Equilibrium size distribution for terminally-molting (i.e. Tanner) crab

Population states (subscripted by size)

in– immature new shell crab (numbers-at-size vector)
io– immature old shell crab (numbers-at-size vector)
mn – mature new shell crab (numbers-at-size vector)
mo – mature old shell crab (numbers-at-size vector)

Population processes (subscripted by size)

S_1 – survival from start of year to time of molting/growth of immature crab (diagonal matrix)
S_2 – survival after time of molting/growth of immature crab to end of year (diagonal matrix)
Φ – probability of an immature crab molting (pr(molt|z), where z is pre-molt size; diagonal matrix)
Θ – probability that a molt is terminal (pr(molt to maturity|z, molt), where z is pre-molt size; diagonal matrix)
T – size transition matrix (non-diagonal matrix)
1 – identity matrix
R –number of recruits by size (vector)

In the following, the above (except for the identity matrix and *R*) are subscripted by population state (*in*, *io*, *mn*, *mo*) for generality. In particular, I wonder if survival of immature crab differs between those that molted and those that skipped.

Dynamics

$$in^{+} = R + S_{2in} \cdot \{T_{in} \cdot (1 - \Theta_{in}) \cdot \Phi_{in} \cdot S_{1in} \cdot in + T_{io} \cdot (1 - \Theta_{io}) \cdot \Phi_{io} \cdot S_{1io} \cdot io\} \quad (1)$$
$$io^{+} = S_{2io} \cdot \{(1 - \Phi_{in}) \cdot S_{1in} \cdot in + (1 - \Phi_{io}) \cdot S_{1io} \cdot io\} \quad (2)$$
$$mn^{+} = S_{2mn} \cdot \{T_{in} \cdot \Theta_{in} \cdot \Phi_{in} \cdot S_{1in} \cdot in + T_{io} \cdot \Theta_{io} \cdot \Phi_{io} \cdot S_{1io} \cdot io\} \quad (3)$$
$$mo^{+} = S_{2mo} \cdot \{S_{1mn} \cdot mn + S_{1mo} \cdot mo\} \quad (4)$$

R above is numbers recruiting-at-size vector (all of which are assumed immature, new shell), "+" indicates year+1.

Equilibrium equations

$$in = R + S_{2in} \cdot \{T_{in} \cdot (1 - \Theta_{in}) \cdot \Phi_{in} \cdot S_{1in} \cdot in + T_{io} \cdot (1 - \Theta_{io}) \cdot \Phi_{io} \cdot S_{1io} \cdot io\} \quad (6)$$
$$io = S_{2io} \cdot \{(1 - \Phi_{in}) \cdot S_{1in} \cdot in + (1 - \Phi_{io}) \cdot S_{1io} \cdot io\} \quad (7)$$
$$mn = S_{2mn} \cdot \{T_{in} \cdot \Theta_{in} \cdot \Phi_{in} \cdot S_{1in} \cdot in + T_{io} \cdot \Theta_{io} \cdot \Phi_{io} \cdot S_{1io} \cdot io\} \quad (8)$$
$$mo = S_{2mo} \cdot \{S_{1mn} \cdot mn + S_{1mo} \cdot mo\} \quad (9)$$

R above is equilibrium number of recruits-at-size vector

Equilibrium solution

Rewrite the above equilibrium equations as:

$$in = R + A \cdot in + B \cdot io \quad (10)$$
$$io = C \cdot in + D \cdot io \quad (11)$$
$$mn = E \cdot in + F \cdot io \quad (12)$$
$$mo = G \cdot mn + H \cdot mo \quad (13)$$

where A, B, C, D, E, F, G, and H are square matrices.

Solving for *io* in terms of *in* in eq. 11, one obtains

$$io = \{1 - D\}^{-1} \cdot C \cdot in \tag{14}$$

Plugging 14 into 10 and solving for *in* yields

$$in = \{1 - A - B \cdot [1 - D]^{-1} \cdot C\}^{-1} \cdot R \tag{15}$$

Equations 14 for *io* and 15 for *in* can simply be plugged into eq. 12 to yield *mn* while eq. 13 can then be solved for *mo*, yielding

$$mo = \{1 - H\}^{-1} \cdot G \cdot mn \tag{16}$$

where (for completeness):

$$A = S_{2in} \cdot T_{in} \cdot (1 - \Theta_{in}) \cdot \Phi_{in} \cdot S_{1in} \tag{17}$$

$$B = S_{2in} \cdot T_{io} \cdot (1 - \Theta_{io}) \cdot \Phi_{io} \cdot S_{1io} \tag{18}$$

$$C = S_{2io} \cdot (1 - \Phi_{in}) \cdot S_{1in} \tag{19}$$

$$D = S_{2io} \cdot (1 - \Phi_{io}) \cdot S_{1io} \tag{20}$$

$$E = S_{2mn} \cdot T_{in} \cdot \Theta_{in} \cdot \Phi_{in} \cdot S_{1in} \tag{21}$$

$$F = S_{2mn} \cdot T_{io} \cdot \Theta_{io} \cdot \Phi_{io} \cdot S_{1io} \tag{22}$$

$$G = S_{2mo} \cdot S_{1mn} \tag{23}$$

$$H = S_{2mo} \cdot S_{1mo} \tag{24}$$