CSC 261
Lab 5: Propositional Logic

Due: 2/28/18

Problem 3: The solutions of Part 3

(a) The propositional knowledge base represented by unicorn-kb is:

$$Mythical \implies \neg Mortal \tag{1}$$
$$\neg Mythical \implies (Mortal \wedge Mammal) \tag{2}$$
$$(\neg Mortal \vee Mammal) \implies Horned \tag{3}$$
$$Horned \implies Magical \tag{4}$$

To rewrite the propositional knowledge base represented by unicorn-kb in CNF, we should apply bidirectional elimination. Since there is no bidirectional implication in the kb, we should then eliminate implications and get:

$$\neg Mythical \vee \neg Mortal \tag{5}$$
$$\neg(\neg Mythical) \vee (Mortal \wedge Mammal) \tag{6}$$
$$\neg(\neg Mortal \vee Mammal) \vee Horned \tag{7}$$
$$\neg Magical \vee Horned \tag{8}$$

Next, we should move negation to literals and get:

$$\neg Mythical \vee \neg Mortal \tag{9}$$
$$Mythical \vee (Mortal \wedge Mammal) \tag{10}$$
$$(Mortal \wedge \neg Mammal) \vee Horned \tag{11}$$
$$\neg Horned \vee Magical \tag{12}$$

At last, we would distribute disjunction over conjuction:

$$\neg Mythical \vee \neg Mortal \tag{13}$$
$$(Mythical \vee Mortal) \wedge (Mythical \vee Mammal) \tag{14}$$
$$(Mortal \vee Horned) \wedge (\neg Mammal \vee Horned) \tag{15}$$
$$\neg Horned \vee Magical \tag{16}$$

We now have CNF that : $(\neg Mythical \vee \neg Mortal) \wedge (Mythical \vee Mortal) \wedge (Mythical \vee Mammal) \wedge (Mortal \vee Horned) \wedge (\neg Mammal \vee Horned) \wedge (\neg Horned \vee Magical)$

(b) Suppose the α that we want to prove is Horned
To show that $KB| = \alpha$, we show that $(KB \wedge \neg\alpha)$ is unsatisfiable, so we have an assumption: $\neg Horned$. We do this by proving a contradiction.
Now the Knowledge base that we have:

$$\neg Mythical \vee \neg Mortal \quad (17)$$
$$Mythical \vee Mortal \quad (18)$$
$$Mythical \vee Mammal \quad (19)$$
$$Mortal \vee Horned \quad (20)$$
$$\neg Mammal \vee Horned \quad (21)$$
$$\neg Horned \vee Magical \quad (22)$$
$$\neg Horned \quad (23)$$

$$Resolve(23) and (20) : Mortal \quad (24)$$
$$Resolve(23) and (21) : \neg Mammal \quad (25)$$
$$Resolve(25) and (19) : Mythical \quad (26)$$
$$Resolve(17) and (24) : \neg Mythical \quad (27)$$

Since two clauses,(26) and (27) resolve to yield the empty clause, we can conclude that KB entails α; Thus, we prove that the unicorn is horned.