



**University
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US Border Services Airport Wait Time Simulation

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Introduction

The waiting line problem has always been a trade-off between minimizing the waiting time for customers and increasing the service efficiency [1]. A waiting line or queue consists of multiple elements including arrivals, servers and waiting lines. In order to determine the best strategy to maximize customers' experience, we need to calculate operating characteristics such as the average number of customers in queue, average time customers spend in the system etc. In this project, we investigated two different models that could be used in the airport border services waiting line. While we harvested real data with some assumption from the U.S. Customs and Border Protection website, we do not expect the result to be close to the real-world scenario. This is because the airport traffic is constantly changing and may be affected by some uncontrollable element like weather. However we do see some differences using two different models to simulate the traffic and gave our conclusion about which model performs better in our simulation.

Project Objective

The objective of this project is to build a simulation system for Washington International Thurgood Marshall Airport at specific time interval to test the feasibility of our simulation and use our model to find a better service mode in order to maximize the service efficiency and reduce the waiting in queue time.

Problem Mapping

We decide to use M/M/6 system in this simulation as the M/M/6 system is made of Poisson arrivals, exponential (Poisson) servers, FIFO (or not specified) queue of unlimited capacity and unlimited customer population, which exactly matches Airport waiting line scenario. In the first simulation we will use the actual data from US Border Services and Protection website (<https://awt.cbp.gov/>) with some assumption to build our simulation model. We will compare this simulated outcome with theoretical outcome to see if our model is feasible.

Simulation Model

Since we assume the passenger arrival as poisson processes with exponential arrival time and the service time is also exponentially distributed. In a regular airport setting, check-in queues are First-In-First-Out. We will use the M/M/C system where C stands for number of

paralleled counters/servers in the system. The passengers with local citizenship will usually receive express custom entry, so we will use priority queue to represent this scenario.

Simulation Goals and Parameters

The goal of our project is to simulate the same airport border check-in time with two different models, and compare their efficiency. The models are described as below: (1) M/M/6 with two counters that is designated only for people with citizenships or permanent resident. These counters will have their own queue and we will call these counters the priority counter. (2) M/M/6 with priority queue with non-preemptive scheduling. This means that when the priority counter is empty, international passengers can go to this counter. The local passengers can join the queue at any time and will skip international passengers but will not interrupt the passenger that is already in service. The number of counters are decided using the data from the website. We also assume that there is only one server at each counter.

Methodology

Tool used

We decided to use python because it has easy structure and rich library and packages like SimPy and Matplot.

SimPy is a process-based discrete-event simulation framework based on standard Python. Simulations can be performed “as fast as possible”, in real time (wall clock time) or by manually stepping through the events which allows us test our result efficiently [3]. This tool allows us to simulate customer arrivals, service process and departure, which exactly fit our needs in this project. After we obtain enough data, we can then use Matplot library to plot the diagrams and confidence intervals.

Simulation Approach

To model the proposed queuing system, a python program with SimPy was developed. It is not feasible for us to collect all actual data in a very short time, so our simulation was using actual data with some liberty taken on certain assumption. We only assumed that the service time for both international passengers and local passengers are the same with 4 minutes per person at the counter. This allows us to model the simulation consistently and it is easier for us to calculate and collect the result.

Modelling

We constructed our system model based on our own experience at the airport border check-in, where the border services have several check-in counters but a single queue. Passengers will usually take a fixed amount of time for service, and waiting passengers will form a single first-in first-out queue. In order to test our model, in our first simulation we assumed that all passengers share the same queue. This is not what the reality is but we would like to see the outcome from this model. In our second simulation, we separated 2 queues only for local passengers with citizenships or permanent residents. This is closer to the real-world scenario but we would like to further reduce the total waiting time for all passengers by introduce the last model. In our last simulation (3), we suggested that the custom services use priority queue. We gave the local passengers a higher priority (marked as VIP customers in code). They can still join the normal counters and will skip all the international passengers. While we still have priority counters, now international passengers can use those counters if there is no one in service. This means that the priority counters will share some of the traffic from the normal counters. We do realize that this approach is difficult to implement in real life, but we were hoping to see improved waiting time through this simulation.

Theoretical Parameters:

The utilization rate is depends on how many Booths we have at that hour, in our case is 6:

$$\rho = \frac{\lambda}{c\mu}$$

For the M/M/c queue,

$$L_q = \frac{P_0(\frac{\lambda}{\mu})^c \rho}{c!(1 - \rho)^2}$$

where

$$P_0 = 1 / \left[\sum_{m=0}^{c-1} \frac{(c\rho)^m}{m!} + \frac{(c\rho)^c}{c!(1-\rho)} \right].$$

We denote the probability that there are 0 passengers in the system so the sum m is starting from 0.

To calculate the Long-run time-average number of customers in system (L),

$$L = c\rho + \frac{(c\rho)^{c+1}P_0}{c(c!)(1-\rho)^2} = c\rho + \frac{\rho P(L(\infty) \geq c)}{1-\rho}$$

For Long-run average time in the system (w) and time in the queue per passenger can be calculated by

$$w = \frac{L}{\lambda}$$

$$w_Q = w - \frac{1}{\mu}$$

Confidence Intervals:

In this project, we calculated 95% confidence intervals for all parameters and plotted as shaded area on the graph. The formula for calculating the CI is as follows:

$$\bar{x} \pm z \frac{s}{\sqrt{n}}$$

The S is the standard deviation of a parameter calculated using the Statistics module from Python library [2]. With 95% confidence, the value of Z is 1.96.

Simulation 1

As mentioned before, we use data from US Border Services and Protection website on September 1st 2018 of Washington International Thurgood Marshall Airport peak hour as our data resource. During that hour, there were 188 passengers that checked in to U.S. border with 6 counters. We assumed that the current situation is all counters serve for both citizens and foreigners with a single queue.

Airport	Terminal	Date	Hour	U.S. Citizen		Non U.S. Citizen		Wait Times		Total	Flights	Booths
				Average Wait Time	Max Wait Time	Average Wait Time	Max Wait Time	Average Wait Time	Max Wait Time			
BWI	International Arrivals	11/1/2018	1600 - 1700	1	9	5	9	1	9	90	1	4
BWI	International Arrivals	11/1/2018	1700 - 1800	3	9	4	8	3	9	47	1	5
BWI	International Arrivals	11/1/2018	1800 - 1900	4	9	4	7	4	9	40	1	4
BWI	International Arrivals	11/1/2018	1900 - 2000	9	67	23	64	12	67	462	4	5
BWI	International Arrivals	11/1/2018	2000 - 2100	18	52	33	56	23	56	188	1	6
BWI	International Arrivals	11/1/2018	2100 - 2200	12	23	18	21	12	23	54	1	2

Table A.1 Real data for Washington International Thurgood Marshall Airport

We use Simpy to build a M/M/6 systems to simulate the scenario. Because of limited time and lack of data resource, in our simulation, we assumed the service rate was 4 minutes per passenger. Based on M/M/c system's formula we calculate Long-run time-average number of customers in system(L), Long-run average time spent in system per customer(w) and Long-run average time spent in queue per customer(wQ).

Here is the table for theoretical value and simulated result for 188 passengers:

	L	Lq	w	wq
Theoretical value calculated by equation	21.7839130	9.806694	7.17809	3.17809
Simulated value	9.02854950393	7.76224509801	28.2894551123	24.322
Actual value from source	N/A	7.2	19.0	23.0

Table A.2 Theoretical result and simulated result

Our simulated Wq is close to the actual data from source, and that is what we cared the most, so we decided to keep the service rate at 4 minutes per passenger for our other simulations.

Simulation 2

In this simulation, we move on to find a mean that can help the waiting process by reduce long-run waiting in queue for each individual passenger. We came up with an idea that we use different queue for different type of passengers: foreigner or citizen. We assume that there are special counters only open for citizens, and has a totally separated queue where only citizens can wait in line. The ratio for counters is based on ratio of passengers on the flight. In this case we assume that the ratio between citizen and foreigner is 2:4, means we have 62 citizens and 126 foreigners for this hour. In our simulation, there are two types of queue,

parameters can be calculated based on a system that combined M/M/4 and M/M/2 queueing model. Noted that 2: 4 ratio is just an example. This ratio depends on the ratio of the passengers. As described before, our goal is to minimize the long-run average wait time per passenger in the system.

In this scenario, the queueing system is not stable because the arrival rate is much bigger than the service rate and the queue will eventually go into infinity. So for the analysis, we will heavily rely on the practical data we collected from our simulation.

The mean W and W_q were calculated and plotted in Figure B1 and B2. The shaded area is the confidence interval calculated using the method stated above. To conveniently compare the result with simulation 3, Figure B1 and B2 are only shown in simulation 3.

	W	W _q	L	L _q
Seed:123221	34.1075088166	30.3438285856	102.32252645	79.0314857569
Seed:1212	36.6283025348	32.4065349576	109.884907604	85.2196048729
Seed:32412	36.3901665235	32.4624827017	109.17049957	85.3874481052
Seed:212121	41.210486964	37.250062525	123.631460892	99.7501875749
Seed:5467761865	37.9697288839	33.9011480096	113.909186652	89.7034440287
Avg:	37.2612387	33.2728114	111.783716	87.8184341

Table A.3 Simulated value for model 2

Simulation 3

In this scenario, we will also use separate queues for citizens and foreigners. However, the difference from previous simulation is foreigners can use counters for citizens if and only if the counter is not in service. That is to say, foreigners can not wait in the queue for citizen-only counter but can directly go into service if nobody waiting in that counter. We need to calculate M/M/c system with priority queue for long-run time-average number of customers in system(L), long-run average time spent in system per customer(W) and long-run average time spent in queue per customer(W_q). We found a study that looked into the waiting time of a non-preemptive M/M/c queue by Offer and Uri [4], which can be applied to our simulation.

In their studies, they suggested that the time spend in system per customer of system that have two classes can be calculated as:

$$\tilde{W}_k(s) = (1-\pi) + \pi \frac{c\mu(1-\sigma_k)(1-\tilde{\gamma}(s))}{s-\lambda_k+\lambda_k\tilde{\gamma}(s)}$$

Where $\tilde{\gamma}(s)$:

$$\tilde{\gamma}(s) \equiv E[e^{-sY}] = \left\{ s + \lambda_a + c\mu - [(s + \lambda_a + c\mu)^2 - 4\lambda_a c\mu]^{\frac{1}{2}} \right\} (2\lambda_a)^{-1}.$$

π :

$$\pi = \frac{(\lambda/\mu)^c}{c!(1-\rho)} \left[\sum_{i=0}^{c-1} \frac{(\lambda/\mu)^i}{i!} + \frac{(\lambda/\mu)^c}{c!(1-\rho)} \right]^{-1}$$

and σ_j :

$$\sigma_j = \sum_{i=1}^j \rho_i$$

In this equation, j is the total class we have in this case equals to 2. After calculation, the average time each customer spend in system is 24.31314969

	W	Wq	L	Lq
Seed:123221	26.4344116062	22.837	79.3032348185	56.5111647159
Seed:1212	39.6353513604	35.103	118.906054081	93.3085728732
Seed:32412	32.4690628255	28.637	97.4071884765	73.9105695942
Seed:212121	29.9031192158	26.236	89.7093576474	66.7071741653
Seed:5467761865	31.912676179	28.052	95.738028537	72.1564810477
Avg:	32.07092	28.173	96.21277	84.21277

Table A.4 Simulated result from model 3

	L	Lq	w	wq
Theoretical value	76.17309797	63.64827986	24.31314969	20.3134969
Simulated value	96.21277	84.21277	32.07092	28.173

Table A.5 Theoretical result and simulated result for model3

For the rest of this section, we provided figures of simulation 3 by comparing all four parameters of international passengers and local citizens.

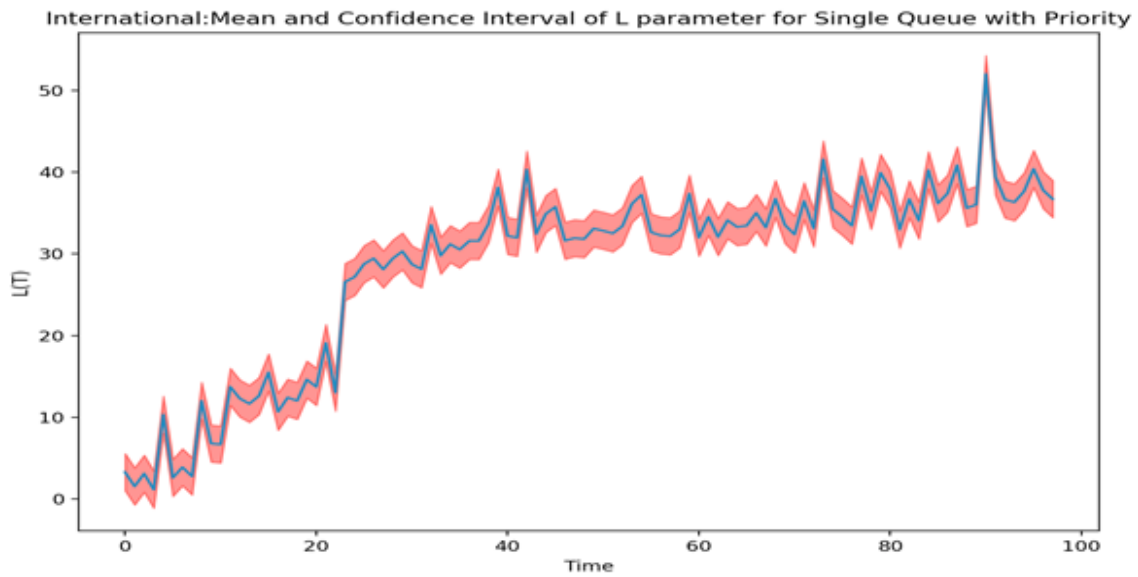


Figure A.1: L parameter of International passengers

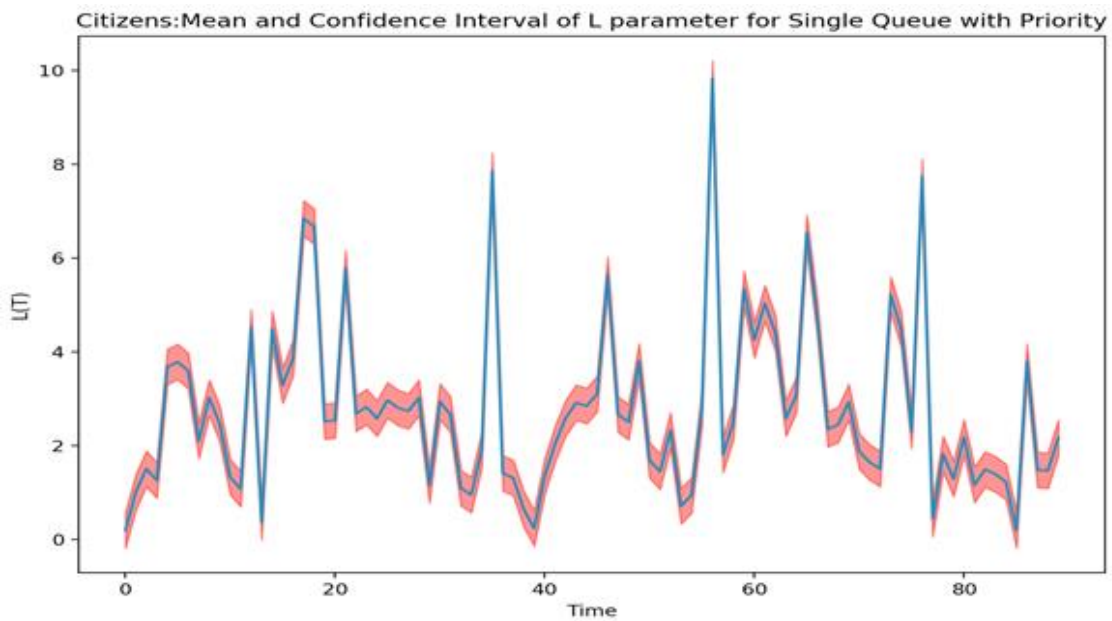


Figure A.2: L parameter of local passengers

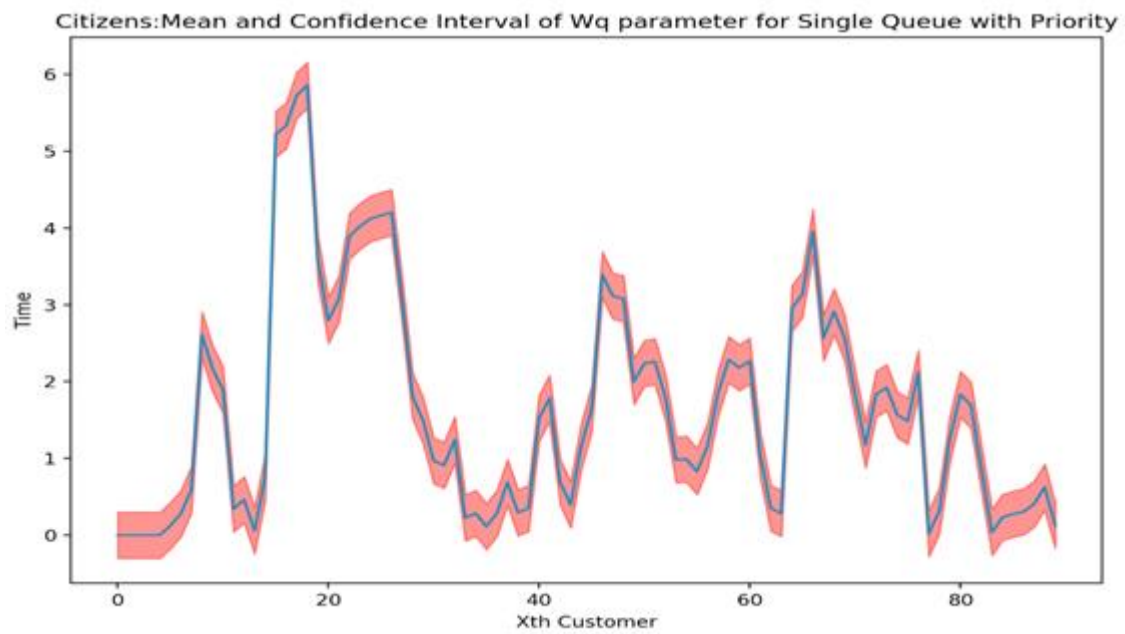


Figure A.3: W_q parameter of local passengers

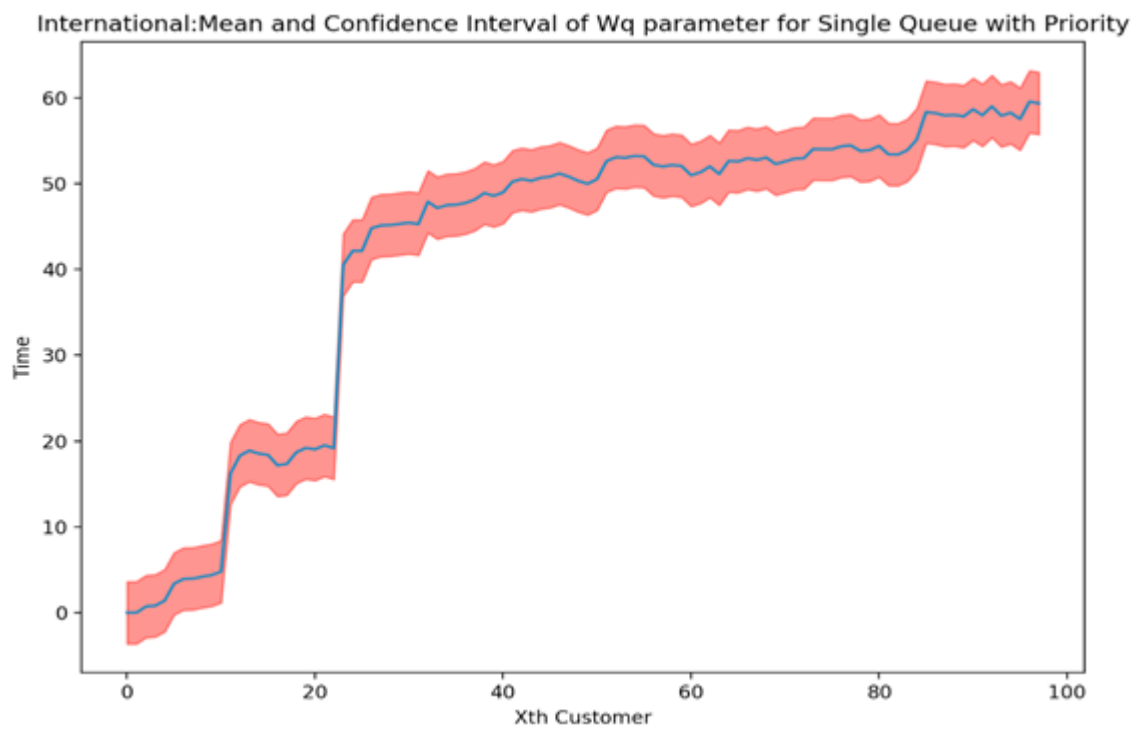


Figure A.4: W_q parameter of International passengers

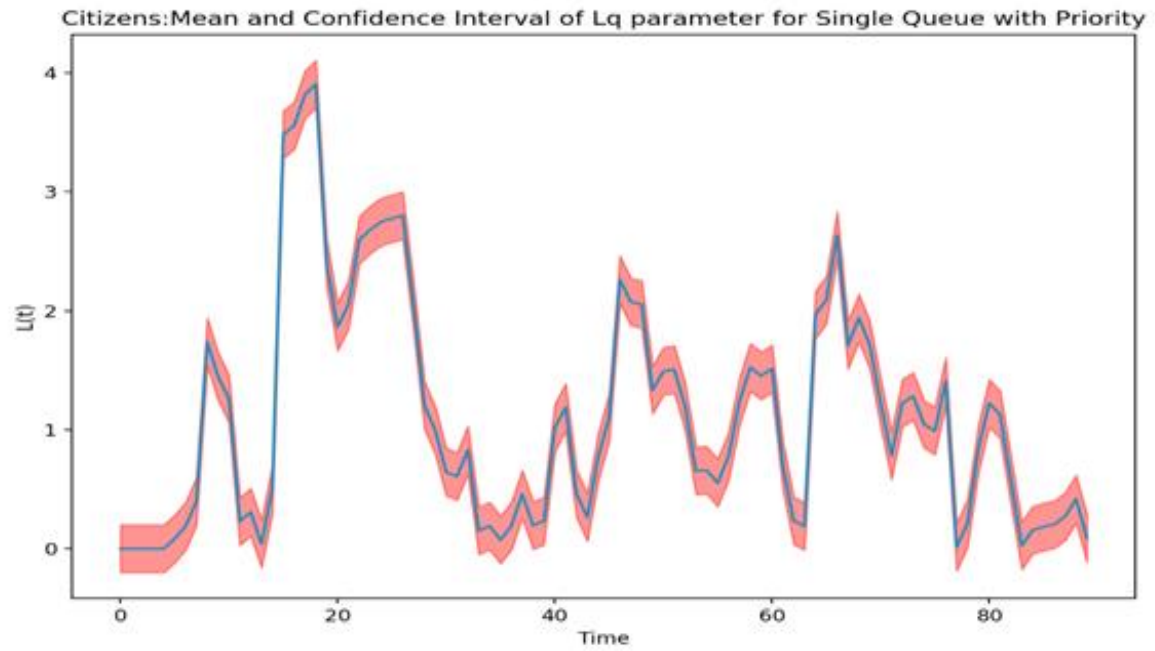


Figure A.5: L_q parameter of local passengers

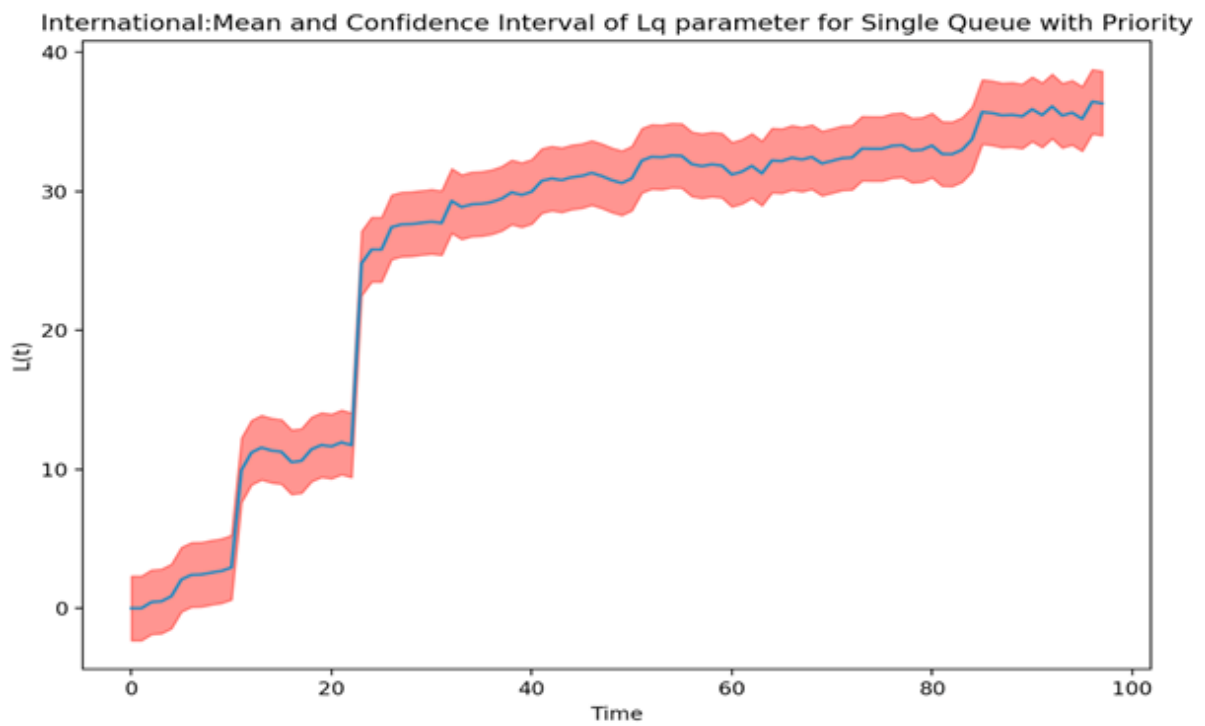


Figure A.6: L_q parameter of International passengers

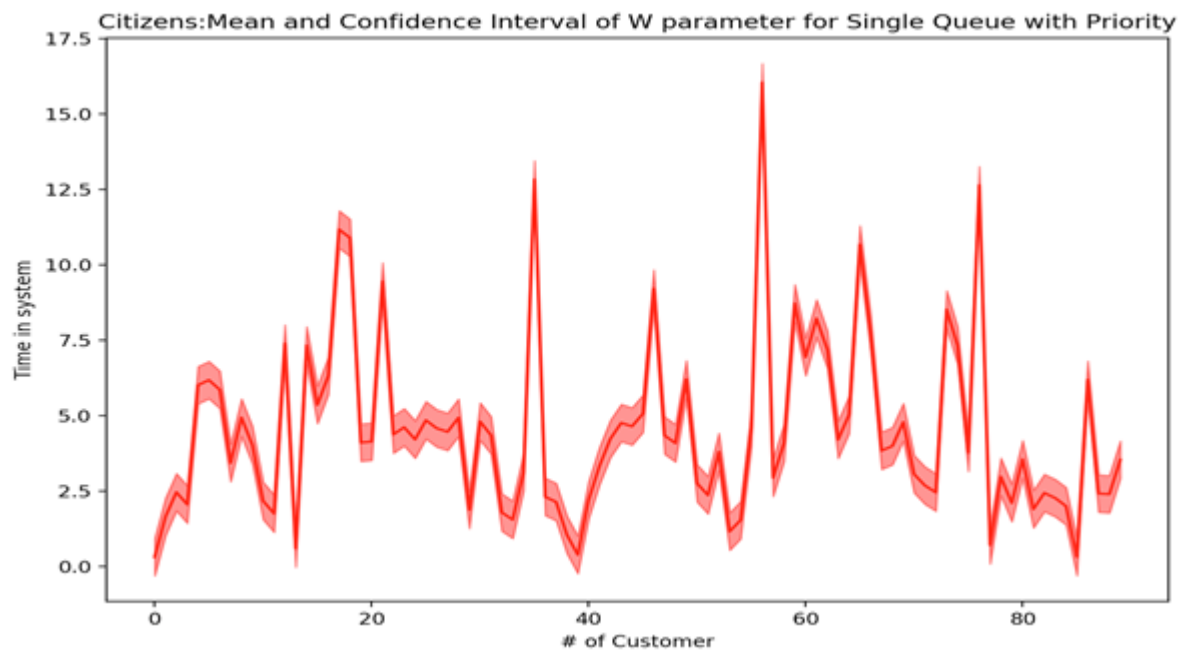


Figure A.7: W parameter of local passengers

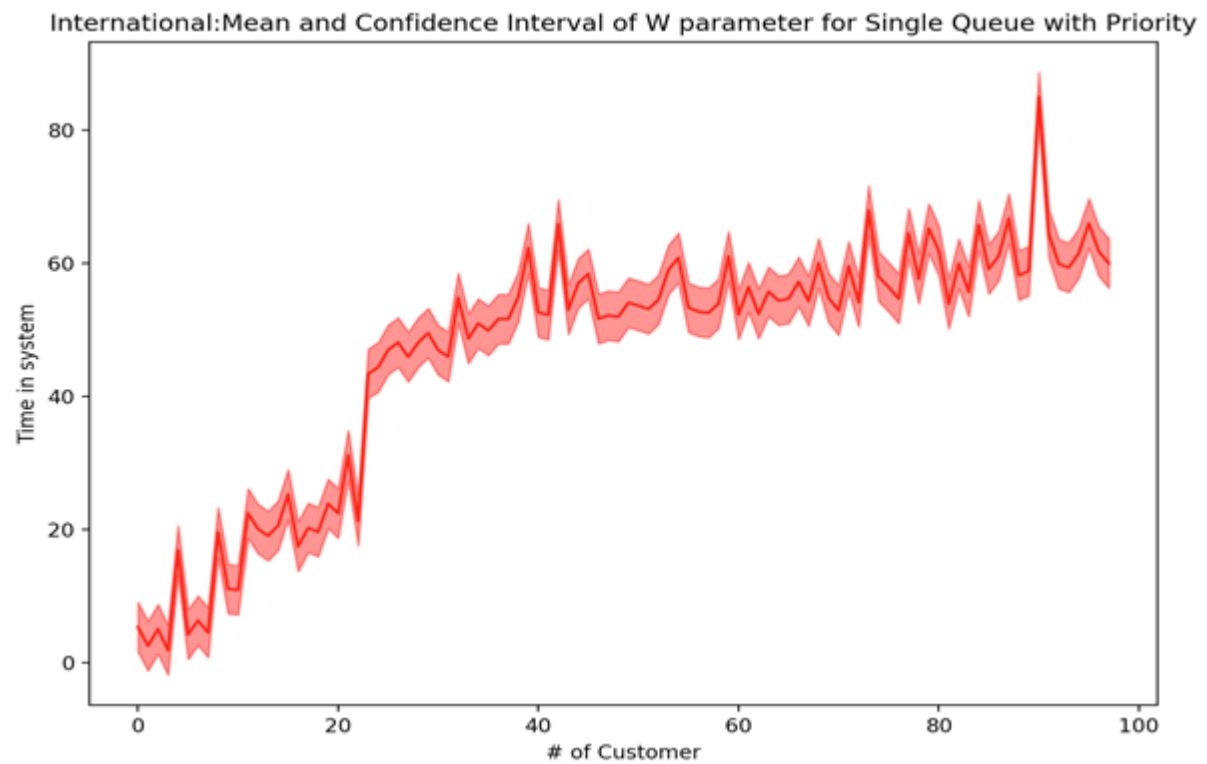


Figure A.8: W parameter of International passengers

The following figures compared the waiting time in system, and the waiting time in queue for all passengers (International + local).

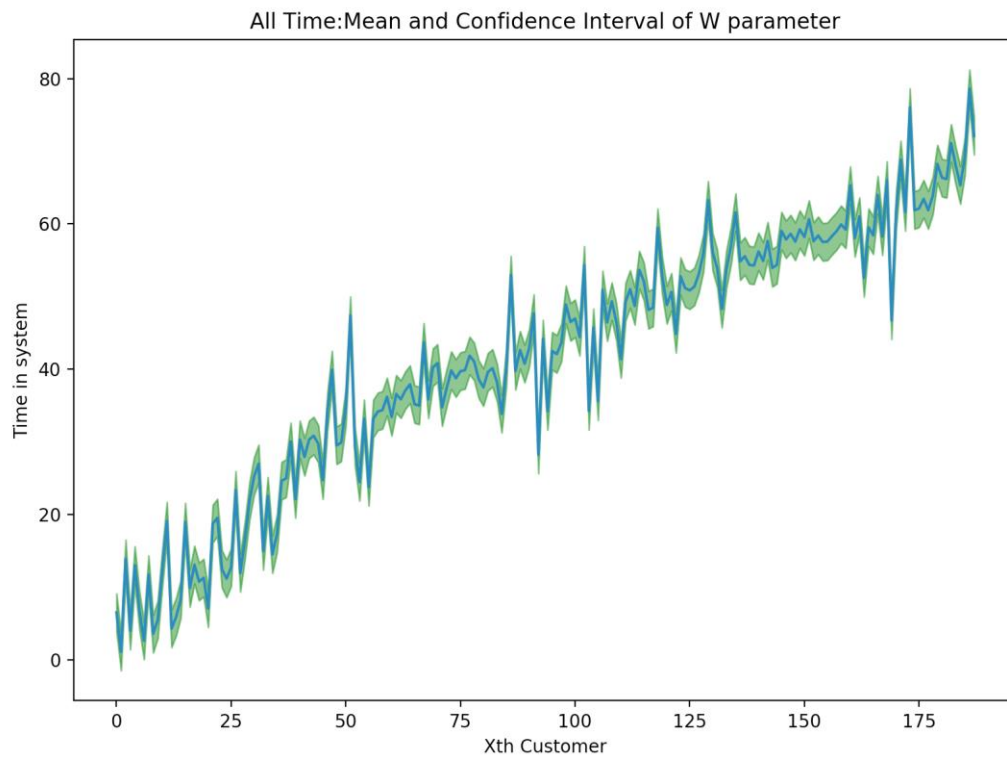


Figure B.1 Average time in system for model 2(All Time): 41.3566930154

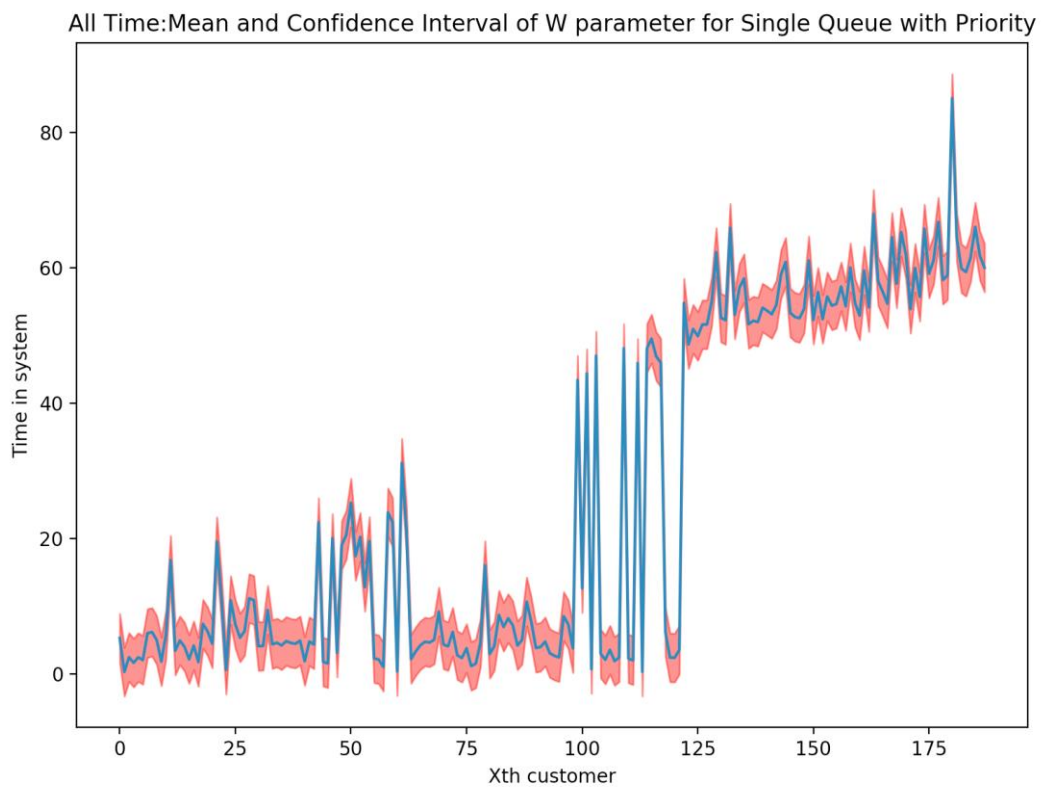


Figure A.8 Average time in system for model 3(All time): 26.4344116062

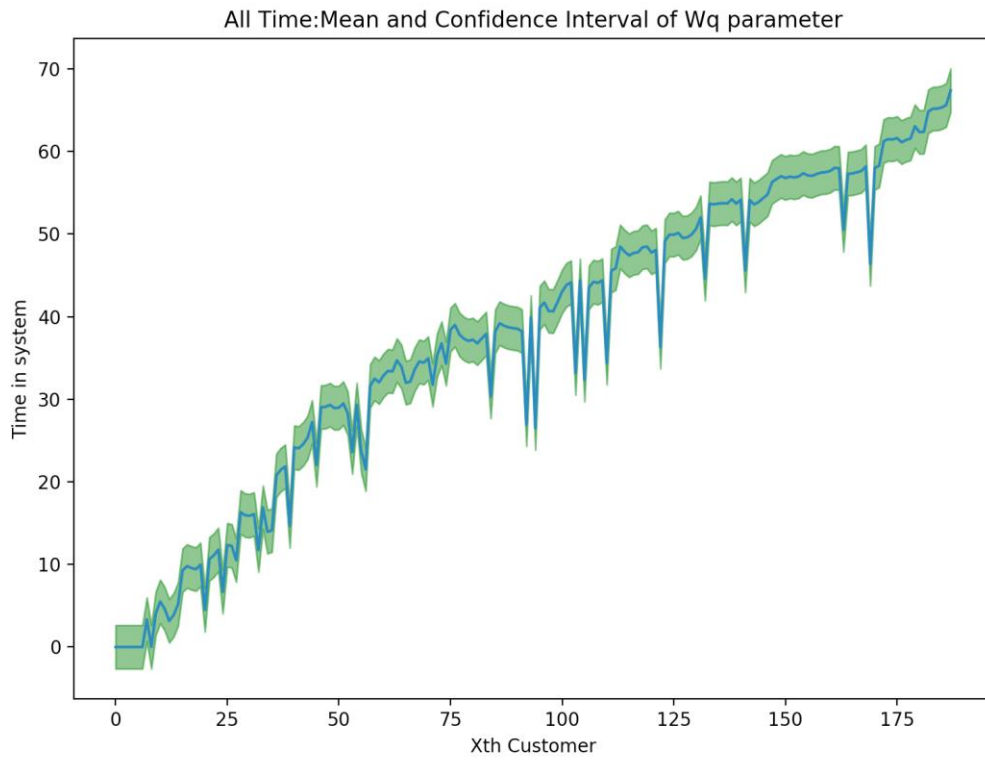


Figure B.2 Average time in queue for model 2(All time): 37.3880956959

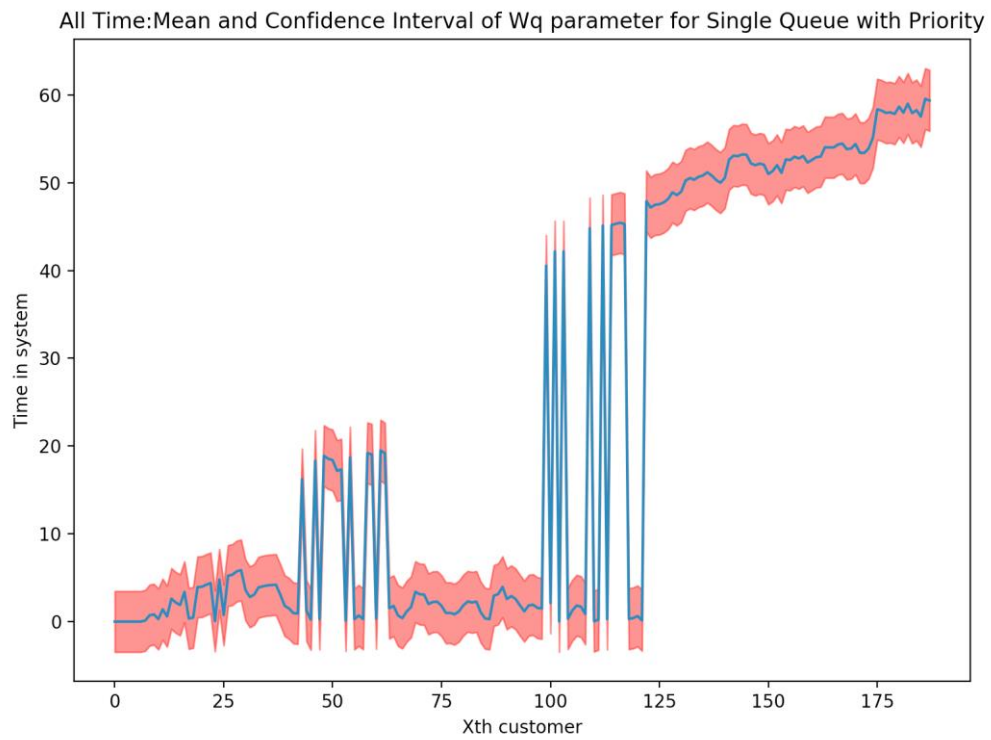


Figure A.9 Average time in queue for model 3(All time): 22.8370549053

Analysis

According to data we received, we compared the result from simulation 2 and 3. The data shows that the overall waiting time from model 3 is significantly lower than the result obtained from model 2. Using model 3, the time spend in the system for all customers (W) and the time spend in the queue for all customers (W_q) are 26.4344116062 and 22.8370549053 respectively. We expected the result to be like this because in model 3 the priority counters share some of the traffic from normal counters. In figure A.8, we see starting from about 100 customers, the average time spend in the system start to jump ups and downs. We think this is because some priority passengers start to jump in line and leave the system. Their waiting time is minimum, so we see the trend goes ups and downs like that. Otherwise the trend is increasing steadily.

Discussion

The goal of this project is to investigate the queueing model at the airport Border Services and compare two models to find out the optimal one. It should be noted that our simulation was built on several assumptions including service rate and check-in scenario. In reality, the check-in procedure might be different, and we only explored data from a limited time frame. The result was in our expectation but we discovered that the initial seed will have a noticeable impact on our result. This might due to the reason that we only use 188 passengers within one-hour time frame and the simulation length is not long enough to produce stable outcome.

We also noticed that the theoretical value calculated for model 3 was quite different than our simulated result. This might again due to the reason of short simulation length, however the performance of model 3 is still better than model 2.

Conclusion

In this project, we build up three different simulation models for airport Border Services. Five replications of two experiments were run. We use the first model as a reference for the other two. We found that the priority queue model(model 3) produce more optimal result than model 2 as the waiting time for all passengers at the border services are lower.

From our discussion section, it was found that the seed value has a noticeable impact on our result, and our simulation length is not long enough to produce stable outcome.

While this work was primarily intended to model the wait line at the border services at the airport, it offered an interesting opportunity to look deeper into this problem from a more sophisticated angle.

Future Work

Since our simulation was limited on small sample size, we could apply larger passenger size with longer period, booths and with different service rate to generate more data to analyze the result. This will make our result closer to the real-life scenario.

We also need to study more on priority queue models and find the best one to apply to our simulation.

Another interesting work that can be done in future is to do cost analysis. This will allow us to know how many counters are needed and the feasibility of increasing number of servers, so we can use that information in our simulation.

References

- [1] B. Taylor, *Introduction to management science*, 10th ed. 2006, p. 573.
- [2] "Statistics". [Online]. Available: <https://docs.python.org/3/library/statistics.html>
- [3] "SimPy". [Online]. Available: <https://simpy.readthedocs.io/en/latest/>
- [4] O. Kella and U. Yechiali, "Waiting Times in the Non-Preemptive Priority M/M/c Queue", *Communications in Statistics. Stochastic Models*, vol. 1, no. 2, pp. 257-262, 1985.