Exercise 23.4.2(*)

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$$\dfrac{\dfrac{\dfrac{\text{A}}{X, Y \ \vdash \ \lambda f : X \to Y.\ (fix\ (\lambda m : List\ X \to List\ Y.\ \lambda l : List\ X.\ if\ isnil\ [X]\ l\ then\ nil\ [Y]\ else\ cons\ [Y]\ (f\ (head\ [X]\ l))\ (m\ (tail\ [X]\ l)))) : T_2}\ \text{T-TABS}}{X \ \vdash \ \lambda Y.\ \lambda f : X \to Y.\ (fix\ (\lambda m : List\ X \to List\ Y.\ \lambda l : List\ X.\ if\ isnil\ [X]\ l\ then\ nil\ [Y]\ else\ cons\ [Y]\ (f\ (head\ [X]\ l))\ (m\ (tail\ [X]\ l)))) : \forall Y.\ T_2}\ \text{T-TABS}}{\vdash \ \lambda X.\ \lambda Y.\ \lambda f : X \to Y.\ (fix\ (\lambda m : List\ X \to List\ Y.\ \lambda l : List\ X.\ if\ isnil\ [X]\ l\ then\ nil\ [Y]\ else\ cons\ [Y]\ (f\ (head\ [X]\ l))\ (m\ (tail\ [X]\ l)))) : \forall X.\ T_1}\ \text{T-TABS}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{X_1 \ \vdash \ double : \forall X_2.\ (X_2 \to X_2) \to X_2 \to X_2}{X_1 \ \vdash \ double\ [X_1 \to X_1] : [X_2 \mapsto X_1 \to X_1]((X_2 \to X_2) \to X_2 \to X_2)}\ \text{T-TAPP}}{X_1 \ \vdash \ double\ [X_1 \to X_1] : ((X_1 \to X_1) \to X_1 \to X_1) \to ((X_1 \to X_1) \to X_1 \to X_1)}\ \text{substitution} \quad \dfrac{\dfrac{X_1 \ \vdash \ double : \forall X_3.\ (X_3 \to X_3) \to X_3 \to X_3}{X_1 \ \vdash \ double\ [X_1] : [X_3 \to X_1]((X_3 \to X_3) \to X_3 \to X_3)}\ \text{T-TAPP}}{X_1 \ \vdash \ double\ [X_1] : (X_1 \to X_1) \to X_1 \to X_1}\ \text{substitution}}{X_1 \ \vdash \ double\ [X_1 \to X_1]\ (double\ [X_1]) : (X_1 \to X_1) \to X_1 \to X_1}\ \text{T-APP}}{\vdash \ \lambda X_1.\ double\ [X_1 \to X_1]\ (double\ [X_1]) : \forall X_1.\ (X_1 \to X_1) \to X_1 \to X_1}\ \text{T-TABS}$$