Exercise 22.3.3(*)

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2019/11/11

$$\cfrac{\cfrac{x:T_{11}\in x:X,y:Y,z:Z}{x:X,y:Y,z:Z\ \vdash\ x:T_{11}\ |_{\chi_{11}}\ C_{11}}\text{CT-VAR}\quad\cfrac{z:T_{12}\in x:X,y:Y,z:Z}{x:X,y:Y,z:Z\ \vdash\ z:T_{12}\ |_{\chi_{12}}\ C_{12}}\text{CT-VAR}}{x:X,y:Y,z:Z\ \vdash\ (x\ z):T_1\ |_{\chi_1}\ C_1}\text{CT-APP}$$

$$\cfrac{\cfrac{y:T_{21}\in x:X,y:Y,z:Z}{x:X,y:Y,z:Z\ \vdash\ y:T_{21}\ |_{\chi_{21}}\ C_{21}}\text{CT-VAR}\quad\cfrac{z:T_{22}\in x:X,y:Y,z:Z}{x:X,y:Y,z:Z\ \vdash\ z:T_{22}\ |_{\chi_{22}}\ C_{22}}\text{CT-VAR}}{x:X,y:Y,z:Z\ \vdash\ (y\ z):T_2\ |_{\chi_2}\ C_2}\text{CT-APP}$$

$$\cfrac{\cfrac{\cfrac{x:X,y:Y,z:Z\ \vdash\ (x\ z)\ (y\ z):S\ |_{\chi}\ C}{x:X,y:Y\ \vdash\ \lambda z:Z.\ (x\ z)\ (y\ z):S\ |_{\chi}\ C}\text{CT-ABS}}{x:X\ \vdash\ \lambda y:Y.\ \lambda z:Z.\ (x\ z)\ (y\ z):S\ |_{\chi}\ C}\text{CT-ABS}}{\varnothing\ \vdash\ \lambda x:X.\ \lambda y:Y.\ \lambda z:Z.\ (x\ z)\ (y\ z):S\ |_{\chi}\ C}\text{CT-ABS}$$

$$\cfrac{\cfrac{x:X\in x:X,y:Y,z:Z}{x:X,y:Y,z:Z\ \vdash\ x:X\ |_{\varnothing}\ \{\}}\text{CT-VAR}\quad\cfrac{z:Z\in x:X,y:Y,z:Z}{x:X,y:Y,z:Z\ \vdash\ z:Z\ |_{\varnothing}\ \{\}}\text{CT-VAR}}{x:X,y:Y,z:Z\ \vdash\ (x\ z):T_1\ |_{\{T_1\}}\ \{X=Z\rightarrow T_1\}}\text{CT-APP}$$

$$\cfrac{\cfrac{y:Y\in x:X,y:Y,z:Z}{x:X,y:Y,z:Z\ \vdash\ y:Y\ |_{\varnothing}\ \{\}}\text{CT-VAR}\quad\cfrac{z:Z\in x:X,y:Y,z:Z}{x:X,y:Y,z:Z\ \vdash\ z:Z\ |_{\varnothing}\ \{\}}\text{CT-VAR}}{x:X,y:Y,z:Z\ \vdash\ (y\ z):T_2\ |_{\{T_2\}}\ \{Y=Z\rightarrow T_2\}}\text{CT-APP}$$

$$\cfrac{\cfrac{\cfrac{x:X,y:Y,z:Z\ \vdash\ (x\ z)\ (y\ z):S\ |_{\{T_1,T_2,S\}}\ \{X=Z\rightarrow T_1,Y=Z\rightarrow T_2,T_1=T_2\rightarrow S\}}{x:X,y:Y\ \vdash\ \lambda z:Z.\ (x\ z)\ (y\ z):Z\rightarrow S\ |_{\{T_1,T_2,S\}}\ \{X=Z\rightarrow T_1,Y=Z\rightarrow T_2,T_1=T_2\rightarrow S\}}\text{CT-ABS}}{x:X\ \vdash\ \lambda y:Y.\ \lambda z:Z.\ (x\ z)\ (y\ z):Y\rightarrow(Z\rightarrow S)\ |_{\{T_1,T_2,S\}}\ \{X=Z\rightarrow T_1,Y=Z\rightarrow T_2,T_1=T_2\rightarrow S\}}\text{CT-ABS}}{\varnothing\ \vdash\ \lambda x:X.\ \lambda y:Y.\ \lambda z:Z.\ (x\ z)\ (y\ z):X\rightarrow(Y\rightarrow(Z\rightarrow S))\ |_{\{T_1,T_2,S\}}\ \{X=Z\rightarrow T_1,Y=Z\rightarrow T_2,T_1=T_2\rightarrow S\}}\text{CT-ABS}$$