Linear discriminant function fusion based on new definition of mean

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${\bf Experiment~2-Imbalanced~datasets}$

 $\textbf{Table 1.} \ \text{Results of the global Wilcoxon rank-sum test for various sizes of the classifier pool regarding BAC}$

Pool size		MV (2)		NM (4)	Pool size	L (1)	MV (2)	M (3)	NM (4)
3	1.859	2.435	2.924	2.783	29	1.652	2.522	2.859	2.967
	_	1	1, 2	1, 2		_	1	1	1, 2
5	1.663	2.511	3.196	2.63	31	1.630	2.522	2.870	2.978
	_	1	all	1		_	1	1	1, 2
7	1.576	2.402	3.120	2.902	33	1.609	2.478	2.891	3.022
	_	1	1, 2	1, 2		_	1	1, 2	1, 2
9	1.489	2.380	3.130	3.000	35	1.630	2.565	2.989	2.815
	_	1	1, 2	1, 2		_	1	1, 2	1
11	1.500	2.457	3.098	2.946	37	1.674	2.543	2.859	2.924
	_	1	1, 2	1, 2		_	1	1	1
13	1.565	2.326	3.011	3.098	39	1.652	2.543	2.837	2.967
	_	1	1, 2	1, 2		_	1	1	1, 2
15	1.511	2.391	2.848	3.250	41	1.652	2.543	2.946	2.859
	_	1	1, 2	1, 2		_	1	1	1
17	1.598	2.500	2.826	3.076	43	1.674	2.478	2.880	2.967
	_	1	1	1, 2		_	1	1	1, 2
19	1.609	2.370	2.859	3.163	45	1.674	2.522	2.859	2.946
	_	1	1, 2	1, 2		_	1	1	1, 2
21	1.609	2.478	2.793	3.120	47	1.717	2.543	2.772	2.967
	_	1	1	1, 2		_	1	1	1, 2
23	1.674	2.500	2.685	3.141	49	1.717	2.587	2.815	2.880
	_	1	1	1, 2		_	1	1	1
25	1.587	2.478	2.815	3.120	51	1.739	2.609		
	_	1	1	1, 2		_	1	1	1
27	1.674	2.478	2.837	3.011	53	1.761	2.609		2.739
	_	1	1	1, 2		_	1	1	1

 $\textbf{Table 2.} \ \text{Results of the global Wilcoxon rank-sum test for various sizes of the classifier pool regarding $_{\rm AUC}$$

Pool size	L (1)	MV (2)	M (3)	NM (4)	Pool size	L (1)	MV (2)	M (3)	NM (4)
3	1.859	2.413	2.924	2.804	29	1.652	2.543	2.837	2.967
	_	1	1, 2	1, 2		_	1	1	1
5	1.685	2.511	3.174	2.63	31	1.630	2.543	2.848	2.978
	_	1	all	1		_	1	1	1, 2
7	1.576	2.402	3.120	2.902	33	1.630	2.543	2.804	3.022
	_	1	1, 2	1, 2		_	1	1	1, 2
9	1.511	2.402	3.087	3.000	35	1.630	2.63	2.967	2.772
	_	1	1, 2	1, 2		_	1	1	1
11	1.565	2.5	2.989	2.946	37	1.674	2.609	2.815	2.902
	_	1	1, 2	1, 2		_	1	1	1
13	1.587	2.348	2.946	3.120	39	1.652	2.587	2.772	2.989
	_	1	1, 2	1, 2		_	1	1	1, 2
15	1.533	2.413	2.848	3.207	41	1.674	2.565	2.859	2.902
	_	1	,	1, 2		_	1	1	1
17	1.598	2.5	2.826	3.076	43	1.674	2.522	2.815	2.989
	_	1	1	1, 2		_	1	1	1, 2
19	1.609	2.37	2.859	3.163	45	1.674	2.565	2.793	2.967
	_	1	1, 2	1, 2		_	1	1	1, 2
21	1.609	2.5	2.793	3.098	47	1.717	2.587	2.728	2.967
	_	1	1	1, 2		_	1	1	1, 2
23	1.696	2.522	2.685	3.098	49	1.761	2.652	2.75	2.837
	_	1	1	1, 2		_	1	1	1
25	1.630	2.543	2.793	3.033	51	1.783	2.63	2.815	2.772
	_	1		1, 2		_	1	1	1
27	1.696	2.543	2.837	2.924	53	1.826	2.652	2.783	
	_	1	1	1		_	1	1	1

4 P. Ksieniewicz et al.

 $\textbf{Table 3.} \ \text{Results of the global Wilcoxon rank-sum test for various sizes of the classifier pool regarding RRSE}$

Pool size		MV (2)	M (3)	NM (4)		L (1)			
3	1.967	2.315		2.685	29	1.696	2.587	2.826	2.891
	_			1, 2		_	1	1	1
5	1.489	2.511	3.207	2.793	31	1.674	2.674	2.837	2.815
	_	1	all	1		_	1	1	1
7	1.511	2.467	3.163	2.859	33	1.696	2.63	2.815	2.859
	_	1	all	1, 2		_	1	1	1
9	1.533	2.489	3.087	2.891	35	1.761	2.674	2.902	2.663
	_	1	1, 2	1, 2		_	1	1	1
11	1.522	2.478	3.087	2.913	37	1.739	2.63	2.902	2.728
	_		1, 2			_	1	1	1
13	1.587	2.391	3.011	3.011	39	1.761	2.674	2.859	2.707
	_	1	1, 2	1, 2		_	1	1	1
15	1.576	2.5	2.793	3.130	41	1.717		2.88	
	_	1		1, 2		_	1	1	1
17	1.576	2.63		3.087	43	1.717	2.674		
	_	1	1	1, 2		_	1	1	1
19	1.565	2.543	2.815	3.076	45	1.739	2.652		
	_	1		1, 2		_	1	1	1
21	1.565	2.609			47	1.761		2.707	
	_	1	1	1, 2		_	1	1	1
23	1.587	2.652	2.826	2.935	49	1.739	2.652	2.815	
		1	1	1			1	1	_
25	1.652	2.63	2.826	2.891	51		2.63		
	_	1	1	1		_	1	1	
27	1.696	2.609		2.891	53	1.826		2.717	
	_	1	1	1		_	1	1	1

 $\textbf{Table 4.} \ \text{Results of the global Wilcoxon rank-sum test for various sizes of the classifier pool regarding G-mean.}$

Pool	L	MV	M	NM	Pool	L	MV	M	
\mathbf{size}	(1)	(2)	(3)	(4)	size	(1)	(2)	(3)	(4)
3	1.924	2.326	3.011	2.739	29	1.717	2.500	2.750	3.033
	_	1	1, 2	1, 2		_	1	1	1, 2
5	1.620	2.424	3.239	2.717	31	1.652	2.587	2.826	2.935
	_	1	all	1		_	1	1	1
7	1.446	2.446	3.185	2.924	33	1.609	2.652	2.761	2.978
	_	1	1, 2	1, 2		_	1	1	1
9	1.446	2.402	3.196	2.957	35	1.630	2.652	2.837	2.880
	_	1	1, 2	1, 2		_	1	1	1
11	1.435	2.587	2.946	3.033	37	1.674	2.652	2.815	2.859
	_	1	1	1, 2		_	1	1	1
13	1.543	2.391	2.924	3.141	39	1.652	2.652	2.815	2.880
	_	1	1, 2	1, 2		_	1	1	1
15	1.511	2.391	2.761	3.337	41	1.696	2.630	2.880	2.793
	_	1	1	all		_	1	1	1
17	1.576	2.500	2.804	3.120	43	1.717	2.609	2.815	2.859
	_	1	1	1, 2		_	1	1	1
19	1.543	2.326	2.924	3.207	45	1.717	2.609	2.815	2.859
	_	1	1, 2	1, 2		_	1	1	1
21	1.543	2.435	2.859	3.163	47	1.717	2.696	2.750	2.837
	_	1	1	1, 2		_	1	1	1
23	1.609	2.565	2.772	3.054	49	1.783	2.652	2.750	2.815
	_	1	1	1, 2		_	1	1	1
25	1.652	2.413	2.793	3.141	51	1.739	2.674	2.859	2.728
	_	1	1	1, 2		_	1	1	1
27	1.674	2.413	2.793	3.120	53	1.761	2.652	2.891	2.696
	_	1	1	1, 2		_	1	1	1

Table 5. Results of the global Wilcoxon rank-sum test for various sizes of the classifier pool regarding F1.

Pool size	L (1)	MV (2)	M (3)	NM (4)	Pool size	L (1)	MV (2)	M (3)	NM (4)
3	1.772	2.315	3.120	2.793	29	1.652	2.543	2.837	2.967
	_	1	all	1, 2		_	1	1	1
5	1.511	2.467	3.283	2.739	31	1.652	2.543	2.870	2.935
	_	1	all	1		_	1	1	1
7	1.424	2.446	3.120	3.011	33	1.674	2.630	2.783	2.913
	_	1	1, 2	1, 2		_	1	1	1
9	1.402	2.337	3.217	3.043	35	1.674	2.696	2.859	2.772
	_	1	1, 2	1, 2		_	1	1	1
11	1.413	2.413	3.120	3.054	37	1.696	2.674	2.783	2.848
	_	1	,	1, 2		_	1	1	1
13	1.391	2.370	3.054	3.185	39	1.717	2.652	2.815	2.815
	_	1	1, 2	1, 2		_	1	1	1
15	1.402	2.446	2.804	3.348	41	1.717	2.696	2.859	2.728
	_	1	1	all		_	1	1	1
17	1.380	2.500	2.891	3.228	43	1.761	2.63	2.793	
	_	1	1	1, 2			1	1	1
19	1.478	2.413	2.924	3.185	45	1.717	2.674	2.728	2.880
		1	1, 2				1	1	1
21	1.435	2.543	2.859	3.163	47	1.761	2.696	2.707	2.837
		1	1	1, 2	4.0		1	1	1
23	1.500	2.522	2.880	3.098	49	1.761	2.717	2.750	2.772
25		1	1	1, 2			1	1	1
25	1.609	2.533	2.837	3.022	51	1.783	2.674	2.859	
		1	1	1, 2			1	1	1
27	1.565	2.565	2.837	3.033	53	1.783	2.696	2.870	
	_	1	1	1, 2		_	1	1	1

 ${\bf Table~6.}~{\bf Results~of~the~global~Wilcoxon~rank-sum~test~for~various~sizes~of~the~classifier~pool~regarding~precision.$

Pool size	L (1)	MV (2)	M (3)			L (1)		M (3)	
3	2.348	2.402	2.685	2.565	29	0.000	2.522	2.109	2.337
	_	_	_	_			3	_	_
5	2.228	2.446	2.674		31		2.576	2.043	2.337
_			1	1	2.2	all	3		
7	2.391	2.598	2.402	2.609	33	3.033	2.663	2.033	2.272
					~~		3, 4		
9	2.424	2.533	2.402	2.641	35		2.641	2.076	2.239
					0.		3, 4		
11	2.587	2.467	2.402	2.543	37	3.065	2.587	2.087	2.261
10		0.470		0.540	9.0		3, 4		
13	2.576	2.478		2.543	39		2.598	2.065	2.261
1.5	0.00	0.500	0.105	0.000	41		3, 4	0.005	
15	2.685	2.522	2.185	$\frac{2.609}{3}$	41	3.076	2.620	2.065	2.239
1.77	3		0.141		49		3, 4	0.011	0.240
17	3	2.543	2.141	$\frac{2.554}{3}$	43	3.054 all	$\frac{2.587}{3}$	2.011	2.348
19	3 2.793	2.533	2.120	$\frac{3}{2.554}$	45	$\frac{an}{3.054}$	_	2.054	2.283
19	2.795 3	2.333	2.120	3	40		3, 4	2.034	2.203
21	$\frac{3}{2.826}$		2.141	$\frac{3}{2.511}$	47		$\frac{3}{2.576}$		2.359
21	3	2.322	2.141	2.311	41	3.011 all	3	2.034	2.559
23	2.913	2.587	2.141	2.359	49	3.033	2.630	2.076	2.261
20	3, 4	3	Z.141 —	4.00 <i>0</i>	40		3, 4	2.070	2.201
25	,	2.565	2.152	2.337	51		2.554		2.293
20	all	2.000			01		3	2.130	
27	3.000	2.511		2.348	53		2.576		2.228
41	all				00		4		

 $\textbf{Table 7.} \ \text{Results of the global Wilcoxon rank-sum test for various sizes of the classifier pool regarding \textit{recall.}}$

Pool size	L (1)	MV (2)	M (3)	NM (4)	Pool size	L (1)	MV (2)	M (3)	NM (4)
3	1.793	2.457	3.011	2.739	29	1.543	2.543	2.924	2.989
	_	1	1, 2	1		_	1	1	1, 2
5	1.598	2.467	3.196	2.739	31	1.543	2.565	2.913	2.978
	_	1	all	1		_	1	1	1, 2
7	1.511	2.337	3.207	2.946	33	1.478	2.543	2.913	3.065
	_	1	1, 2	1, 2		_	1	1	1, 2
9	1.359	2.467	3.217	2.957	35	1.522	2.630	2.967	2.880
	_	1	1, 2	1, 2		_	1	1	1
11	1.370	2.478	3.141	3.011	37	1.522	2.587	2.902	2.989
	_	1	1, 2	1, 2		_	1	1	1, 2
13	1.413	2.370	3.054	3.163	39	1.500	2.587	2.891	3.022
	_	1	1, 2	1, 2		_	1	1	1, 2
15	1.370	2.478	2.880	3.272	41	1.543	2.565	2.935	2.957
	_	1	1	1, 2		_	1	1	1, 2
17	1.446	2.457	2.957	3.141	43	1.587	2.543	2.870	3.000
	_	1	1, 2	1, 2		_	1	1	1, 2
19	1.478	2.348	2.946	3.228	45	1.565	2.565	2.870	3.000
	_	1	1, 2	1, 2		_	1	1	1, 2
21	1.478	2.478	2.902	3.141	47	1.630	2.565	2.815	2.989
	_	1	1	1, 2		_	1	1	1, 2
23	1.500	2.500	2.772	3.228	49	1.652	2.587	2.826	2.935
	_	1	1	1, 2		_	1	1	1
25	1.478	2.522	2.815	3.185	51	1.630	2.587	2.957	2.826
		1		1, 2		_	1	1	1
27	1.413	2.478	2.880	3.228	53	1.652	2.620	2.957	2.772
		1	1	1, 2		_	1	1	1

 $\textbf{Table 8.} \ \text{Results of the global Wilcoxon rank-sum test for various sizes of the classifier pool regarding \textit{specificity}.}$

Pool size		MV (2)				L (1)			
3	2.174	2.446			29	2.587	2.587	2.163	
	_	_	1	1		_	_	_	3
5	2.033	2.435		2.63	31	2.630		2.098	
	_	_	1, 2	1		3	3	_	3
7	2.054	2.424		2.576	33	2.565	2.652	2.076	2.707
	_	_	-, -	1		_	3	_	3
9	2.022	2.413		2.728	35	2.630	2.696	2.196	2.478
	_	_	1, 2	1		_	3	_	_
11	2.120	2.348	2.685	2.848	37	2.630		2.196	2.500
	_	_	1	1, 2		_	3	_	_
13	2.152	2.380	2.761	2.707	39	2.696	2.652	2.152	2.500
	_	_	1	1		3	3	_	_
15	2.250	2.478	2.326	2.946	41	2.652	2.674	2.217	2.457
	_	_	_	all		_	3	_	_
17	2.293	2.543	2.348	2.815	43	2.652	2.674	2.109	2.565
	_	_	_	1, 3		3	3	_	_
19	2.239	2.522	2.446	2.793	45	2.630	2.674	2.174	2.522
	_	_	_	1		_	3	_	_
21	2.391	2.457	2.283	2.870	47	2.609	2.609	2.217	2.565
	_			3		_			_
23	2.500	2.543	2.337	2.620	49	2.587	2.652	2.217	2.543
	_			_		_			_
25	2.478	2.543	2.402	2.576	51	2.630	2.652	2.174	2.543
	_	_	_	_		_	3	_	_
27	2.587	2.587	2.217	2.609	53	2.641	2.674	2.239	2.446
	_	_				_	_		_

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