

Exercise 23.4.1(*)

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$$\begin{array}{c}
 \frac{X_1 \vdash \text{double} : \forall X_2. (X_2 \rightarrow X_2) \rightarrow X_2 \rightarrow X_2}{X_1 \vdash \text{double} [X_1 \rightarrow X_1] : T_2 \rightarrow T_1} \text{T-TAPP} \quad \frac{X_1 \vdash \text{double} : \forall X_3. (X_3 \rightarrow X_3) \rightarrow X_3 \rightarrow X_3}{X_1 \vdash \text{double} [X_1] : T_2} \text{T-TAPP} \\
 \frac{\quad}{X_1 \vdash \text{double} [X_1 \rightarrow X_1] (\text{double} [X_1]) : T_1} \text{T-APP} \\
 \frac{\quad}{\vdash \lambda X_1. \text{double} [X_1 \rightarrow X_1] (\text{double} [X_1]) : T} \text{T-TABS}
 \end{array}$$

$$\begin{array}{c}
 \frac{X_1 \vdash \text{double} : \forall X_2. (X_2 \rightarrow X_2) \rightarrow X_2 \rightarrow X_2}{X_1 \vdash \text{double} [X_1 \rightarrow X_1] : [X_2 \mapsto X_1 \rightarrow X_1] ((X_2 \rightarrow X_2) \rightarrow X_2 \rightarrow X_2)} \text{T-TAPP} \quad \frac{X_1 \vdash \text{double} : \forall X_3. (X_3 \rightarrow X_3) \rightarrow X_3 \rightarrow X_3}{X_1 \vdash \text{double} [X_1] : [X_3 \rightarrow X_1] ((X_3 \rightarrow X_3) \rightarrow X_3 \rightarrow X_3)} \text{T-TAPP} \\
 \frac{\quad}{X_1 \vdash \text{double} [X_1 \rightarrow X_1] : ((X_1 \rightarrow X_1) \rightarrow X_1 \rightarrow X_1) \rightarrow ((X_1 \rightarrow X_1) \rightarrow X_1 \rightarrow X_1)} \text{substitution} \quad \frac{\quad}{X_1 \vdash \text{double} [X_1] : (X_1 \rightarrow X_1) \rightarrow X_1 \rightarrow X_1} \text{substitution} \\
 \frac{\quad}{X_1 \vdash \text{double} [X_1 \rightarrow X_1] (\text{double} [X_1]) : (X_1 \rightarrow X_1) \rightarrow X_1 \rightarrow X_1} \text{T-APP} \\
 \frac{\quad}{\vdash \lambda X_1. \text{double} [X_1 \rightarrow X_1] (\text{double} [X_1]) : \forall X_1. (X_1 \rightarrow X_1) \rightarrow X_1 \rightarrow X_1} \text{T-TABS}
 \end{array}$$