Questions: - it is not obvious why angular momentum at 1 and 2 is $r_1 v_1 = \underline{b} v_2$
how was b derived?
- will we be able to convert all first order diff eq.s we encounter into difference equations?

$$F_g = \frac{GM_S M_E}{r^2} \Rightarrow \frac{d^2x}{dt^2} = \frac{F_{G,x}}{M_E} + \frac{d^2y}{dt^2} = \frac{F_{G,y}}{M_E}$$

method of least squares - type of data fit for noisy data

$$= F_{G,x} = -\frac{GM_S M_E}{r^2}\cos\theta = \boxed{-\frac{GM_S M_E x}{r^3}}$$

towards sun

now write 2nd order as 1st order

$$\frac{dv_x}{dt} = -\frac{GM_S x}{r^3} \qquad \frac{dv_y}{dt} = -\frac{GM_S y}{r^3}$$

$$\frac{dx}{dt} = v_x \qquad \frac{dy}{dt} = v_y$$

For circular motion $\frac{M_E r^2}{r} = F_G = \frac{GM_S M_E}{r^2}$ so $GM_S = r^2 r = 4\pi^2 AU^3/yrs^2$

now convert first order diff eq into difference equations

- Keplers laws all planets move in elliptical orbits with the Sun at the focus
- The line joining a planet to the Sun sweeps out equal areas in equal areas in equal times
- If T is the period and a the semimajor axis of orbit then T^2/a^3 (yr^2/AU^3) is constant for all planets

Computational solution (difference equations)

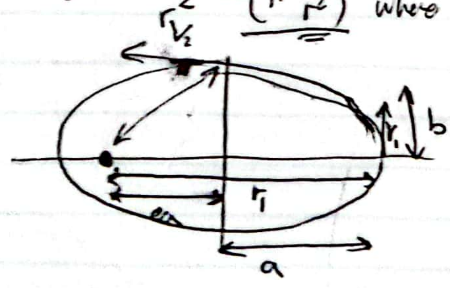
$$v_{x\,i+1} = v_{x_i} - \frac{4\pi^2 x_i}{r_i^3}\Delta t \qquad v_{y\,i+1} = v_{y_i} - \frac{4\pi^2 y_i}{r_i^3}\Delta t$$

$$x_{i+1} = x_i + (v_{i+1})\Delta t \qquad y_{i+1} = y_i + v_{y_{i+1}}\Delta t$$

Precession of the perihelion of Mercury
- the perihelion of Mercury ~~changes~~ rotates (perihelion is the point nearest the Sun for a planets orbit)
- General relativity explains this strange behavior in precession when the distance between planets is "small" enough

The force law for general-relativistic effects on gravity is

$$F_G \approx \frac{GM_S M_M}{r^2}\left(1 + \frac{\alpha}{r^2}\right)$$ where M_M is mass of mercury and $\alpha = 1.1E{-8}\,AU^2$



conservation of energy states

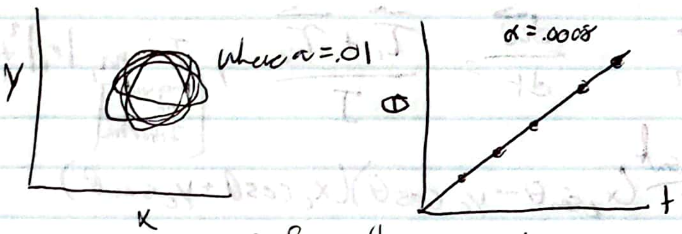
$$-\frac{GM_S M_M}{r_1} + \frac{1}{2}M_M v_1^2 = -\frac{GM_S M_M}{r^2} + \frac{1}{2}M_M v_2^2$$

$$r_1 v_1 = b v_2$$

Questions: ~~I understand why is the equation~~ $\cancel{\frac{d\omega}{dt} = \frac{\vec{T}_1 + \vec{\tau}_2}{I}}$
~~do the units match?~~

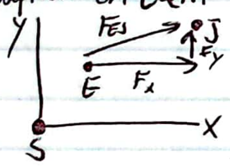
$$r_1 = \sqrt{2GM_S\left[\frac{b^2}{a^2(1+e)^2 - b^2}\right]\left[\frac{1}{\sqrt{e^2a^2+b^2}} - \frac{1}{a+ea}\right]} = \sqrt{\frac{GM_S(1-e)}{a(1+e)}}.$$

where $b = a\sqrt{1-e^2}$ next calculate the angle of precession ~~at~~ as a function of time
the plots for this should resemble



Three body problem and Effect of Jupiter on Earth

- Include $F_{ES} \Rightarrow \frac{GM_J M_E}{r^2_{EJ}}$



$r_e(i) = \sqrt{x_e(i)^2 + y_e(i)^2}$

$r_j(i) = \sqrt{x_j(i)^2 + y_j(i)^2}$

$r_{EJ} = \sqrt{[x_e(i) - x_j(i)]^2 + [y_e(i) - y_j(i)]^2}$

$v_{ex}(i+1) = v_{ex} - a_{ES} - a_{EJ}$

$v_{Jx}(i+1) = v_{Jx} - a_{ES} - a_{EJ}$

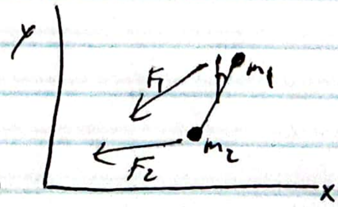
F_{EJ} in terms of x & y components is...

$$F_{EJ_x} = -\frac{GM_J M_E}{r^2_{EJ}}\cos\theta_{EJ} = -\frac{GM_J M_E(x_e - x_j)}{r^2_{EJ}}$$

$$\Sigma F_x = -\frac{GM_S x_e}{r^3} - \frac{GM_J(x_e - x_j)}{r^3_{EJ}}$$

Resonances in solar system: kirkwood gaps & planetary rings
Kirkwood gaps are locations that contain no asteroids for a given orbital radius
- placing an asteroid in a kirkwood gap means it is in resonance with Jupiter meaning the orbit will be some integer multiple of Jupiter's orbit like 2x or $\frac{1}{2}$x orbit for every orbit Jupiter makes
- for an object in resonance with Jupiter the orbit will be affected by Jupiter significantly, creating a "smear"

Chaotic tumbling of Hyperion



- Hyperion has egg shaped orbit
θ = angle of rod w/ ~~related axis~~ x axis
$\omega = \frac{d\theta}{dt}$

$$\vec{F}_1 = -\frac{GM_{sat} m_1}{r_1^3}(x_1\hat{i} + y_1\hat{j})$$ M_{sat} = mass saturn

Questions:

r_1 = distance from Saturn to m_1
center of mass coordinates (x_c, y_c)
Vector from center of mass to m_1: $(x_1 - x_c)\hat{i} + (y_1 - y_c)\hat{j}$

$\tau_1 = [(x_1 - x_c)\hat{i} + (y_1 - y_c)\hat{j}] \times \vec{F}_1$ = Torque on m_1

$\vec{\tau}_{total} = \vec{\tau}_1 + \vec{\tau}_2$ $\quad \frac{d\vec{\omega}}{dt} = \frac{\vec{\tau}_1 + \vec{\tau}_2}{I}$, $I = m_1 |r_1|^2 + m_2 |r_2|^2$ [moment of Inertia]

$$\frac{d\omega}{dt} \approx -\frac{3GM_{sat}}{r_c^5}(x_c \sin\theta - y_c \cos\theta)(x_c \cos\theta + y_c \sin\theta)$$