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Weighted Aging Ensemble

Supplementary materials of experimental evaluation

I. RESEARCH QUESTIONS

The experiments presented in this paper are designed to answer the following questions:

- Q1. How do the values of individual hyperparameters of the WAE algorithm affect its ability to classify data streams with concept drift?
- Q2. What set of hyperparameter values of the proposed WAE algorithm works best for classifying drifting data streams with both full labeling and restricted label access?
- Q3. How does the WAE algorithm compare to the *state-of-the-art* algorithms known from the literature, designed specifically for the task of drifting data streams classification, both in the case of full and limited label access?

II. DATA STREAMS & EVALUATION PROTOCOL

In order to perform the experimental evaluation of the WAE algorithm, data streams – both synthetic and based on real concepts – with various characteristics were used.

Synthetic data streams were generated using the Python stream-learn library. Three balanced streams were prepared, differing in the concept drift type, and replicated 5 times based on a random state value to stabilize the results and to enable statistical analysis. These streams were characterized by the following parameters:

- data chunks number 200,
- chunk size 250,
- global label noise − 1%,
- concept drift type sudden, gradual, and incremental,
- drifts number 10,
- *number of features* 8.

The second data source was a generator that allowed to create data streams based on static concepts from known benchmark static datasets available on repositories such as UCI or KEEL. The following data sets were used to use the streams (Table I). Finally, the experiments were also carried out using real streams (Table II).

All experiments were conducted using the *stream-learn* library and based on the Test-Than-Train evaluation protocol. In the case of synthetic streams, *accuracy score* values are reported, while in experiments containing streams based on real concepts – in order to eliminate the impact of possible data imbalance – a *balanced accuracy score* was used. The statistical analysis was performed using the *student's t-test*. The experiments presented in this article can be replicated using the code available in the *GitHub* repository.

 $\label{thm:table in the stream} TABLE\ I$ Static datasets used for data stream generation.

Dataset	#I	#F	#C
banknote	1372	4	2
heart	270	13	2
liver	345	6	2
monkone	556	6	2
sonar	208	60	2
soybean	47	35	2
wisconsin	683	9	2

TABLE II
REAL DATA STREAMS CHARACTERISTICS.

Data stream	#I	#F	#C
NSECTS-abrupt_balanced_norm	1372	4	2
INSECTS-gradual_balanced_norm	1372	4	2
INSECTS-incremental-abrupt_balanced_n or m	1372	4	2
NSECTS-incremental-reoccurring_balanced_norm	1372	4	2
INSECTS-incremental_balanced_norm	1372	4	2
airlines	1372	4	2
covtype	1372	4	2
elec	1372	4	2
poker-lsn	1372	4	2

III. FULL RESULTS OF EXPERIMENTS

A. Experiment 1 – Hyperparametrization

The experiment presents streams from the stream-learn package. The impact of the values of individual WAE hyperparameter pairs on the quality of classification in the event of a given type of concept drift is presented.

Finally, the found pseudo-optimal hyperparameter values for WAE are presented for scenarios with full label access (Tables III-IX and Figure 1) and with limited labeling (only final hyperparameterization in Table X).

			GNB			НТ			MLP	
		PROP	CONST	GAUSS	PROP	CONST	GAUSS	PROP	CONST	GAUSS
SUDDEN	POST	0.839	0.815	0.736	0.839	0.815	0.74	0.908	0.879	0.812
SUD	PRE	0.838	0.814	0.727	0.839	0.814	0.724	0.908	0.878	0.804
AD.	POST	0.829	0.811	0.752	0.829	0.811	0.755	0.918	0.895	0.851
GRAD.	PRE	0.827	0.810	0.752	0.827	0.810	0.751	0.916	0.893	0.841
INC.	POST	0.862	0.841	0.778	0.862	0.841	0.773	0.925	0.900	0.829
Ż	PRE	0.861	0.84	0.775	0.860	0.839	0.768	0.922	0.898	0.823

			GNB			нт			MLP	
		PROP	CONST	GAUSS	PROP	CONST	GAUSS	PROP	CONST	GAUSS
	SAME	0.839	0.813	0.629	0.839	0.815	0.636	0.908	0.875	0.714
SUDDEN	KUNCH.	0.839	0.842	0.825	0.839	0.842	0.823	0.908	0.915	0.902
SUD	PTA	0.839	0.832	0.721	0.839	0.832	0.719	0.908	0.900	0.801
	BELL	0.839	0.769	0.749	0.839	0.770	0.75	0.908	0.823	0.815
	SAME	0.828	0.820	0.697	0.828	0.820	0.702	0.917	0.911	0.81
AD.	KUNCH.	0.828	0.831	0.788	0.828	0.831	0.787	0.917	0.915	0.861
GRAD.	PTA	0.828	0.825	0.736	0.828	0.825	0.735	0.917	0.914	0.829
	BELL	0.828	0.767	0.787	0.828	0.766	0.789	0.917	0.836	0.883
	SAME	0.862	0.851	0.714	0.861	0.851	0.703	0.924	0.909	0.755
c.	KUNCH.	0.862	0.862	0.819	0.861	0.861	0.817	0.924	0.925	0.877
INC.	PTA	0.862	0.857	0.76	0.861	0.857	0.75	0.924	0.918	0.811
	BELL	0.862	0.792	0.812	0.861	0.791	0.813	0.924	0.843	0.861

 $\begin{tabular}{ll} TABLE\ V\\ Experiment\ 1 & --- Weight\ calculation\ method\ (rows)\ vs\ pruning\ type\ (columns). \end{tabular}$

		G	NB	I	ΙΤ	M	LP
		POST	PRE	POST	PRE	POST	PRE
	SAME	0.764	0.757	0.768	0.758	0.834	0.830
SUDDEN	KUNCH.	0.836	0.835	0.836	0.834	0.908	0.908
SUD	PTA	0.799	0.796	0.799	0.794	0.871	0.869
U 1	BELL	0.788	0.784	0.789	0.783	0.851	0.846
	SAME	0.782	0.781	0.785	0.782	0.883	0.876
D.	KUNCH.	0.816	0.815	0.816	0.815	0.899	0.896
GRAD.	PTA	0.796	0.796	0.797	0.795	0.889	0.884
	BELL	0.795	0.793	0.796	0.793	0.881	0.877
	SAME	0.810	0.808	0.808	0.802	0.864	0.861
Ċ.	KUNCH.	0.848	0.847	0.847	0.846	0.910	0.907
INC	PTA	0.827	0.825	0.823	0.822	0.886	0.883
	BELL	0.824	0.820	0.823	0.820	0.879	0.873

 $\begin{tabular}{ll} TABLE~VI\\ Experiment~l~--Theta~(rows)~vs~aging~method~(columns). \end{tabular}$

	1	1						<u> </u>		
			GNB			HT			MLP	
		PROP	CONST	GAUSS	PROP	CONST	GAUSS	PROP	CONST	GAUSS
	0%	0.839	0.811	0.731	0.839	0.812	0.732	0.908	0.872	0.808
Z	2.5%	0.839	0.828	0.731	0.839	0.829	0.732	0.908	0.894	0.808
SUDDEN	5%	0.839	0.837	0.731	0.839	0.837	0.732	0.908	0.906	0.808
s_0	7.5%	0.839	0.839	0.731	0.839	0.839	0.732	0.908	0.910	0.808
	10%	0.839	0.755	0.731	0.839	0.755	0.732	0.908	0.808	0.808
AL	0%	0.828	0.813	0.752	0.828	0.813	0.753	0.917	0.903	0.846
	2.5%	0.828	0.825	0.752	0.828	0.825	0.753	0.917	0.913	0.846
GRADUAL	5%	0.828	0.830	0.752	0.828	0.830	0.753	0.917	0.917	0.846
GR_{λ}	7.5%	0.828	0.833	0.752	0.828	0.833	0.753	0.917	0.919	0.846
	10%	0.828	0.752	0.752	0.828	0.752	0.753	0.917	0.817	0.846
L	0%	0.862	0.844	0.776	0.861	0.844	0.771	0.924	0.900	0.826
NTA	2.5%	0.862	0.857	0.776	0.861	0.856	0.771	0.924	0.918	0.826
3ME.	5%	0.862	0.862	0.776	0.861	0.861	0.771	0.924	0.924	0.826
INCREMENTAL	7.5%	0.862	0.865	0.776	0.861	0.864	0.771	0.924	0.929	0.826
Z	10%	0.862	0.775	0.776	0.861	0.775	0.771	0.924	0.824	0.826
					•			•		

		G	NB	H	IT	М	LP
		POST	PRE	POST	PRE	POST	PRE
	0%	0.796	0.791	0.798	0.791	0.864	0.861
Z	2.5%	0.801	0.798	0.803	0.797	0.871	0.869
SUDDEN	5%	0.804	0.800	0.806	0.800	0.875	0.873
SU	7.5%	0.805	0.801	0.806	0.801	0.877	0.874
	10%	0.777	0.773	0.778	0.773	0.843	0.840
	0%	0.799	0.797	0.800	0.797	0.892	0.886
ΑL	2.5%	0.802	0.801	0.803	0.801	0.895	0.890
GRADUAL	5%	0.804	0.803	0.805	0.802	0.896	0.891
GR	7.5%	0.805	0.804	0.806	0.804	0.896	0.892
	10%	0.778	0.777	0.779	0.777	0.862	0.858
	0%	0.829	0.826	0.827	0.823	0.886	0.881
NTA	2.5%	0.833	0.830	0.831	0.828	0.891	0.887
INCREMENTAL	5%	0.834	0.832	0.832	0.829	0.893	0.889
ICRI	7.5%	0.835	0.833	0.833	0.831	0.894	0.891
4	10%	0.805	0.804	0.803	0.801	0.859	0.856

 $\begin{tabular}{ll} TABLE\ VIII \\ Experiment\ 1\ --\ Theta\ (rows)\ vs\ weight\ calculation\ method\ (columns). \end{tabular}$

			Gl	NB			Н	T			M	LP	_
		SAME	KUN.	PTA	BELL	SAME	KUN.	PTA	BELL	SAME	KUN.	PTA	BELL
	0%	0.755	0.835	0.793	0.792	0.758	0.835	0.792	0.793	0.825	0.908	0.864	0.854
Z	2.5%	0.757	0.835	0.795	0.809	0.760	0.835	0.795	0.810	0.828	0.908	0.868	0.877
SUDDEN	5%	0.761	0.835	0.797	0.815	0.763	0.835	0.797	0.816	0.832	0.908	0.870	0.885
s_0	7.5%	0.763	0.836	0.799	0.815	0.766	0.835	0.799	0.815	0.836	0.908	0.873	0.884
	10%	0.766	0.836	0.801	0.697	0.769	0.835	0.800	0.697	0.841	0.908	0.875	0.742
	0%	0.778	0.814	0.793	0.807	0.780	0.813	0.792	0.808	0.876	0.896	0.883	0.900
AL	2.5%	0.78	0.815	0.795	0.818	0.781	0.815	0.794	0.819	0.877	0.897	0.885	0.909
GRADUAL	5%	0.782	0.816	0.796	0.819	0.783	0.816	0.796	0.819	0.879	0.898	0.886	0.909
GR.	7.5%	0.784	0.817	0.798	0.819	0.785	0.817	0.798	0.819	0.881	0.899	0.888	0.908
	10%	0.786	0.818	0.800	0.706	0.788	0.817	0.799	0.707	0.883	0.899	0.890	0.768
	0%	0.805	0.846	0.822	0.837	0.801	0.844	0.819	0.837	0.857	0.907	0.879	0.890
NTA	2.5%	0.807	0.847	0.824	0.849	0.803	0.845	0.821	0.848	0.859	0.908	0.882	0.908
3ME	5%	0.809	0.848	0.826	0.850	0.805	0.846	0.823	0.849	0.862	0.909	0.885	0.910
INCREMENTAL	7.5%	0.811	0.848	0.828	0.850	0.807	0.847	0.824	0.848	0.866	0.909	0.887	0.909
í	10%	0.813	0.849	0.83	0.726	0.809	0.848	0.826	0.726	0.869	0.910	0.889	0.763

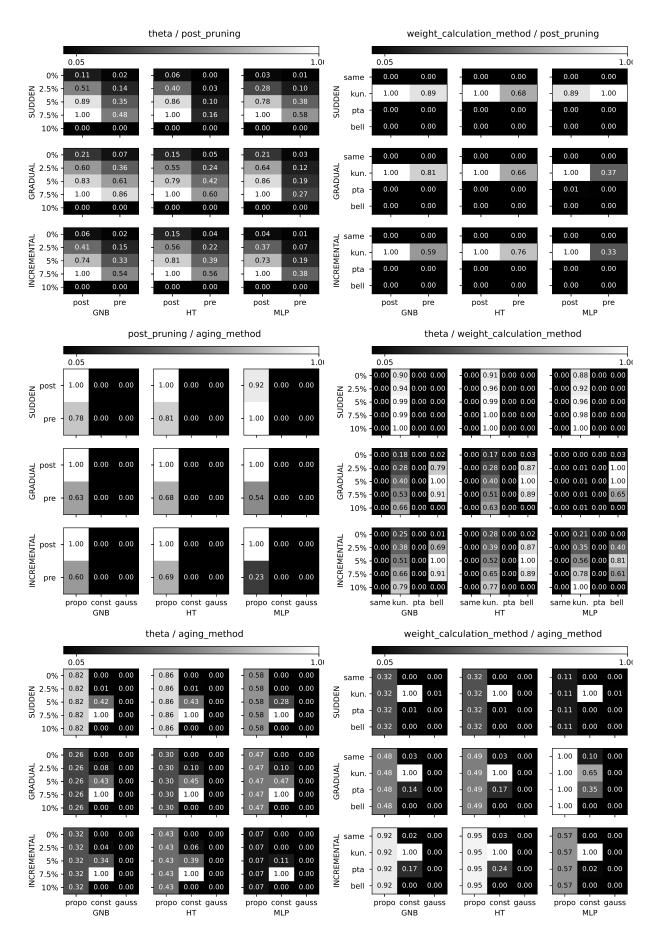


Fig. 1. EXPERIMENT 1 — Graphs of p-values achieved by various configurations of hyperparameter optimization.

 $\label{eq:table_ix} \textbf{TABLE IX} \\ \textbf{Experiment 1 } \textbf{—} \textbf{ was optimization for regular scenarios}.$

	Drift type	Accuracy	Pruning	θ	WCM	Aging
	SUDDEN	0.858	_	5%	BELL	CONST
GNB	GRADUAL	0.843	_	5%	BELL	CONST
Ū	INCREMENTAL	0.876	_	5%	BELL	CONST
	SUDDEN	0.858	_	5%	BELL	CONST
HT	GRADUAL	0.841	_	5%	BELL	CONST
	INCREMENTAL	0.873	_	5%	BELL	CONST
	SUDDEN	0.934	_	5%	BELL	CONST
MLP	GRADUAL	0.928	_	5%	BELL	CONST
	INCREMENTAL	0.944	_	5%	BELL	CONST

 $\label{eq:table x} \text{TABLE X} \\ \text{Experiment 1} \ -\text{wae optimization for active learning scenarios}.$

	Drift type	Accuracy	Pruning	θ	WCM	Aging
	SUDDEN	0.806	_	5%	BELL	CONST
GNB	GRADUAL	0.788	_	5%	BELL	CONST
Ū	INCREMENTAL	0.821	_	5%	BELL	CONST
	SUDDEN	0.791	POST	5%	PTA	CONST
HT	GRADUAL	0.778	PRE	7.5%	SAME	CONST
	INCREMENTAL	0.812	POST	7.5%	SAME	CONST
	SUDDEN	0.871	_	5%	BELL	CONST
MLP	GRADUAL	0.859	_	5%	BELL	CONST
~	INCREMENTAL	0.887	_	5%	BELL	CONST

B. Experiment 2 – Comparison with state-of-the-art on sythetic streams

The averaged results of the WAE comparison with the default and optimized hyperparameterization with reference methods for a given drift type and base classifier are presented, supplemented with statistical analysis using the student's pair T-test. Results for scenarios with full (Table XI) and limited labeling (Table XII).

TABLE XI Experiment 1 — final results on synthetic problems for regular scenarios.

		SUDDEN			GRADUAL			INCREMENTAL	r
	GNB	HT	MLP	GNB	HT	MLP	GNB	НТ	MLP
(1) SEA	0.801	0.802	0.859 5	0.813	0.813	0.904 5	0.843	0.843	0.896 5
(2) AWE	0.802	0.803	0.860 5	0.813	0.813	0.904 5	0.843	0.840	0.896 5
(3) AUE	0.832 $1,2$	0.872	0.874 $1,2,5$	0.833 1,2,5	0.858 $1,2,4,6,7$	0.909 5	0.865 $1,2,5$	0.892 all	0.904 5
(4) NSE	0.860 $1,2,3,5,6$	0.860 $1,2,6$	$0.922 \\ 1,2,3,5,6$	0.841 $1,2,5,6$	0.841 $1,2,6$	0.920 $1,2,3,5$	0.872 $1,2,5,6$	0.871 $1,2,6$	0.929 $1,2,3,5$
(5) OALE	0.827 $1,2$	0.861 $1,2,6$	0.703	0.817 -	$0.854 \\1,2,4,6,7$	0.722	0.850	0.874 $1,2,6$	0.712
(6) WAE^D	0.839 $1,2,5$	0.839 $1,2$	0.908 $1,2,3,5$	0.829 $1,2,5$	0.829 $1,2$	0.918 $1,2,3,5$	0.862 1,2,5	$0.862 \atop 1,2$	0.925 $1,2,3,5$
(7) WAE ^O	$0.858 \\1,2,3,5,6$	0.858 $1,2,6$	$0.934\\ all$	$0.843 \\1,2,3,5,6$	0.841 $1,2,6$	$0.928\\ {\it all}$	$0.876 \\1,2,3,5,6$	0.873 $1,2,6$	$0.944\\ {\it all}$

 $\begin{tabular}{l} TABLE~XII\\ EXPERIMENT~2 — FINAL~RESULTS~ON~SYNTHETIC~PROBLEMS~FOR~ACTIVE~LEARNING~SCENARIOS~. \end{tabular}$

		SUDDEN			GRADUAL			INCREMENTAL	
	GNB	HT	MLP	GNB	HT	MLP	GNB	HT	MLP
(1) SEA	0.842 4,5,7	0.843 4,5,7	0.871 4,5	0.856 $4,5,7$	0.855 4,5,7	0.909 $4,5,7$	0.886 4,5,7	0.885 $4,5,7$	0.920 $4,5,7$
(2) AWE	0.841 $4,5,7$	0.842 $4,5,7$	$0.870 \\ 4,5$	0.855 $4,5,7$	0.854 $4,5,7$	0.908 $4,5,7$	0.885 $4,5,7$	0.886 $4,5,7$	0.919 $4,5,7$
(3) AUE	$0.874 \\1,2,4,5,7$	0.884	0.871 $4,5$	0.863 $4,5,7$	$0.873 _{1,2,4,5,7}$	0.909 $4,5,7$	0.898 $1,2,4,5,7$	$0.903 \\1,2,4,5,7$	0.920 $4,5,7$
(4) NSE	0.536	0.540	0.586	0.540	0.550	0.595 -	0.539	0.541	0.598 -
(5) OALE	0.788 4	0.789	0.604	0.773 4	0.780 4	0.608	0.812	0.814	0.608
(6) WAE D	$0.866 \\1,2,4,5,7$	0.868 $1,2,4,5,7$	0.912	0.866 $4,5,7$	0.866 $4,5,7$	0.923 all	0.896 $2,4,5,7$	0.896 $4,5,7$	0.941
(7) WAE ^O	0.810 4,5	0.807 $4,5$	0.891 $1,2,3,4,5$	0.798 $4,5$	0.795 $4,5$	0.877 $4,5$	0.823 4,5	$0.826 \\ 4,5$	0.904 4,5

IV. EXPERIMENT 3 – COMPARISON WITH STATE-OF-THE-ART ON REAL STREAMS

Results comparing the efficiency on (a) real streams (Figure 2) and (b) streams with real concepts (Figures 3-4), presented using the accumulative sum of the flow visualization.

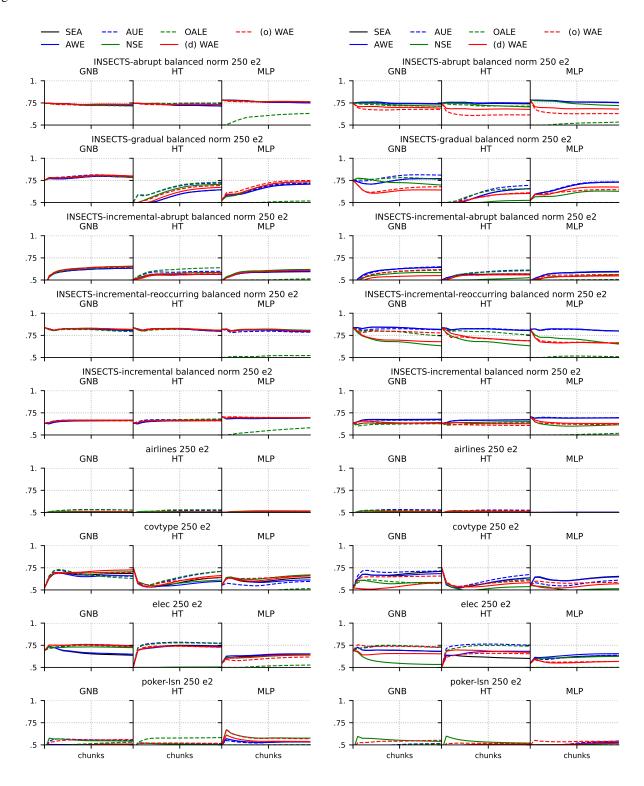


Fig. 2. EXPERIMENT 3 — final results on real streams for regular (left) and active learning scenarios (right).

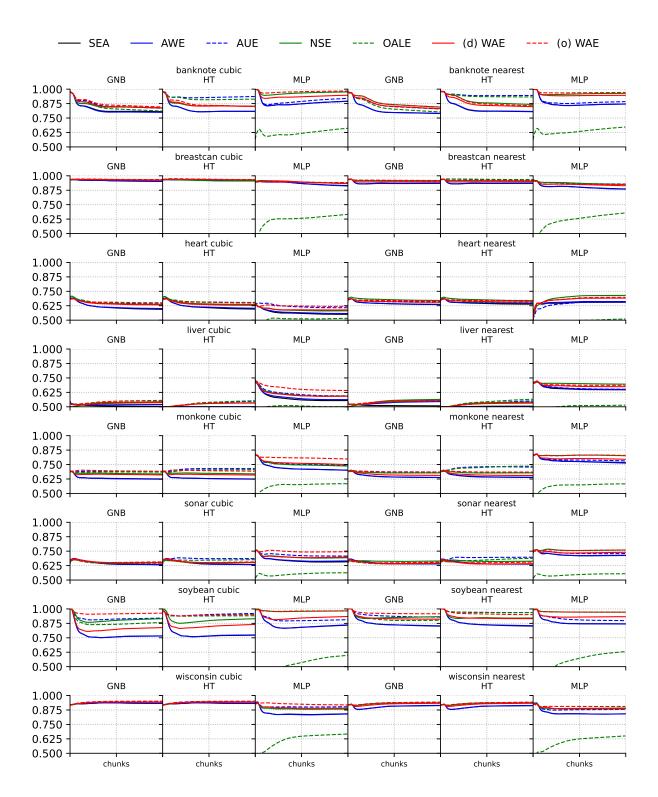


Fig. 3. EXPERIMENT 3 — final results on streams with real concepts for regular scenarios.

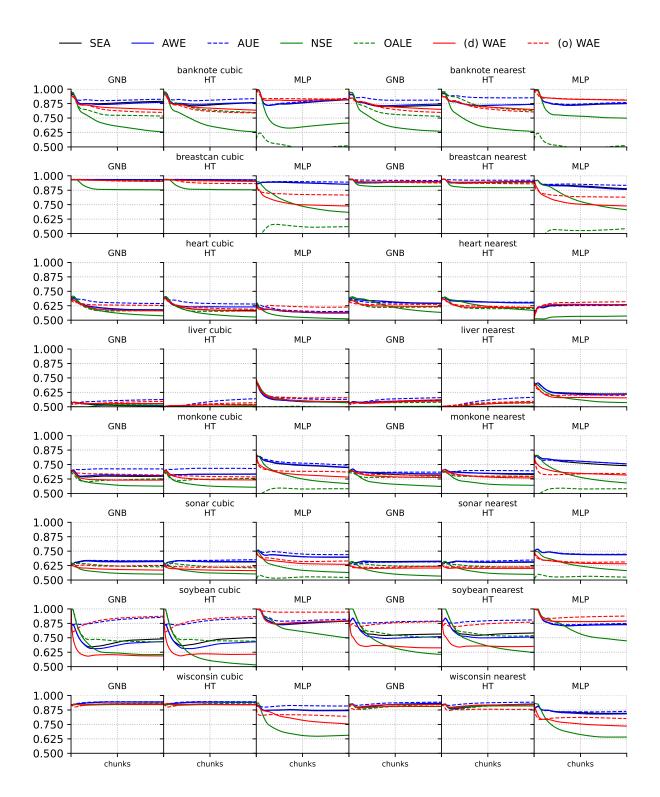


Fig. 4. EXPERIMENT 3 — final results on streams with real concepts for active learning scenarios.