# Trajectory planning - basic applied algorithms

Waddah Ali

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**Abstract**

So far, we have four basic trajectory planning algorithms to research; trapezoidal velocity profile, S-curve (7 segments) profiles, cubic/ cuintic polynomial interpolation and online motion profiles with PID feed forward/ feedback.

## 1 Trapezoidal velocity profile

First we implement the equations for a single DoF, $j$ then we develop to account mltiple DoFs.

### 1.1 Scalar Trapezoidal Trajectory Planner

INPUTS:

$$j_0 - \text{initial position.}$$
$$j_d - \text{desired position}$$
$$v_{max} - \text{maximum velocity}$$
$$a_{max} - \text{maximum acceleration}$$

OUTPUTS:

$$s(t) - \text{current position}$$

PARAMETERS:

$$t_{acc} = \frac{v_{max}}{a_{max}};\ \text{acceleration time} \tag{1}$$
$$t_{dec} = \frac{v_{max}}{a_{max}}; \text{deceleration time} \tag{2}$$
$$t_{cr} = \frac{|\Delta j| - \Delta j_{acc} - \Delta j_{dec}}{v_{max}}; \text{cruise time} \tag{3}$$
$$t_{total} = t_{acc} + t_{dec} + t_{cr}; \text{total time} \tag{4}$$
$$\Delta j = j_d - j_0; \text{distance to go , DTG} \tag{5}$$
$$\Delta j_{acc} = \frac{1}{2} \cdot a_{max} \cdot t_{acc}^2; \text{accelerating distance to go} \tag{6}$$
$$\Delta j_{dec} = \frac{1}{2} \cdot a_{max} \cdot t_{dec}^2; \text{decelerating distance to go} \tag{7}$$
$$\delta t - \text{time step} \tag{8}$$

Three phases (segments): acceleration (acc), cruise and deceleration (dec):

1. Acceleration phase $t \leq t_{acc}$:

$$\dot{s}(t) = a_{max} \cdot t \tag{9}$$
$$s(t) = j_0 \pm \frac{1}{2} \cdot a_{max} \cdot t^2 \tag{10}$$

2. Constant velocity phase $t_{acc} t \leq t_{acc} + t_{cr}$:

$$\dot{s}(t) = v_{max} \tag{11}$$
$$t_1 = t - t_{acc} \tag{12}$$
$$s(t) = j_0 \pm (\Delta j_{acc} + v_{max} \cdot t_1) \tag{13}$$

ps: $\pm$ refers to the direction of the motion i.e., the sign of the distance to go $\Delta j = j_d - j_0$

3. Deceleration phase $t_{cr} t \leq t_{total}$:

$$t_2 = t - (t_{acc} + t_{cr}) \tag{14}$$
$$\dot{s} = v_{max} - a_{max} \cdot t_2 \tag{15}$$
$$s(t) = j_0 \pm \left( \Delta j_{acc} + v_{max} \cdot t_{cr} + v_{max} \cdot t_2 - \frac{1}{2} \cdot a_{max} \cdot t_2^2 \right) \tag{16}$$

4. Finally:

$$s(t) = j_d \tag{17}$$
$$\dot{s}(t) = 0.0 \tag{18}$$

## 2 Multi-Axis Trapezoidal Trajectory Planner

we need first to calculate the distance between the initial and target pos in joint space where we have $N$ DoF:

$$\Delta s = \sqrt{\sum_{i=0}^{N} (j_{id} - j_{i0})^2} \tag{19}$$

then for each joint/axis (DoF),

$$j_i(t) = j_{i0} + \frac{\Delta j_i}{\Delta s} \cdot s(t) \tag{20}$$

where $s(t)$ was calculated in the previous section under 3 phases.

Why This Is Important

- All axes move in sync — they start and stop at the same time.
- No need to plan separate acceleration/cruise/deceleration phases for each axis or DoF.
- Ensures motion follows a straight line in N-dimensional space (Cartesian path).

This is the same principle used in CNC, robotics, and motion control — sometimes called **scalar path parametrization**.