# HW #3

### 3.13 LABORATORY EXERCISES AND PROJECTS

- Design a Wiener filter to remove the artifacts in the ECG signal in the file ecg_hfn.dat. (See also the file ecg_hfn.m.) The equation of the desired filter is given in Equation 3.101. The required model PSDs may be obtained as follows:

  Create a piece-wise linear model of the desired version of the signal by concatenating linear segments to provide P, QRS, and T waves with amplitudes, durations, and intervals similar to those in the given noisy ECG signal. Compute the PSD of the model signal.

- Redo the above exp by using the ECG filtered by the Comb filter as the template.
- Also Compare the results with the results of the lowpass filter.
- Due date of PPT report: 10/25 2023

(3.13 of **Biomedical Signal Analysis, Rangaraj M. Rangayyan, Wiley, ISBN: 0-471-20811-62.**)

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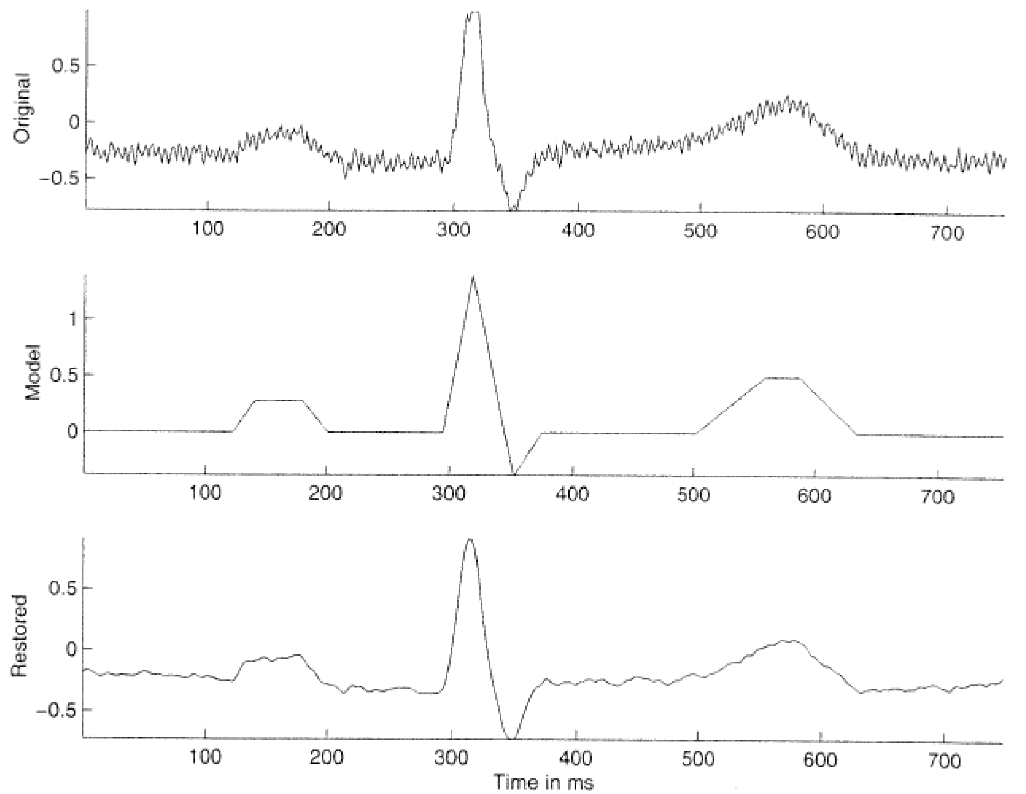


**Figure 3.47** From top to bottom: one cycle of the noisy ECG signal in Figure 3.5 (labeled as Original); a piece-wise linear model of the desired noise-free signal (Model); and the output of the Wiener filter (Restored).

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# 補充說明-I

$$\sum_{i=0}^{M-1} w_{oi}\,\phi(k-i) = \theta(k),\ k = 0, 1, 2, \ldots, M-1. \qquad (3.88)$$

Thus we have the convolution relationship

$$w_{ok} * \phi(k) = \theta(k). \qquad (3.89)$$

Applying the Fourier transform to the equation above, we get

$$W(\omega)S_{xx}(\omega) = S_{xd}(\omega), \qquad (3.90)$$

which may be modified to obtain the Wiener filter frequency response $W(\omega)$ as

$$W(\omega) = \frac{S_{xd}(\omega)}{S_{xx}(\omega)}, \qquad (3.91)$$

where $S_{xx}(\omega)$ is the PSD of the input signal and $S_{xd}(\omega)$ is the cross-spectral density (CSD) between the input signal and the desired signal.

# 補充說明

Let us now consider the problem of removing noise from a corrupted input signal. For this case, let the input $x(n)$ contain a mixture of the desired (original) signal $d(n)$ and noise $\eta(n)$, that is,

$$x(n) = d(n) + \eta(n). \tag{3.92}$$

Using the vector notation as before, we have

$$\mathbf{x}(n) = \mathbf{d}(n) + \boldsymbol{\eta}(n), \tag{3.93}$$

where $\boldsymbol{\eta}(n)$ is the vector representation of the noise function $\eta(n)$. The autocorrelation matrix of the input is given by

$$\mathbf{\Phi} = E[\mathbf{x}(n)\mathbf{x}^T(n)] = E[\{\mathbf{d}(n) + \boldsymbol{\eta}(n)\}\{\mathbf{d}(n) + \boldsymbol{\eta}(n)\}^T]. \tag{3.94}$$

If we now assume that the noise process is statistically independent of the signal process, we have

$$E[\mathbf{d}(n)\boldsymbol{\eta}^T(n)] = E[\boldsymbol{\eta}^T(n)\mathbf{d}(n)] = \mathbf{0}. \tag{3.95}$$

Then,

$$\mathbf{\Phi} = E[\mathbf{d}(n)\mathbf{d}^T(n)] + E[\boldsymbol{\eta}(n)\boldsymbol{\eta}^T(n)] = \mathbf{\Phi}_d + \mathbf{\Phi}_\eta, \tag{3.96}$$

where $\mathbf{\Phi}_d$ and $\mathbf{\Phi}_\eta$ are the $M \times M$ autocorrelation matrices of the signal and noise,

# 補充說明

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where $\mathbf{\Phi}_d$ and $\mathbf{\Phi}_\eta$ are the $M \times M$ autocorrelation matrices of the signal and noise, respectively. Furthermore,

$$\mathbf{\Theta} = E[\mathbf{x}(n)d(n)] = E[\{\mathbf{d}(n) + \boldsymbol{\eta}(n)\}d(n)] = E[\mathbf{d}(n)d(n)] = \mathbf{\Phi}_{1d}, \tag{3.97}$$

where $\mathbf{\Phi}_{1d}$ is an $M \times 1$ autocorrelation vector of the desired signal. The optimal Wiener filter is then given by

$$\mathbf{w}_o = (\mathbf{\Phi}_d + \mathbf{\Phi}_\eta)^{-1}\mathbf{\Phi}_{1d}. \tag{3.98}$$

The frequency response of the Wiener filter may be obtained by modifying Equation 3.91 by taking into account the spectral relationships

$$S_{xx}(\omega) = S_d(\omega) + S_\eta(\omega) \tag{3.99}$$

and

$$S_{xd}(\omega) = S_d(\omega), \tag{3.100}$$

which leads to

$$W(\omega) = \frac{S_d(\omega)}{S_d(\omega) + S_\eta(\omega)} = \frac{1}{1 + \frac{S_\eta(\omega)}{S_d(\omega)}}, \tag{3.101}$$