Fourier Transform

-can represent any function using the sum of sine and cosine waves

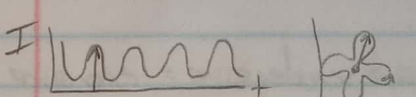
frequency (f, ν) = $\frac{cyc}{sec}$

Period (T) = seconds/cyc

$f = \frac{1}{T}$, $T = \frac{1}{f}$

to visualize a fourier transform

- Imagine vector representing amplitude graph of $I(t)$ vs t, imagine it is moving along (intensity)
- as it moves linearly, imagine in our new transformed circular plot it is rotating at a constant rate so that 2 seconds is representative of 1 rotation



3 beats per second

winding frequency = .5 cycl/sec ← variable

when winding frequency = f of wave

this almost Fourier transform allows decomposition of summed cosine/sine waves because in adjusting winding frequency you will get a spike for the original frequencies of the waves

$g(t) \rightarrow \hat{g}(f)$ where the domain of the new function is frequency. the output will be a complex number that corresponds to the strength of the original frequency in the original signal

Formula

why use complex numbers on a complex plane

$e^{2\pi i t}$ where t is the amount of time that has passed or $e^{2\pi i f t}$ for a different frequency e^{it} is inherently related to sine and cosine via Euler's theory

$e^{-2\pi i t}$ makes CW rotation now take $f(t)e^{-2\pi i f t}$ and you have the transformed function so $\hat{g}(f) = \int_{-\infty}^{\infty} g(t)e^{-2\pi i f t}\,dt$

the reason you we interpret this as a "center of mass" for a basic understanding is that we can write $g(t)$ as $\hat{g}(f)$ using the average of the points on the plot/total points $\frac{1}{N}\sum_{k=1}^{N} g(t_k)e^{-2\pi i f t_k}$ the approximation becomes more accurate for more points

we also typically analyse a finite time so $\left(\frac{1}{t_2 - t_1}\right)\int_{t_1}^{t_2} g(t)e^{-2\pi f i t}\,dt$

but with a regular Fourier integral if the signal persists then the frequency of $\hat{g}(f)$ is just increasingly scaled