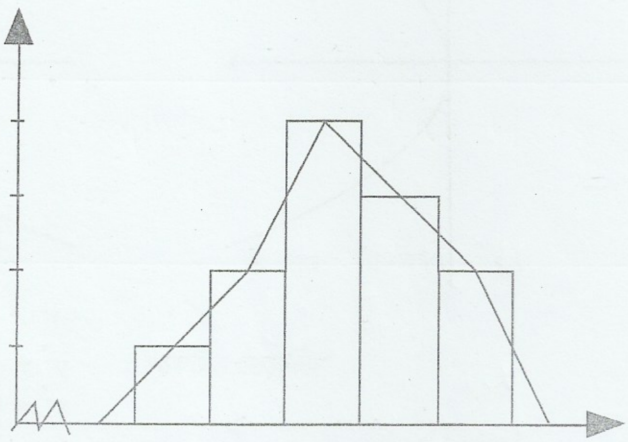
বৃত্তের সমীকরণ ঃ $(x+h)^2 + (y+k)^2 = a^2$



অধিবৃত্তের সমীকরণ ঃ $y^2 = 4ax$
$x^2 = 4ay$



উপবৃত্তের সমীকরণ ঃ $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$



গণিতের ব্যবহৃত কতিপয় গুরুত্বপূর্ণ সূত্রাবলী

জ্যামিতি

জ্যামিতির ইতিহাস ঃ জ্যামিতির আলোচনা প্রথমে প্রাচীন মিশর দেশে শুরু হয় কিন্তু আনুমানিক খৃষ্টপূর্ব ৩০০ অব্দে গ্রীক পণ্ডিত ইউক্লিড জ্যামিতির ইতস্ততঃ বিক্ষিপ্ত সূত্রগুলি সর্ব প্রথম বিধিবদ্ধভাবে সুবিন্যস্ত করে এক বিখ্যাত গ্রন্থ রচনা করেন। এই গ্রন্থের নাম "Endid's elments" ইহাই আধুনিক জ্যামিতির ভিত্তি স্বরূপ।

প্রাচীন কালে জ্যামিতি অর্থে "ভূমির পরিমাপ" নির্ণয় শাস্ত্রকে বোঝাত। জ্যা শব্দের অর্থে ভূমি এবং মিতি শব্দের অর্থ পরিমাপ। অতএব জ্যামিতি শব্দের অর্থ ভূমির পরিমাপ।

বর্তমানে জ্যামিতির সংজ্ঞা দু'ভাবে দেয়া যেতে পারে।

(i) গণিত শাস্ত্রের যে শাখায় ভূমির পরিমাপ সম্বন্ধে আলোচনা করা হয়। সে শাখাকেই জ্যামিতি বলে।

(ii) গণিত শাস্ত্রের যে শাখায় বিন্দু, রেখা, তল এবং ঘনবস্তুর বিভিন্ন গুণাগুণ নিয়ে ধারাবাহিক যুক্তিপূর্ণ আলোচনা করা হয়, তাকেই জ্যামিতি বলা হয়।

ক্ষেত্রফল নির্ণয়ের সূত্র ঃ

(i) ত্রিভুজের ক্ষেত্রফল $= \frac{1}{2} \times$ ভূমি $\times$ উচ্চতা

অথবা $= \sqrt{s\,(s-a)\,(s-b)\,(s-c)}$

এখানে s পরিসীমার অর্থেক a, b ও c যথাক্রমে তিন বাহু।

(ii) চতুর্ভূজের ক্ষেত্রেফল = যে কোন কর্ণের অর্ধেক x বিপরীত শীর্ষদ্বয় হতে উহার দূরত্বের সমষ্টি।

(iii) আয়তক্ষেত্রের ক্ষেত্রফল = দৈর্ঘ্য x প্রস্থ; $(a \times b)$

(iv) সামন্তরিকের ক্ষেত্রফল = ভূমি x উচ্চতা; $(a \times b)$

(v) বর্গক্ষেত্রের ক্ষেত্রেফল = যে কোন এক বাহুর বর্গ; a^2

(vi) রম্বসের ক্ষেত্রফল $= \frac{1}{2}$ কর্ণদ্বয়ের গুণফল

(vii) ট্রাপিজিয়ামের ক্ষেত্রফল $= \frac{1}{2}$সমান্তরাল বাহুদ্বয়ের দৈর্ঘ্যের সমষ্টি x উচ্চতা।

(viii) বৃত্তের ক্ষেত্রফল $= \pi$ (ব্যাসার্ধ), এখানে $\pi \frac{22}{7}$

(ix) গোলীয় পৃষ্ঠের ক্ষেত্রফল $= 4 \times \pi$ (ব্যাসার্ধ)

আয়তন নির্ণয়ের সূত্র =

(i) ঘনবস্তুর আয়তন = দৈর্ঘ্য x প্রস্থ x উচ্চতা

(ii) গোলকের আয়তন $= \frac{3}{4} \times \pi \times$ (ব্যাসার্ধ)

(iii) সিলিন্ডারের আয়তন $= \pi \times$ (ব্যাসার্ধ) x উচ্চতা

পরিসীমা নির্ণয়ের সূত্র ঃ

(i) আয়তক্ষেত্রের পরিসীমা = ২ x (দৈর্ঘ্য + প্রস্থ); 2 (a+b)

(ii) বর্গক্ষেত্রের পরিসীমা = ৪ x বর্গক্ষেত্রের এক বাহু; 4a

(iii) ত্রিভুজের পরিসীমা = তিন বাহুর সমষ্টি; (a+b+c)

(iv) বৃত্তের পরিসীমা $= \frac{7}{22} \times$ ব্যাস; $\frac{7}{22} \times d$

(v) চতুর্ভূজের পরিসীমা = চার বাহুর সমষ্টি; (a+b+c+d)

জ্যামিতিক চিহ্ন বা প্রতীক ঃ

নাম	প্রতীক
রশ্মি	$\rightarrow$
রেখাংশ	
রেখা	$\leftrightarrow$
লম্ব	$\perp$
সমান্তরাল	$\parallel$
সর্বসম	$\cong$
কোণ	$\angle$
বৃহত্তর	$>$
ক্ষুদ্রতর	$<$
বৃহত্তর বা সমান	$\geq$
ক্ষুদ্রতর বা সমান	$\leq$
ত্রিভুজ	Δ
চতুর্ভূজ	$\square$
বৃত্ত	$\odot$
পরিধি	O
সমান	$=$
সমান নয়	$\neq$
বৃহত্তর নয়	$\not>$
ক্ষুদ্রতর নয়	$\not<$
ক্ষুদ্রতর	$<$
সুতরায়/ অতএব	$\therefore$
যেহেতু	$\because$
রেখাংশ	AB

প্রতিজ্ঞা ঃ বিন্দু, রেখা, ক্ষেত্র ইত্যাদি বিষয়ক কোন প্রমাণ করা বা কোন কিছু অঙ্কন করার প্রস্তাবের নাম প্রতিজ্ঞা। প্রতিজ্ঞা দুই প্রকার ঃ (১) উপপাদ্য ও (২) সম্পাদ্য।

উপপাদ্য ঃ যে প্রতিজ্ঞায় জ্যামিতিক কোন সত্য বা ক্ষেত্র বিশেষের কোন বৈশিষ্ট্য যুক্তি দ্বারা প্রমাণ করার প্রস্তাব করা হয়, তাকে উপপাদ্য বলে।

সম্পাদ্য ঃ যে প্রতিজ্ঞায় কোন জ্যামিকি বিন্দু রেখা বা ক্ষেত্রাদি অঙ্কন করার প্রস্তাব করা হয়, তাকে সম্পাদ্য বলে। প্রতিজ্ঞার চারটি অংশ থাকে ঃ

(i) সাধারণ নির্বাচন (ii) বিশেষ নির্বাচন (iii)অঙ্কন (iv) প্রমাণ।

বর্গ নির্ণয়ের সূত্র ঃ

$(a + b)^2 = a^2 + 2ab + b^2$

$(a - b)^2 = a^2 - 2ab + b^2$

$(a + b + c)^2 = a^2 + b^2 + c^2 + 2 (ab + bc + ca)$

ঘন নির্ণয়ের সূত্র ঃ

$(a + b)^3 = a^3 + 3a^2b + 3ab^2$

$= a^3 + b^3 + 3ab (a + b)$

$(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

$= a^3 - b^3 - 3ab (a - b)$

$(a + b + c)^3 = a^3 + b^3 + c^3 + 3 (a + b) (b + c) c + a)$

মান নির্ণয়ের সূত্র ঃ

$(a + b)^2 = (a - b)^2 + 4ab$

বা $(a + b) = \sqrt{a - b^2 + 4ab}$

$(a - b)^2 = (a + b)^2 - 4ab$

বা $(a - b) = \sqrt{a + b^2 - 4ab}$

$a^2 + b^2 = (a + b)^2 - 2ab$

$= (a - b)^2 + 2ab$

$= \frac{1}{2} \{(a + b)\}^2 + a - b)^2$

$ab = \left(\frac{a + b}{2}\right)^2 - \left(\frac{a - b}{2}\right)^2$

$4ab = (a + b)^2 - (a - b)^2$

$a^2 + b^2 + c^2 = (a + b + c)^2 - 2 (ab + bc + ca)$

$ab + bc + ca = (a + b + c)^2 - 2 (ab + bc + ca)$

$ab + bc + ca = \frac{(a + b + c)^2 - 2 (a^2 + b^2 + c^2)}{2}$

$a^3 + b^3 + c^3 = (a + b + c)^3 - 3 (a + b) (b + c) (c + a)$

$(a + b) (b + c) \frac{(c + a) = (a + b + c)^3 - a^3 + b^3 + c^3}{3}$

$a^3 + b^3 = (a + b)^3 - 3ab (a + b)$

$a^3 - b^3 = (a - b)^3 + 3ab (a - b)$

$2 (a^2 + b^2) = (a + b)^2 + (a - b)^2$

গুণনের সূত্র ঃ

$(a + b) (a - b) = a^2 - b^2$

$(a + b) (a^2 - ab + b^2) = a^2 + b^2$

$(a - b) (a^2 + ab + b^2) a^3 - b^3) x + ab$

$(x - a) (x + b) = x^2 + (a + b) x + ab$

$(x - a) (x - b) = x^2 - (a + b) x + ab$

$(x + a) (x - b) = x^2 + (a + b) x - ab$

$(x - a) (x - b) = x^2 - (a - b) x - ab$

$(x + a) (x + b) (x + c) = x^3 + (a + b + c)$

$(x - a) (x - b) (x - c) = x^3 - (a + b + c)$

$x^2 + (ab + bc + ca) x - abc$

$(a + b) (b + c) (c + a) = a^2b + a^2b + ab^2 + b^2c + bc^2 + c^2a + ca^2 + 2abc$

উৎপাদকের সূত্র ঃ

$a^2 - b^2 = (a + b) (a - b)$

$a^3 + b^3 = (a + b) (a^2 - ab + b^2)$

$a^3 - b^3 = (a - b) (a^2 + ab + b^2)$

$a^3 + b^3 + c^3 - 3abc = (a + b + c) (a^2 + b^2 + c^2 - ab - bc - ca)$

$a^3 + b^3 + c^3 - 3abc$

$= \frac{1}{2} (a + b + c) (a + b + c) \{(a - b)^2 + (b - c)^2 + (c - a)^2\}$

$= \frac{1}{2} (a - b)^3 + (b - c)^3 + (c - a)^3$

$= 3 (a - b) (b - c) (c - a)$

সূচক ও শক্তি সম্পর্কীয় সূত্র ঃ

a^n, এখানে হল ভিত্তি এবং হলো শক্তি সূচক।

যদি হয় তবে নিম্নের প্রতি ক্ষেত্রেই

(i) $a^m \times a^n = a^{m+n}$

(ii) $a^m \div a^n = a^{m-n}$; $m > n$

(iii) $a^m \div a^n = a^{m-n} = ao \neq 1$; যখন 1; যখন $m=n$

(iv) $a^m \div a^n = m < n$

(v) $(am)^n = a^{mn}$ (vi) $(a.b)^m - a^m . b^m$

সমাধান সম্পর্কীয় সূত্র ঃ

(1) $ax^2 + b^2 + c = 0$ দ্বিঘাত ধরনের সমীকরণের সমাধান ঃ

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

(2) বজ্রগুণন পদ্ধতিতে সমাধান নির্ণয় ঃ

$a_1x + b_1y + c_1 = 0$

$a_2x + b_2y + c_2 = 0$ হলে,

$x = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}$ $\quad$ $y = \frac{a_2c_1 - a_1c_2}{a_1b_2 - a_2b_1}$

মূলের প্রকৃতি (দ্বিঘাত সমীকরণ) ঃ

(1) $ax^2 + bx + c = 0$

সুতরাং $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

(i) যদি $b^2 > 4ac$ হয়, তবে সমীকরণে দুটি মূলই বাস্তব ও অসমান হবে।

(ii) যদি $b^2 = 4ac$ হয়, তবে সমীকরণে দুটি মূলই বাস্তব ও সমান হবে।

(iii) যদি $b^2 < 4ac$ হয়, তবে সমীকরণটির মূলগুলি যুগল জটিল হবে।

(2) **সমান্তর ধারা n পদের সমষ্টি** $S=\frac{n}{2}$ (2a+(n-1)d

(3) $1+2+3$ .... $+ n$ হলে, $S=\frac{n\,(n+1)}{2}$

(4) $1^2+2^2+3^3+ .... + n$ হলে, $S=\frac{n\,(n+1)\,(2n+1)}{6}$

(5) $1^3+2^3+3^3$ .... n^3 হলে, $S=\left\{\frac{n\,(n+1)}{2}\right\}^2$

(6) a, ar, ar^3, ar^3 .... n তম পদ হলে (গুণত্তোর প্রগমন)
n তম পদ ar^{n-1}

(7) $a + ar + ar2 + ar3 +$ .... $+ ar^{n-1}$ হলে, S

(8) $a+ ar + ar2 + ar3 +$ ....অসীম পর্যন্ত হলে।

$S = a\,\frac{r^n-1}{r-1} \quad 1>1$

(9) a, b এর সমান্তর মধ্যক $\frac{a+b}{2}$

(10) a, b এর গুণোত্তর মধ্যক $+\sqrt{ab}$

(11) দুইটি অসমান ধনাত্মক সংখ্যার সমান্তর মধ্যক এবং গুণোত্তর মধ্যক হতে সংখ্যা নির্ণয়,

$a=M+\sqrt{m^2-G^2}$, $b=M-\sqrt{m^2-G^2}$

লগারিদমের সূত্র ঃ

(i) logr M= logaM

(ii) $\text{logbM} = \frac{\text{logaM}}{\text{logab}}$

(iii) loga M= logaM X logab

(iv) logab X logba = 1

(v) loga (M X N) = logaM + logaN

(vi) $\text{loga}\left(\frac{m}{n}\right) = \text{logaM} - \text{logaN}$

(vii) log 1 = 0

(viii) loga 1 = 0

(ix) logaa = 1

(x) logaxn = nlogxl

(xi) $\log\sqrt{x = \frac{1}{n}}\,\text{logx}$

(xii) $\text{logab} = \frac{1}{\text{logba}}$

(xiii) log 10 = 1

(xiv) log 100 = 2

(xv) log 1000 = 3

কতগুলো ধ্রুব মান ঃ

$\log\pi = .4972.\ \log\pi^2 = .9943$

$\log 10 = 2.3026.\ \sqrt{2} = 1.41, \sqrt{3} = 1.73$

$\pi = \frac{22}{7}\,3.142,\ \pi^2 = 9.87,\ \sqrt{\pi} = 1.773$

$\sqrt{5} = 2{,}236, \sqrt{6} = 2.449,\ \log^2 = .30103,$

$\log 3 = .47712,\ \log 5 = .69897,\ \log\sqrt{\pi} = .2485749$

ত্রিকোণমিতি

(i) $\sin\theta = \frac{\text{লম্ব}}{\text{অতিভূজ}}$ $\text{cosec}\,\theta = \frac{\text{অতিভূজ}}{\text{লম্ব}}$

(iii) $\cos\theta = \frac{\text{অতিভূজ}}{\text{ভূমি}}$ $\sec\theta = \frac{\text{ভূমি}}{\text{অতিভূজ}}$

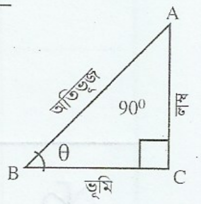
(iii) $\tan\theta = \frac{\text{লম্ব}}{\text{ভূমি}}$ $\cot\theta = \frac{\text{ভূমি}}{\text{লম্ব}}$

(iv) $\sin\theta = \frac{1}{\text{cosec}\,\theta}$ $\therefore \text{cosec} = \theta\ \frac{1}{\sin\theta}$

(v) $\cos\theta = \frac{1}{\sin\theta}$ $\therefore \sec = \theta\ \frac{1}{\cos\theta}$

(vi) $\tan\theta = \frac{1}{\cot\theta}$ $\therefore \cot\theta = \frac{1}{\tan\theta}$

(vii) $\tan\theta = \frac{\sin\theta}{\cos\theta}$ $\therefore \cot = \frac{\cos\theta}{\sin\theta}$



(viii) $\sin\theta$, $\text{cosec}\,\theta = 1$

$\cos\theta$, $\sec\theta = 1$

$\tan\theta$, $\cot\theta = 1$

(ix) $\sin^2\theta + \cos^2\theta = 1$

(x) $\text{cosec}^2\theta = 1\ \cot^2\theta\ \cot^2\theta = \text{cosec}^2\theta\ {-}{-}1$

(xi) $\sec^2\theta = 1 + \tan^2\theta$, $\tan^2 = \sec^2\theta\ {-}{-}1$

(xii) $\cos^2\theta = 1 - \sin\theta$, $\sin\theta = 1\cos^2\theta$

(xiii) $\sec^2\theta - \tan^2\theta = 1$

(xiv) $\text{cosec}^2\theta - \cot^2\theta = 1$

❑ sin 2 A = 2 sin A cos, A

$\cos^2 A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 2 - 2\sin^2 A$

$2\sin^2 A = 1 - \cos^2 A$

$2\cos^2 A = 1 + \cos^2 A$

$\tan^2 A = \frac{1-\cos 2A}{1+\cos 2A}$; $\tan 2A = \frac{2\tan A}{1-\tan 2A}$

$\sin^2 A = \frac{2\tan A}{1+\tan 2A}$; $\cos 2A = \frac{1-\tan 2A}{1+\tan 2A}$

$\sin 3A = 3\sin A - 4\sin^3 A$

$\cos 3A = 4\cos 3A - 3\cos A$

$\tan 3A = \frac{3\tan A - \tan^3 A}{1 - 3\tan 2A}$

Index

Sl No.	Date	Name of the Experiment	Page No.	Initial	Remarks
01	29-03-2022	With the help of graph minimize the objective function $z=2x+3y$.	01, 02, 03, 04		
02	05-04-2022	With the help of graph maximize the objective function $z=20x+10y$ in linear programming.	05, 06, 07, 08		
03	06/04/2022	The focus and equation of directrix of a parabola are $(10,0)$ and $x-6=0$. Draw the graph of the parabola.	09, 10		
04	06/04/2022	The focus, equation of directrix and eccentricity of an ellipse are $S(3,4)$, $y+5=0$ and $e=\frac{1}{2}$ respectively. Draw the graph of the ellipse.	11, 12		
05	06/04/2022	Draw the graph of $y=\cos^{-1}x$.	13, 14		
06	06/04/2022	Draw the graph of $y=\sin x$.	15, 16		
07	07/04/2022	Draw the graph of $y=\sin^{-1}x$.	17, 18		
08	07/04/2022	Draw the graph of $y=\tan^{-1}x$.	19, 20		
09	07/04/2022	Two forces 70N and 50N are acting at a point with an angle 55°. Find the magnitude and direction of the resultant.	21, 22, 23		

NAME OF THE EXPERIMENT With the help of graph minimize the objective function $z = 2x+3y$. Subject to constraints: $2x+7y \geq 22$, $x+y \geq 6$, $5x + y \geq 10$, $x \geq 0$, $y \geq 0$.

DATE 29/03/2022

PAGE NO. 01

EXPT. NO. 01

Theory:

Drawing the graph of the given inequality $2x+7y \geq 22$, $x+y \geq 6$, $5x+y \geq 10$, $x \geq 0$, $y \geq 0$, the corner points of feasible region are determined. Substituting the coordinates of the corner points into the objective function $z = 2x + 3y$, the maximum value of z is selected.

Necessary Equipment:

A pointed pencil, scale, graph paper, eraser, sharpner, pencil-compass, scientific calculator, protractor.

Procedure:

1) Corresponding linear equations of the given inequalities are :-

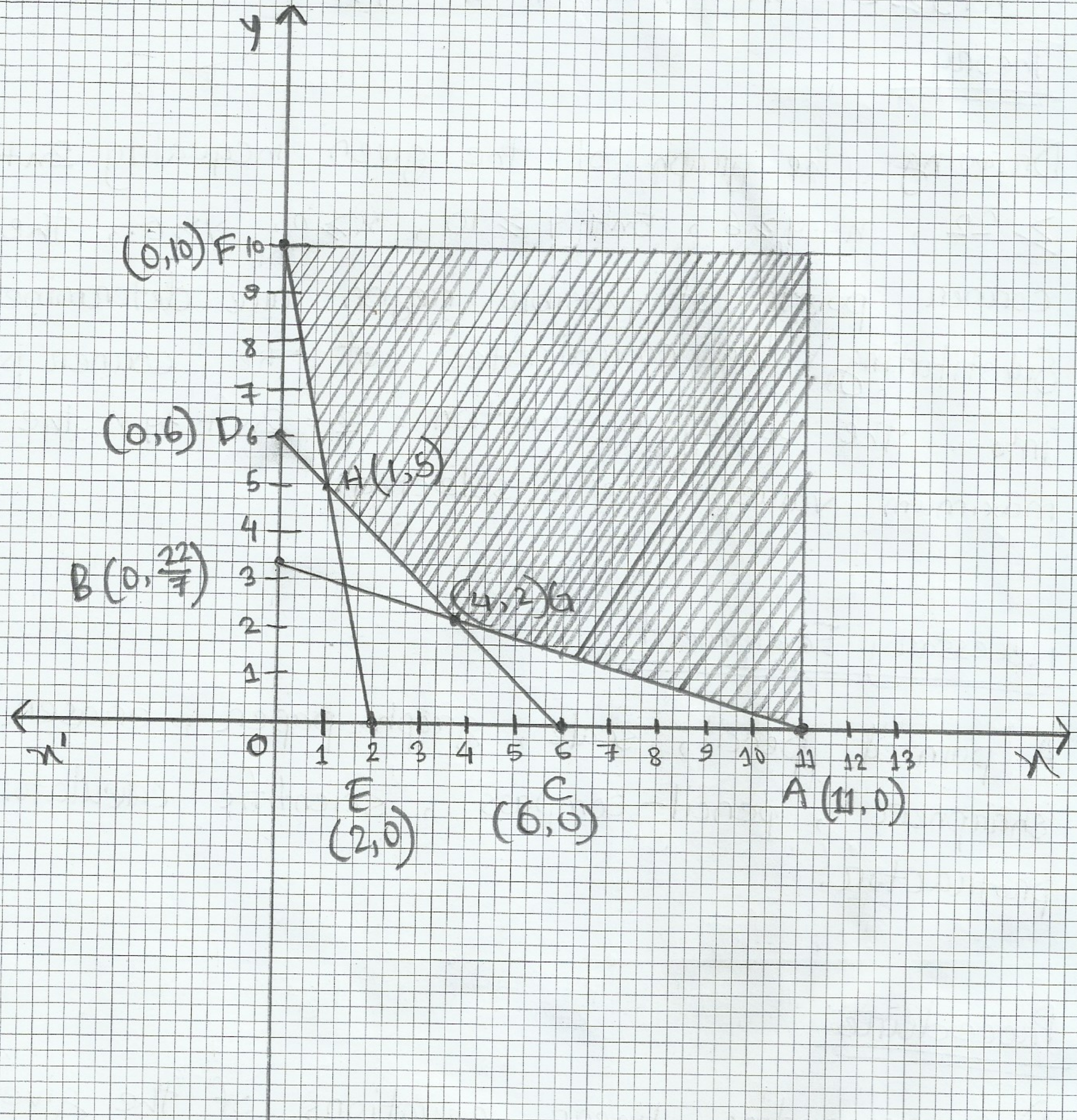
$$2x + 7y = 22$$

$$\text{or, } \frac{x}{11} + \frac{7}{22}y = 1 \quad \ldots\ldots (i)$$

$$x + y = 6$$

$$\text{or, } \frac{x}{6} + \frac{y}{6} = 1 \quad \ldots\ldots (ii)$$

FIGURE NO. 01



NAME OF THE EXPERIMENT With the help of graph minimise the objective function $z = 2x+3y$. Subject to contraints : $2x+7y \geq 22$, $x+y \geq 6$, $5x+y \geq 10$, $x \geq 0$, $y \geq 0$.

DATE 03/04/2022

EXPT. NO. 01

$$5x + y = 10$$

$$\text{or, } \frac{x}{2} + \frac{y}{10} = 1 \quad \ldots\ldots \text{(iii)}$$

2) XOX' and YOY' are drawn as the x-axis and y-axis respectively on a graph paper and O indicates the origin (0,0).

3) By choosing a scale such that length of 3 small squares = 1 unit along x-axis and 3 small squares = 1 unit unit along y-axis also, the points (11, 0) and $(0, \frac{22}{7})$ on (i), (6,0) and (0, 6) on (ii) and (2,0) and (0,10) on (iii) are plotted on the graph paper. By joining each pair of points, the graphs of straight line (i) is AB, that of (ii) is CD and that of (iii) is EF are drawn.

4) Since the points (0,0) satisfies the inequality $2x+7y \geq 22$. ($\because 0 \leq 22$ is true), the set of all points on the straight line (i) and the points towards the origin in the

solution set. Similarly, the sets of all points on (ii) and (iii) and the points toward the origin are the solution set. Moreover, any points satisfying the condition $x \geq 0$ and $y \geq 0$ lies in first quadrant.

5) All points of the polygon bounded by the quadrilateral OAGHF satisfy all of the inequality. So, this area is the feasible region.

6) Coordinates of the corner points OAGHF are: O(0,0), A(11,0), G(4,2), H(1,5) and F(0,10).

7) Substituting the coordinates of corner points into the object function, we can find the maximum value of Z.

NAME OF THE EXPERIMENT With the help of graph minimise the objective function $z = 2x+3y$. Subject to constraints: $2x+7y \geq 22$, $x+y \geq 6$, $5x+y \geq 10$, $x \geq 0$, $y \geq 0$.

DATE 04/04/2022

PAGE NO. 04

EXPT. NO. 01

Compilation of result:

Corner points	$Z = 2x + 3y$	Z min
O(0,0)	$Z = 2\times0 + 3\times0 = 0$	
A(11,0)	$Z = 2\times11 + 3\times0 = 22$	
G(4,2)	$Z = 2\times4 + 3\times2 = 14$	14
H(1,5)	$Z = 2\times1 + 3\times5 = 17$	
F(0,10)	$Z = 2\times0 + 3\times10 = 30$	

Result:- Required minimum value of $z = 14$.

NAME OF THE EXPERIMENT With the help of graph maximise the objective function $z = 20x + 10y$ in linear programming. Subject to constraints: $9x + 17y \leq 153$, $x+y \leq 10$, $13x+8y \leq 104$, $x \geq 0$, $y \geq 0$.

DATE 05/04/2022

PAGE NO. 05

EXPT. NO. 02

Theory:

Drawing the graph of the given inequality $9x+17y\leq153$, $x+y \leq 10$, $13x+8y \leq 104$, $x \geq 0$, $y \geq 0$, the corner points if feasible region are determined. Substituting the coordinates on the corner points into the objective function, $z = 20x + 10y$, the maximum value of z is selected.

Necessary equipments:

A pointed pencil, scale, graph paper, eraser, sharpner, pencil compass, scientific calculator, protractor.

Procedure:

1) Corresponding linear equation of the given inequalities are:

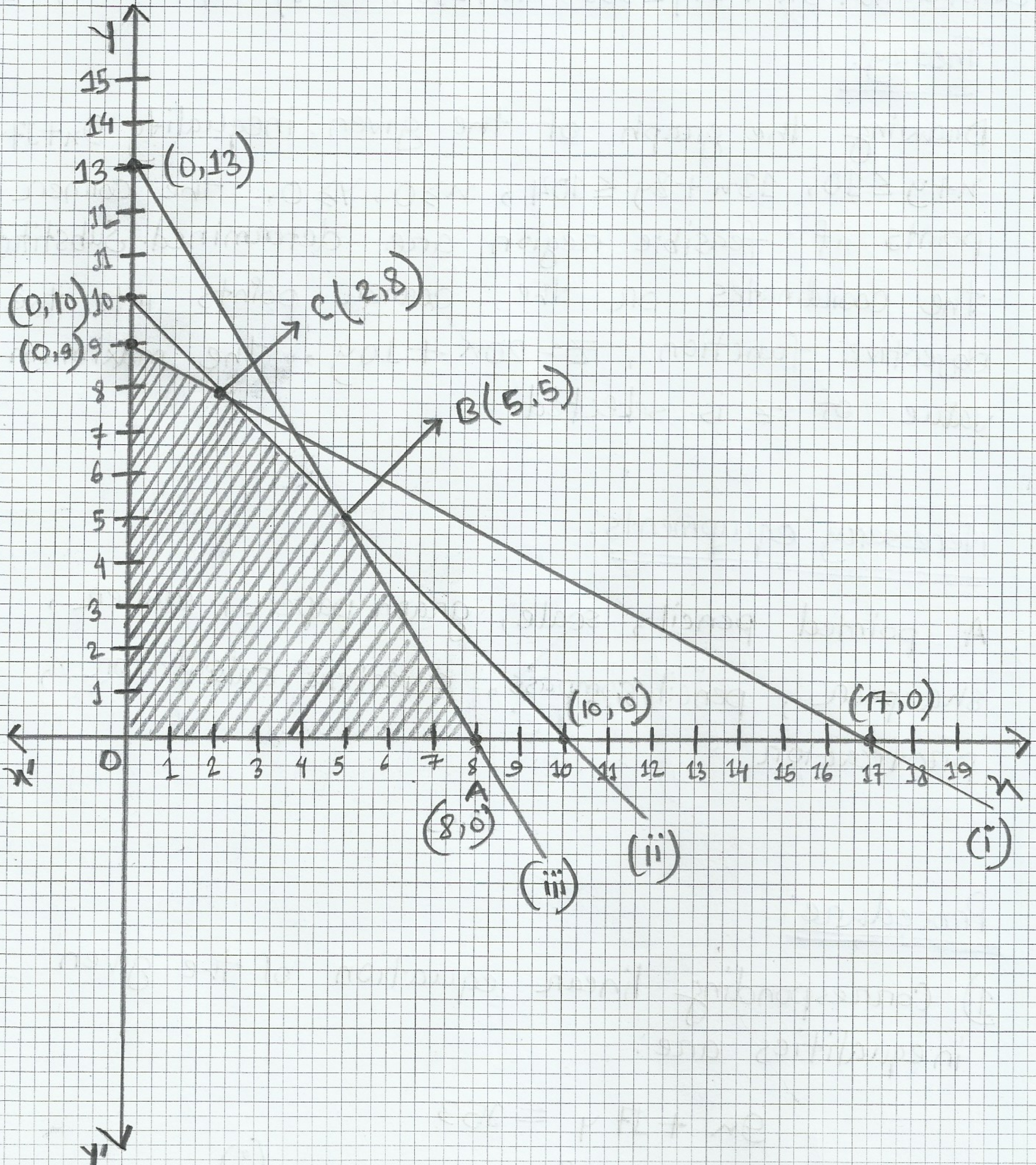
$$9x + 17y = 153$$

$$\text{or, } \frac{x}{17} + \frac{y}{9} = 1 \quad \cdots\cdots \text{(i)}$$

$$x + y = 10$$

$$\text{or, } \frac{x}{10} + \frac{y}{10} = 1 \quad \cdots\cdots \text{(ii)}$$

FIGURE NO. 02



and $13x + 8y = 104$

$$\text{or, } \frac{x}{8} + \frac{y}{13} = 1 \quad \cdots\cdots \text{(iii)}$$

2) XOX' and YOY' are drawn as the x-axis and y-axis respectively on a graph paper and O indicates the origin (0,0).

3) By choosing a scale such that length of 3 small square = 1 unit, the points (17,0) and (0,9) on (i), (10,0) and (0,10) on (ii) and (8,0) and (0,13) on (iii) are plotted on the graph paper.
By joining each pair of points, the graphs of straight lines (i), (ii) and (iii) are drawn.

4) Since the point (0,0) satisfies the inequality $9x + 17y \leq 153$ (as $0 \leq 153$ is true), the set of all points on the straight line (i) and the points towards the origin is the solution set. Similarly, the sets of all points on (ii) and (iii)

and the points towards the origin are the solution sets. Moreover, any point satisfying the first conditions $x \geq 0$ and $y \geq 0$ lies in the first quadrant.

5) All points of the polygon bounded by the quadrilateral OABCD satisfy all of the inequalities. So, this area is the feasible region.

6) Coordinates of corner points of OABCD : O(0,0), A(8,0), (ii) and (iii) intersect at B(5,5); (i) and (ii) intersect at C(2,8) and D(0,9).

7) Substituting the coordinates of the corner points into the objective function, we find the maximum value of Z.

NAME OF THE EXPERIMENT With the help of graph maximise the objective function $z = 20x + 10y$ in linear programming subject to constraints: $9x + 17y \le 153$, $x + y \le 10$; $13x + 8y \le 104$, $x \ge 0$, $y \ge 0$.

DATE 06/04/2022

PAGE NO. 08

EXPT. NO. 02

Compilation of result:

Corner Points	$Z = 20 + 10y$	Z max
O(0,0)	$Z = 20\times0 + 10\times0 = 0$	
A (8,0)	$Z = 20\times8 + 10\times0 = 160$	
B (5,5)	$Z = 20\times5 + 10\times5 = 150$	160
C (2,8)	$Z = 20\times2 + 10\times8 = 120$	
D (0,9)	$Z = 20\times0 + 10\times9 = 90$	

Result:

Required maximum value of $z = 160$.

NAME OF THE EXPERIMENT The focus and equation of directrix of a parabola are (10,0) and $x-6=0$. Draw the graph of the parabola.

DATE 06/04/2022

PAGE NO. 09

EXPT. NO. 03

Theory:

If P be any point on the parabola and PM be the perpendicular distance from P to the directrix and if S be the focus, then eccentricity,

$$e = \frac{SP}{PM} = 1$$

or, $SP = PM$.

Necessary Equipment:

A pointed pencil, scale, graph paper, eraser, sharpener, pencil-compass, scientific calculator, protractor.

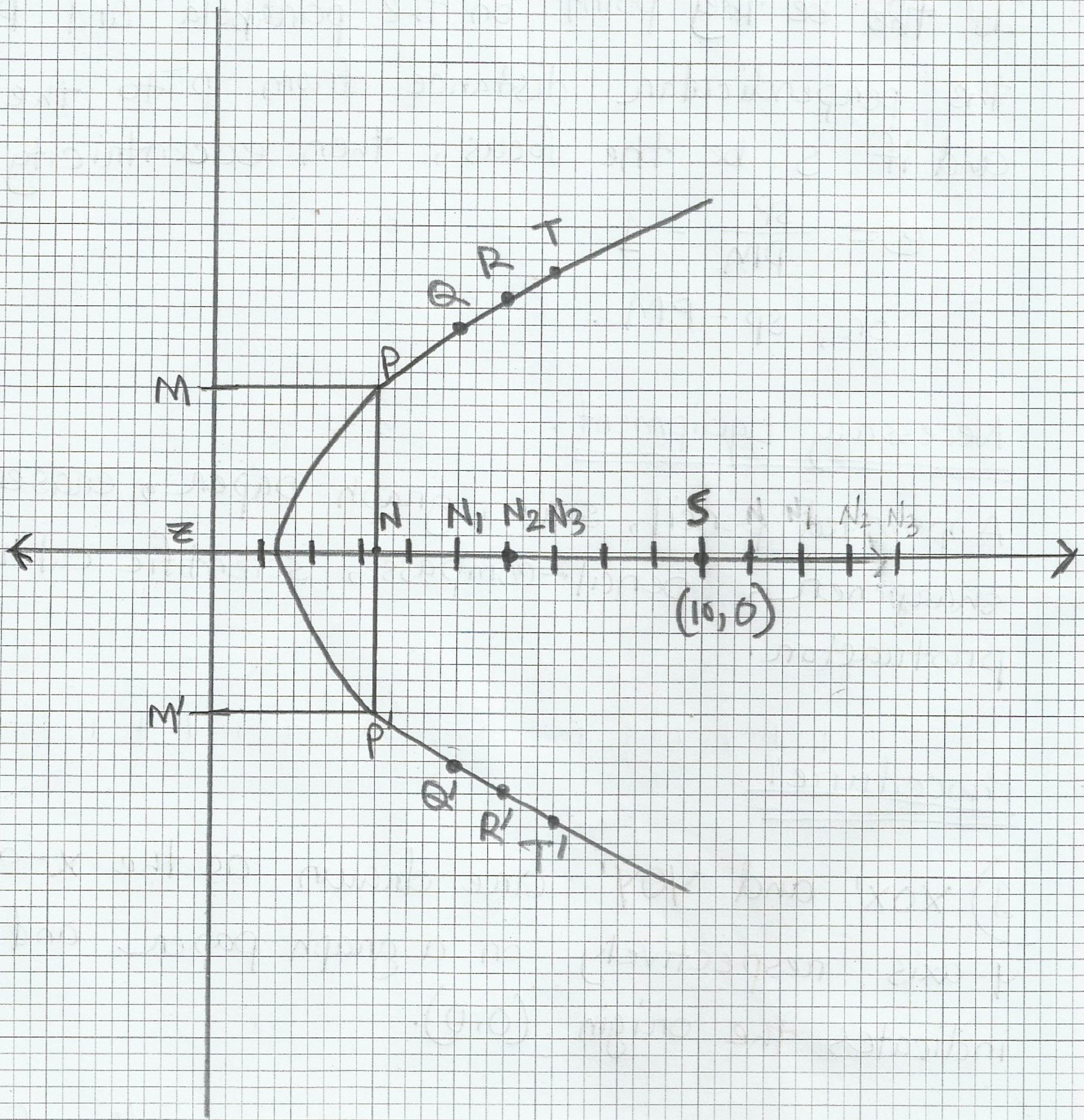
Procedure:

1) XOX' and YOY' are drawn as the x-axis and y-axis respectively on a graph paper and O indicates the origin (0,0).

2) By choosing a scale such that length of 3 small square = 1 unit along x-axis and y-axis, the focus S(10,0) is plotted.

3) Now, SZ perpendicular to the directrix is drawn and SZ is bisected by A. Then SA = AZ.

FIGURE NO. 03



Therefore, A is a point on the parabola.

4) Now, take any point N on ZA produced and draw $PNP' \perp ZN$. Now with center S and radius ZN an arc of a circle is drawn which meets PNP' at P and P'. SP' and SP is joined. PM and PM' perpendicular to the directrix is drawn. Then, $SP' = ZN = P'M'$ and $SP = ZN = PM$. Therefore, P, P' are two points on the parabola.

5) In the similar way, taking point N_1, N_2, N_3 ------ on ZA produced we get the points Q, Q', R, R', T, T' ------

6) Now one free-hand joining the points A, P, Q, R, T, P', Q', R', T', ... we get the required parabola.

NAME OF THE EXPERIMENT The focus, equation of directrix and eccentricity of an ellipse are $S(3,4)$, $y+5=0$ and $e=\frac{1}{2}$ respectively. Draw the graph of the ellipse.

DATE 06/04/2022

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EXPT. NO. 04

Theory:

Let, S be the focus and PM be the perpendicular drawn from P to the directrix, then $\frac{SP}{PM}=e$, where $e<1$.

Necessary Equipment:

A pointed pencil, scale, graph paper, eraser, sharpener, pencil-compass, scientific calculator, protractor.

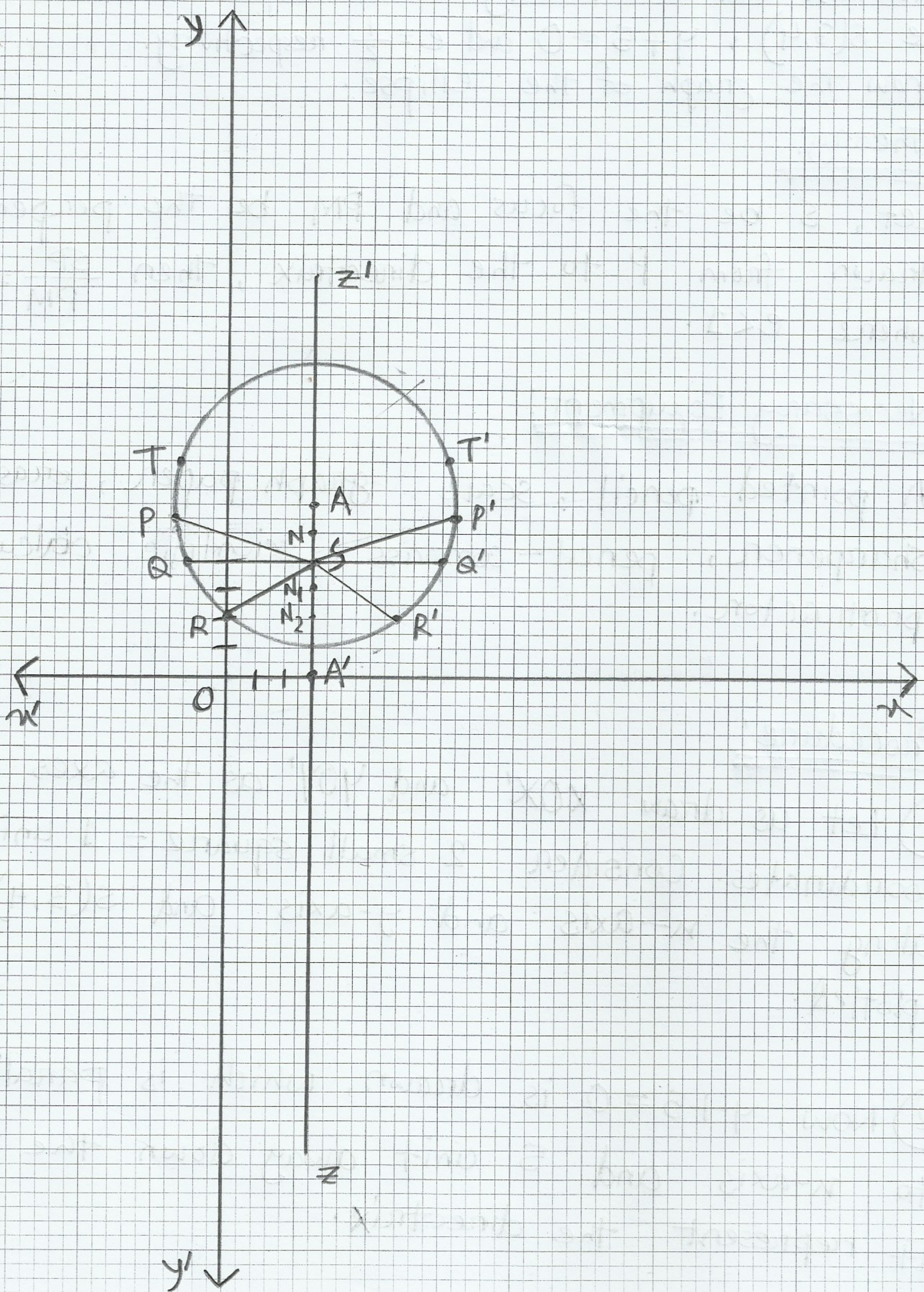
Procedure:

1) Let us draw XOX' and YOY' as the axes of coordinates. Consider 2 small square = 1 unit along the x-axis and y-axis and $S(3,4)$ is plotted.

2) Now, $y+5=0$ is drawn, which is parallel to x-axis and 5 unit away down the x-axis. It represent the directrix.

3) Now, SZ perpendicular to the directrix is drawn and SZ is divided by A and A' in the ratio 1:2 internally and externally. Then,

FIGURE NO. 04



NAME OF THE EXPERIMENT The focus, equation of directrix and eccentricity of an ellipse are $S(3,4)$, $y+5=0$ and $e=\frac{1}{2}$ respectively. Draw the graph of the ellipse.

DATE 06/04/2022

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EXPT. NO. 04

$\frac{SA}{AZ}=\frac{1}{2}$ and $SA'/A'Z'=\frac{1}{2}$. Therefore, A and A' are two points on the ellipse.

4) Now any point N is taken on AA' and PNP' $\perp$ AA' is drawn. Now with center S and radius 1/2 ZN an arc of a circle is drawn which meet PNP' at P and P'. SP and SP' are joined and perpendiculars PM and PM' are drawn. Then, SP/PM = 1/2, and SP/PM' = 1/2. Therefore, PP' are two points on the ellipse.

5) In the similar way, taking points N_1, N_2, N_3 .... on AA' we get the points Q, Q', R, R', T, T' on the ellipse.

6) Now, joining the points, the required ellipse can be found.

NAME OF THE EXPERIMENT Draw the graph of $y = \cos^{-1} x$.

DATE 06/04/2022

PAGE NO. 13

EXPT. NO. 05

Theory:

$$y = \cos^{-1} x \quad (-1 \leq x \leq 1).$$

Necessary Equipment:

A pointed pencil, scale, graph paper, eraser, sharpener, pencil-compass, scientific calculator, protractor.

Procedure:

1) Some values of y for some respective values of x within the interval [-1, 1] are calculated and the ordered pair are formed.

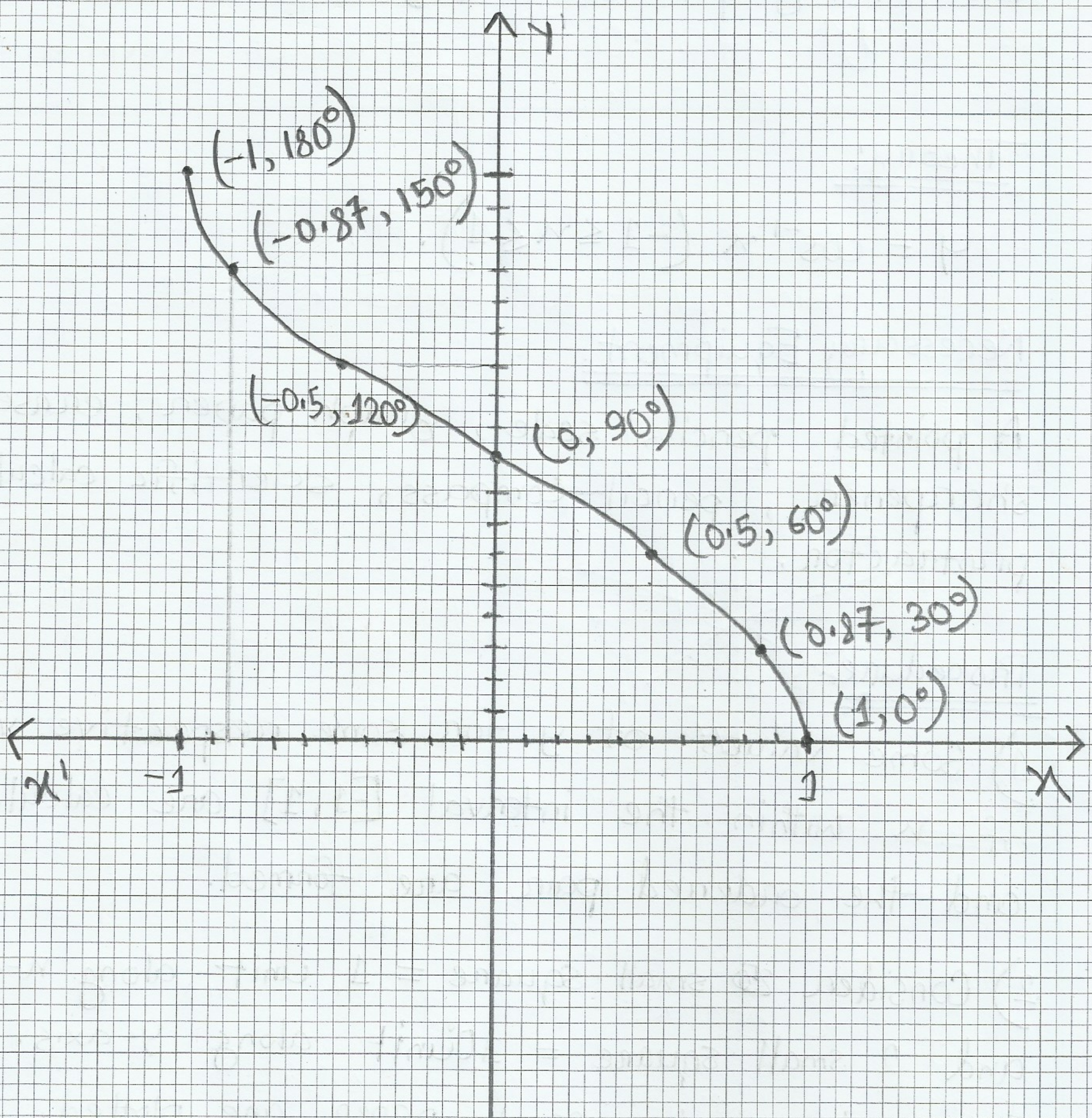
2) Consider 20 small square = 1 unit along x-axis and 2 small square = 10° along y-axis and the points on the graph paper are plotted.

3) The points are joined which is the required graph.

Calculation:

x	-1	-0.87	-0.5	0	0.5	0.87	1
y	$180°$	$150°$	$120°$	$90°$	$60°$	$30°$	0

FIGURE NO. 04



NAME OF THE EXPERIMENT Draw the graph of $y = \cos^{-1} x$.

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EXPT. NO. 05

Characteristics of the graph of $y = \cos^{-1} x$:

i) The graph cuts the x-axis at $(1, 0)$.

ii) Periodically it cuts the y-axis.

NAME OF THE EXPERIMENT Draw the graph of $y = \sin x$.

DATE 06/04/2022

PAGE NO. 15

EXPT. NO. 06

Theory:

$Y = \sin x.$

$$\sin x = \sin(2\pi + x) = \sin(4\pi + x) = \sin(6\pi + x) \cdots\cdots$$

$$= \sin(-2\pi + x) = \sin(-4\pi + x) \cdots\cdots$$

i.e. $\sin x$ is a periodic function of period 2π.

Necessary Equipment:

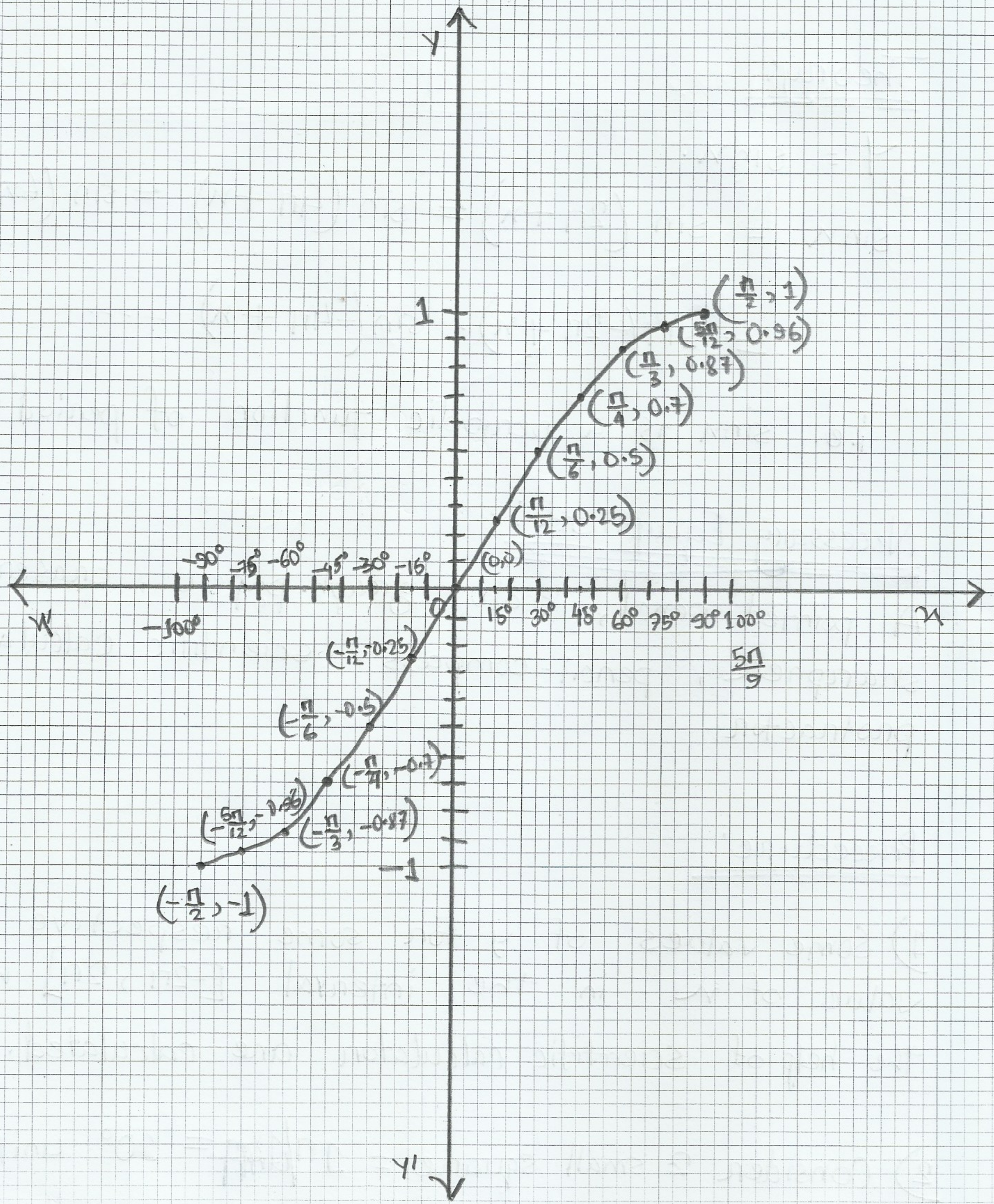
A pointed pencil, scale, graph paper, eraser, sharpner, pencil-compass, scientific calculator, protractor.

Procedure:

1) Some values of y for some respective value of x in the interval $[-2\pi, 2\pi]$ with the help of scientific calculator are calculated.

2) Consider 2 small square $= \pi/18 = 10°$ unit along x-axis and 20 small square $= 1$ unit along the y-axis. On the graph paper, the points are plotted.

FIGURE NO. 06



Y
X'
X
Y'
1
-1
0
-90°
-75°
-60°
-45°
-30°
-15°
-100°
15°
30°
45°
60°
75°
90°
100°
5π/9
(π/2, 1)
(5π/12, 0.96)
(π/3, 0.87)
(π/4, 0.7)
(π/6, 0.5)
(π/12, 0.25)
(0,0)
(-π/12, -0.25)
(-π/6, -0.5)
(-π/4, -0.7)
(-π/3, -0.87)
(-5π/12, -0.96)
(-π/2, -1)

NAME OF THE EXPERIMENT Draw the graph of $y = \sin x$.

DATE 06/04/2022

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EXPT. NO. 06

3) The points with pointed pencil are joined which gives the graph.

Calculation:

x	0	$\frac{\pi}{12}$	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{5\pi}{12}$	$\frac{\pi}{2}$	$-\frac{\pi}{12}$	$-\frac{\pi}{6}$	$-\frac{\pi}{4}$	$-\frac{\pi}{3}$	$-\frac{5\pi}{12}$	$-\frac{\pi}{2}$
y	0	0.25	0.5	0.7	0.87	0.96	1	−0.25	−0.5	−0.7	−0.87	−0.96	−1

Characteristics of the graph of $y = \sin x$:

i) The graph is not discontinuous at any point.

ii) From the graph it is seen that the maximum and minimum value of $\sin x$ is 1 and −1 respectively.

iii) The maximum and minimum value of y is got for the value of odd multiple of $\frac{\pi}{2}$.

iv) The value of $\sin x$ is zero for $x = 0$ or the value of x even multiple of $\frac{\pi}{2}$.

v) Since $\sin x = \sin(2n\pi + x)$. So, the graph extends from left to right.

NAME OF THE EXPERIMENT Draw the graph of $y = \sin^{-1} x$

DATE 07/04/2022

PAGE NO. 17

EXPT. NO. 07

Theory:

$y = \sin^{-1} x \quad (-1 \leq x \leq 1)$.

Necessary Equipment:

A pointed pencil, scale, graph paper, eraser, sharpener, pencil-compass, scientific calculator, protractor.

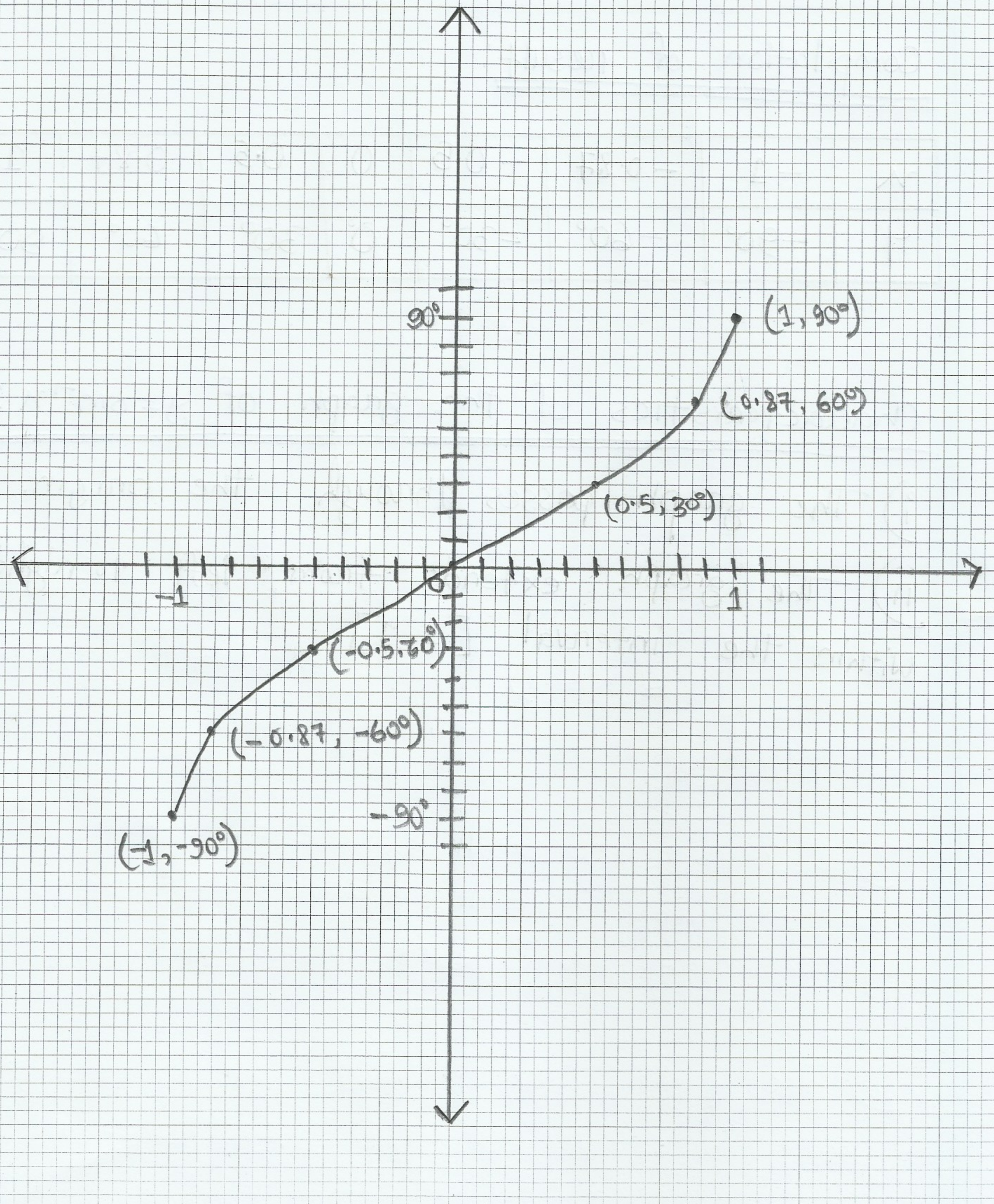
Procedure:

1) Some values of y for some respective values of x $(-1 \leq x \leq 1)$ are calculated and the ordered pair are formed.

2) Consider 20 small square = 1 unit along x-axis and 2 small square $= \frac{\pi}{18} = 10°$ along y-axis and the points are plotted.

3) The point with pointed pencil are joined which is graph of the given function.

FIGURE NO. 07



NAME OF THE EXPERIMENT Draw the graph of $y = \sin^{-1} x$.

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EXPT. NO. 07

Compilation of result:

x	-1	-0.87	-0.5	0	0.5	0.87	1
y	$-90°$	$-60°$	$-30°$	$0°$	$30°$	$60°$	$90°$

Characteristics of the graph of $y = \sin^{-1} x$:

i) The graph passes through the origin.

ii) The graph extends from left to right within the interval $[-1, 1]$.

NAME OF THE EXPERIMENT Draw the graph of $y=\tan^{-1}x$. DATE 07/04/2022

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EXPT. NO. 08

Theory:

In the interval $x \in [-1,1]$, the graph of $y=\tan^{-1}x$ is drawn and the characteristic of the graph is maintained.

Necessary Equipment:

A pointed pencil, scale, graph paper, eraser, sharpener, pencil-compass, scientific calculator, protractor.

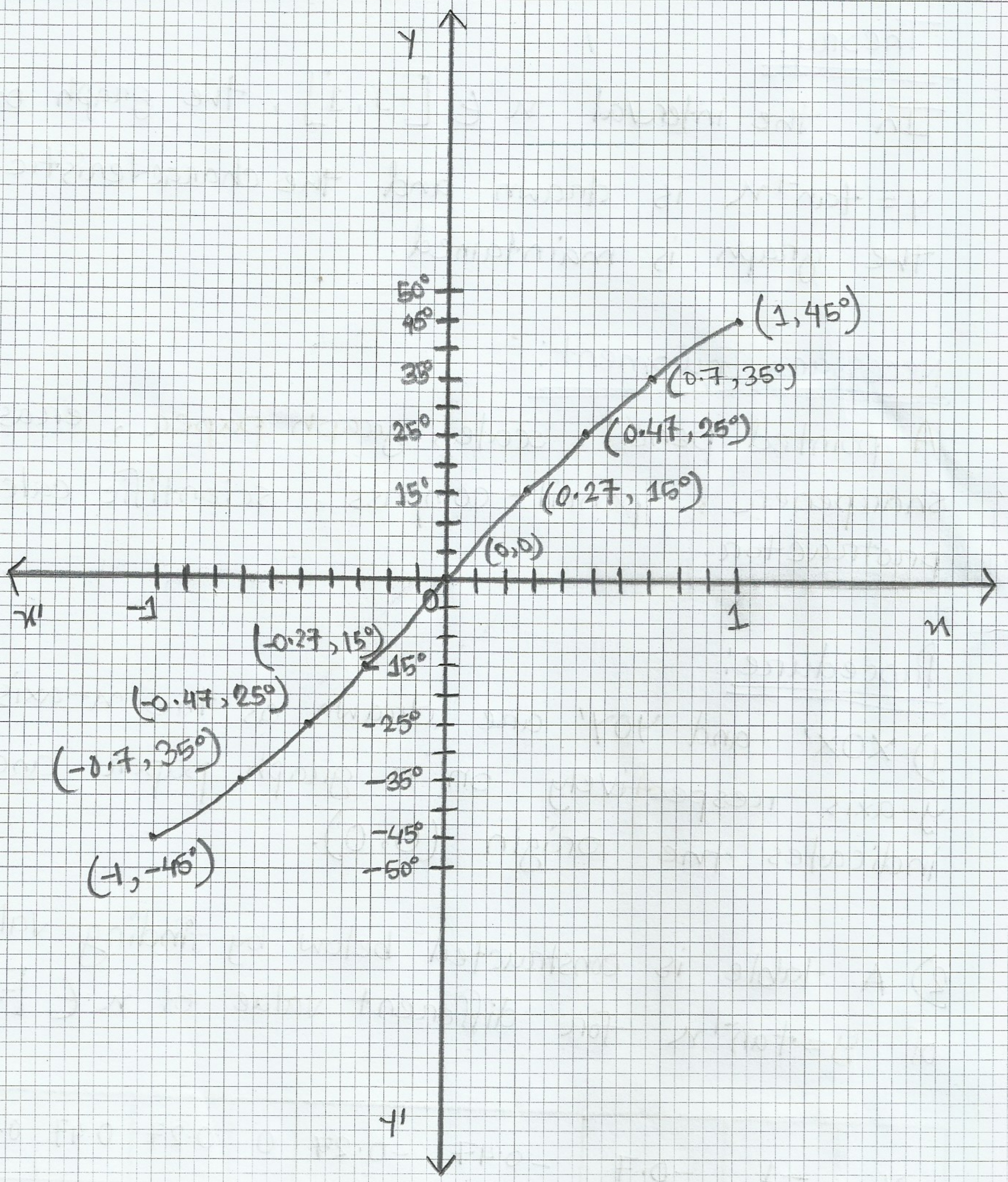
Procedure:

1) XOX' and YOY' are drawn as the x-axis and y-axis respectively on a graph paper and O indicates the origin (0, 0).

2) A table is constructed below by finding values of $y=\tan^{-1}x$ for different value of $x \in [-1,1]$.

x	-1	-0.7	-0.47	-0.27	0	0.27	0.47	0.7	1
$y=\tan^{-1}x$	$-45°$	$-35°$	$-25°$	$-15°$	$0°$	$15°$	$25°$	$35°$	$45°$

FIGURE NO. 08



y
50°
45°
35°
25°
15°
(1, 45°)
(0.7, 35°)
(0.47, 25°)
(0.27, 15°)
(0,0)
x'
-1
0
1
x
(-0.27, 15°)
(-0.47, 25°)
(-0.7, 35°)
(-1, -45°)
-15°
-25°
-35°
-45°
-50°
y'

NAME OF THE EXPERIMENT Draw the graph of $y = \tan^{-1} x$

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EXPT. NO. 08

3) By choosing a scale along the x-axis such that length of 20 small square = 1 unit and along the y-axis such that length of 2 small square = 5°, all points (x, y) are plotted on the graph paper. Joining these points in in the shape of a curve with the help of free hand, the graph of $y = \tan^{-1} x$ is drawn.

Characteristics of the graph of $y = \tan^{-1} x$:

i) The graph is wave-like.

ii) It passes through the origin.

iii) It lies in the 1st and 3rd quadrant.

NAME OF THE EXPERIMENT Two forces 70N and 50N are acting at a point with an 55°. Find the magnitude and direction of the resultant.

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EXPT. NO. 09

Theory:

If two force P, Q are acting at a point with an angle α and if their resultant is R which makes an angle θ with the direction of P then $R = \sqrt{P^2 + Q^2 + 2PQ\cos\alpha}$ and $\theta = \tan^{-1}\frac{Q\sin\alpha}{P+Q}$

Necessary Equipment:

A pointed pencil, scale, graph paper, eraser, sharpener, pencil- compass, scientific calculator, protractor.

Procedure:

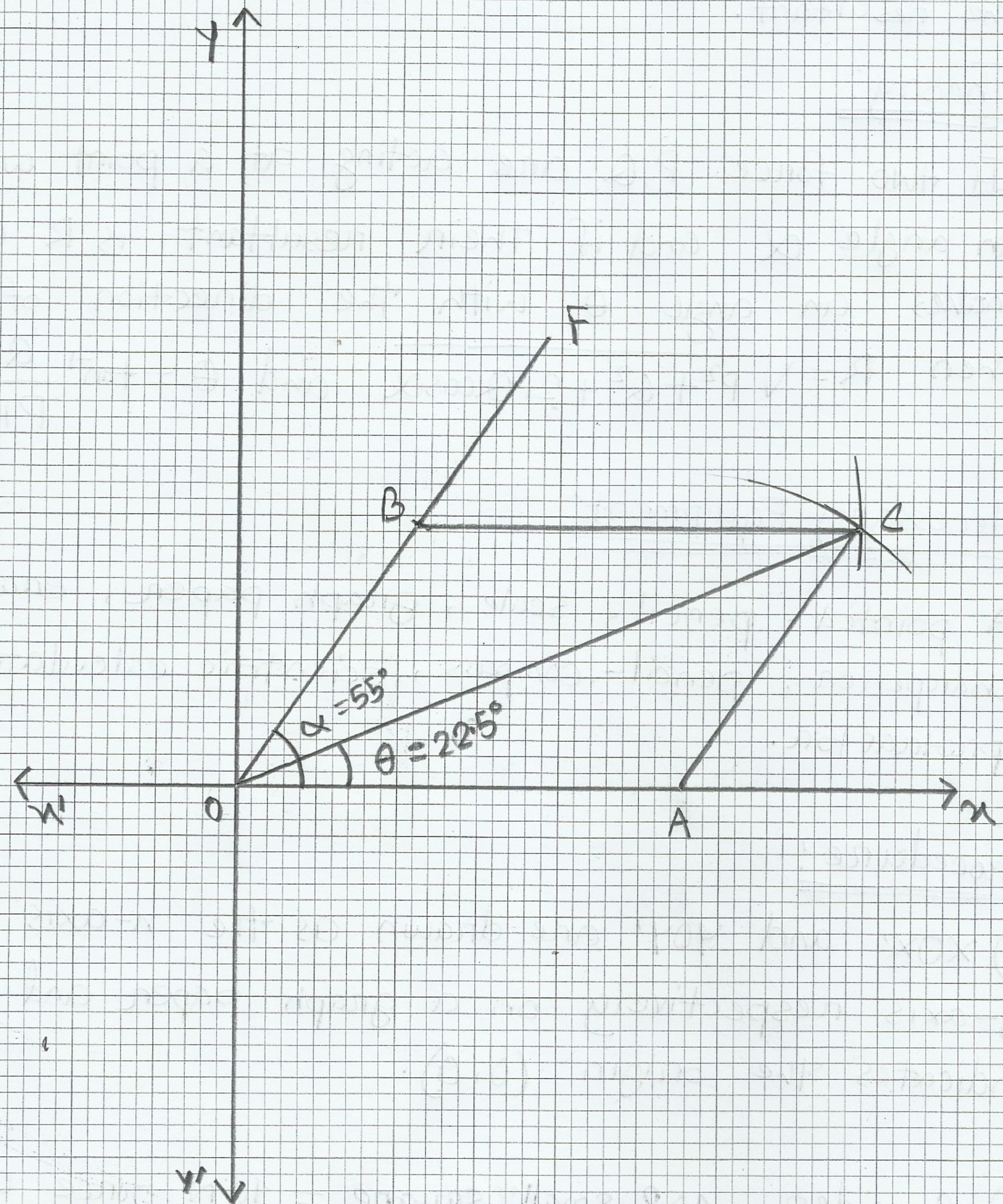
1) XOX' and YOY' are drawn as the x-axis and y-axis respectively on a graph paper and O indicates the origin (0, 0).

2) Consider 10 mm = 10N force along OX where OA = 70 mm represents 70N.

3) ∠XOF = 55° is drawn and OB = 0 mm is cut from OF which represents 50N force.

4) Now, considering OA and OB two adjacent sides

FIGURE NO. 09



NAME OF THE EXPERIMENT Two forces 70N and 50N are acting at a point with an angle 55°. Find the magnitude and direction of the resultant.

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EXPT. NO. 09

complete the parallelogram OACB and OC is joined.. Therefore OC represents the resultant.

5) Now, the length of OC is calculated by scale and angle ∠COA by protractor.

Compilation of result:

P	Q	α	$R=\sqrt{P^2+Q^2+2PQ\cos\alpha}$	Length of OC	Resultant (graphical value)
70N	50N	55°	$\sqrt{70+50^2+2\cdot 70\cdot 50\cos 55}$ $= 106.84$ N	107 mm	107 N

Error of resultant $= (107-106.84)$ N

$= 0.16$ N

P	Q	α	$\theta=\tan^{-1}\frac{Q\sin\alpha}{P+Q\cos\alpha}$	θ from graph
70N	50N	55°	$\tan^{-1}\left(\frac{50\sin 55°}{70+50\cos 55°}\right)$ $= 22.54°$	22.5°

Error of θ $= 22.54° - 22.5°$

$= 0.04°$

NAME OF THE EXPERIMENT Two forces 70N and 50N are acting at a point with an angle 55°. Find the magnitude and direction of the resultant.

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Result:

Required average magnitude of resultant is 106.92 N and average direction is 22.52°.