I、方程求解器：

① 一元有界 非线性方程解：

$x = fzero(@fun, x_0)$：x_0为向量为界，为标量为点。

$\arg\min_{x \in x_0} |fun| = 0$

② 非线性系统求解器：

$x = fsolve(@fun, x_0)$：x_0为搜索起点，

：$fun = 0$，start at x_0.

II、优化器.

① 一元有界非线性函数极小优化器：

$x = fminbnd(@fun, x_1, x_2)$：$x_1, x_2$为界.

：$\min_x f(x)$ s.t.: $x_1 < x < x_2$

② 非线性规划优化器：（无约束最小优化器）

$x = fminsearch(@fun, x_0)$：$x_0$为搜索起点，

$x = fminunc(@fun, x_0)$

$*x = patternsearch(@fun, x_0)$

：$\min_x f(x)$

③ 非负最小二乘曲线拟合问题优化器：

$x = lsqnonneg(C, d)$

：$\min_x \|Cx - d\|^2$，where $x \geq 0$

III、优化算法：Optimization Algorithm.

i) Nonlinear Optimization.：nonlinear function: @fun. @nlfun.

① Unconstrained Optimization

$\min_x f(x)$，$f(x)$ is a fun return scalar. x is a vector or matrix.

：$x = fminsearch(@fun, x_0)$

$x = fminunc(@fun, x_0)$

② Constrained Optimization.

a、$x = fminbnd(@fun, x_1, x_2)$

~~Sec~~ $\min_x f(x)$ s.t. $x_1 < x < x_2$

☆ b、$x = fmincon(@fun, x_0, A, b, A_{eq}, b_{eq}, lb, ub, nonlcon)$

SQP.

$\min_x f(x)$ s.t.: $\begin{cases} C(x) \leq 0 \\ Ceq(x) = 0 \end{cases}$ > nonlcon = @nlfun

$Ax \leq b$ →

$A_{eq}x = b_{eq}$ →

$lb \leq x \leq ub$ →

c、$x = fseminf(@fun, x_0, num, seminfcon, A, b, A_{eq}, b_{eq}, lb, ub)$

$\min_x f(x)$ s.t.: $\begin{cases} Ax \leq b \\ A_{eq}x = b_{eq} \\ lb \leq x \leq ub \\ C(x) \leq 0 \\ Ceq(x) = 0 \\ K_i(x, w_i) \leq 0, 1 \leq i \leq num \end{cases}$ ☆ } num, seminfcon.

③ Multiobjective Optimization.

a、$x = fgoalattain(@fun, x_0, goal, weight, A, b, A_{eq}, b_{eq}, lb, ub, nonlcon)$

$\min_{x,\gamma} \gamma$, s.t.: $\begin{cases} f(x) - weight \cdot \gamma \leq goal \\ C(x) \leq 0 \\ Ceq(x) = 0 \\ A \cdot x \leq b \\ A_{eq} \cdot x = b_{eq} \\ lb \leq x \leq ub. \end{cases}$

b、$x = fminimax(@fun, x_0, A, b, A_{eq}, b_{eq}, lb, ub, nonlcon)$

$\min_x \max_i f_i(x)$ s.t.: $\begin{cases} C(x) \leq 0 \\ Ceq(x) = 0 \\ Ax \leq b \\ A_{eq}x = b_{eq} \\ lb \leq x \leq ub. \end{cases}$

*: $\max_x \min_i f_i(x) = -\min_x \max_i (-f_i(x))$

ii) Programming.（Linear Optimization）

a. Linear Programming.

$x = linprog(f, A, b, A_{eq}, b_{eq}, lb, ub)$

$\min_x f^T \cdot x$ s.t.: $\begin{cases} Ax \leq b \\ A_{eq}x = b_{eq} \\ lb \leq x \leq ub. \end{cases}$

b. Quadratic Programming. LLS.

$x = quadprog(H, f, A, b, A_{eq}, b_{eq}, lb, ub, x_0)$

$\min_x \frac{1}{2}x^T H x + f^T x$ s.t.: $\begin{cases} Ax \leq b \\ A_{eq}x = b_{eq}. \\ lb \leq x \leq ub. \end{cases}$

c. problem-based Optimization

sol = solve(prob, x_0).

prob = optimproblem

x = optimvar('x'), ...

prob.Objective = -x-y/3; # fun. 非矩阵.

prob.Constraints.cons1 = x+y<=2; # con. 可写作矩阵形式.

⋮

sol = solve(prob, x_0).

Pattern search:

X = patternsearch(@fun, x_0, A, b, Aeq, beq, lb, ub, nonlcon)

= patternsearch(problem).

iii) Least Squares.

a. Linear Least Squares: $\min \|Cx-d\|^2$, with {bounds / Lin const}

- X = lsqnonneg(C, d)

$\min_x \|Cx-d\|^2$, s.t.: $x \geq 0$ C, d real.

- X = lsqlin(C, d, A, b, Aeq, beq, lb, ub, x_0)

$\min_x \frac{1}{2}\|Cx-d\|^2$ s.t.: $\begin{cases} Ax \leq b \\ Aeq\,x = beq \\ lb \leq x \leq ub. \end{cases}$

等效: $\frac{1}{2}(u-u_0)^T W(u-u_0) = \frac{1}{2}u^TWu - u_0^TWu + \frac{1}{2}u_0^TWu_0$. $\Rightarrow \begin{cases} W = C^TC \\ d = Cu_0 \end{cases} \Rightarrow$ QP: $\begin{cases} H = C^TC \\ f^T = -d^TC \end{cases}$

$\frac{1}{2}(Cx-d)^T(Cx-d) = \frac{1}{2}x^TC^TCx - d^TCx + \frac{1}{2}d^Td$.

b. Nonlinear least-Squares: $\min(\sum \|f(x_i)-y_i\|^2)$. ↳nl ↳data.

- X = lsqnonlin(@fun, x_0, lb, ub)

$\min_x \sum_i f_i^2(x)$ s.t.: $lb \leq x \leq ub$.

- X = lsqcurvefit(@fun, x_0, xdata, ydata, lb, ub)

$\min \sum (f(x, xdata(i)) - ydata(i))^2$ 用于拟合非线性参数的数据.