

Abstract—

Index Terms—

I. NOTATIONS

The Transmitter energy arrival instants are marked by t_i 's with energy E_i^T while the receiver energy arrivals are marked by r_i 's with energy E_i^R for $i \in \{0, 1, \dots\}$. The receiver spends P amount of power to be *on* and no power when it is *off*. Hence each energy arrival E_i^R can be viewed as it adds $T_i^R = \frac{E_i^R}{P}$ amount of time for which the receiver can be *on*. The maximum amount of time for which the receiver (and hence the Transmitter) can be *on* till time ' t ' is given by function $T^R(\cdot)$. It can be easily seen that $T^R(t) = \sum_{i=0}^{r_i \leq t} T_i^R$. Similarly the maximum energy harvested at the transmitter till time ' t ' is given by function $E^T(t) = \sum_{i=0}^{t_i \leq t} E_i^T$. The rate of bits transmission with power ' p ', given by function $g(\cdot)$ is assumed to follow the following properties as proposed in [1]

$$P1) g(0) = 0 \text{ and } \lim_{x \rightarrow \infty} g(x) \rightarrow \infty. \quad (1)$$

$$P2) g(x) \text{ is concave in nature with } x. \quad (2)$$

$$P3) g(x) \text{ is increasing with } x. \quad (3)$$

$$P4) g(x)/x \text{ is monotonically decreasing with } x. \quad (4)$$

For convenience of presentation, we also follow the following convention : we use the notation $\stackrel{L1}{=}$ or $\stackrel{(1)}{=}$ or $\stackrel{P1}{=}$ or $\stackrel{T1}{=}$ to indicate that the equality " $=$ " follows from Lemma 1 / Equation (1) / Property 1 / Theorem 1 respectively (same for inequalities).

II. OPTIMAL OFFLINE ALGORITHM

Before describing and proving the optimal algorithm we state the following lemmas which would be useful in later proofs

Lemma 1. *The transmitted power in an optimal solution is non-decreasing with time whenever the receiver is on.*

Proof. We prove this by contradiction. The following two cases arise according to the receiver being *on* or *off*.

case - 1 : Assume that the transmit power is p_1 form time A to B and then p_2 for from B to C with $p_1 > p_2$ and the receiver is *on* throughout time A to C as shown in figure 1. In this case suppose we transmit at a power $p' = \frac{p_1(B-A) + p_2(C-B)}{C-A}$ then the number of bits transmitted would be more over the same time duration due

to concavity of $g(p)$ as shown below.

$$g(p_1) \frac{B-A}{C-A} + g(p_2) \frac{C-B}{C-A} \leq g\left(\frac{p_1(B-A) + p_2(C-B)}{C-A}\right) \quad (5)$$

$$\implies g(p')(C-A) \geq g(p_1)(B-A) + g(p_2)(C-B) \quad (6)$$

As we can transmit more number of bits during $C-A$ with power p' we could save total transmission time since we would have lesser number of bits left to transmit after time C . Hence this case cannot be optimal.

case - 2 : The receiver is *off* for certain duration (say from B to C) of time during A to D as shown in figure 1. The transmission power is p_1 from A to B and p_2 from C to D . Now consider the case where keeping everything else intact we put the receiver *off* from instant A to $A+C-B$ and keep transmission from $A+C-B$ to D . This would always be feasible from the receiver point as energy with the receiver can only be non-decreasing with time. This scenario now boils down to *case - 1* from time $A+C-B$ to D and hence cannot be optimal. \square

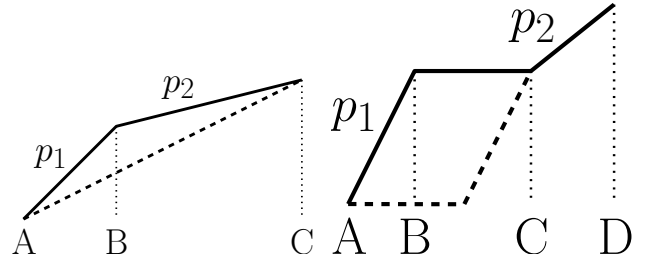


Fig. 1. Figure showing the two cases of Lemma 1, case 1[left] case2 [right] with $p_1 > p_2$

Lemma 2. *In an optimal solution once transmission has started the receiver is never off until transmission is complete.*

Proof. This is equivalent to saying there is no-breaks during transmission in optimal solution. We again prove this by contradiction. Keeping intact Lemma 1 the only case in which this can occur is the transmitter transmits with power p_1 from time A to B and then the receiver is *off* from B to C , again the transmitter is *on* with power p_2 from time C to D with $p_1 < p_2$ as shown in figure . Consider the case where we keep the receiver *off* from time A to $B' = A+C-B$. Now, a energy arrival can occur at the transmitter anywhere between A to D . If there is no energy arrival then transmitting at a constant rate from B' to D would transmit more number of bits.

case - 1 : If the energy arrival is between A and B' , then it can be easily seen that transmitting at a constant rate from B' to D would be better due to concavity of $g(p)$.

TABLE I
ON-LINE ALGORITHM FOR ENERGY HARVESTING TRANSMITTER
AND RECEIVER

Input: Bits to transmit B_0 ; E_i^T, E_i^R for $t_i, r_i < t$ where t is the present time instant

Initialize: $T_{start} = \min_{T^R(t)g\left(\frac{E^T(t)}{T^R(t)}\right) \geq B_0} t$
 $B_{left} = B_0, E_{left} = E^T(T_{start})$

While $B_{left} \geq 0$
 Transmit at power $\frac{E_{left}}{T}$ s.t. $Tg\left(\frac{E_{left}}{T}\right) = B_{left}$
 if $t = t_i$ for any i
 $B_{left} = B_{left} - (t - \max(t_{i-1}, T_{start}))g\left(\frac{E_{left}}{T}\right)$
 $E_{left} = E_{left} - (t - \max(t_{i-1}, T_{start}))\frac{E_{left}}{T}$
 end
end

case - 2 : If the arrival is between B' and C (say C'), then it can be easily seen that transmitting at a same rate p_1 from B' to C' and at a constant rate from C' to D would deliver more number of bits. (At worst case energy arrival occurring at C would make this scenario transmit equal number of bits as the original scenario).

case - 3 : If there is an energy arrival from C to D (say D'), then transmitting at a constant power from B' to D' and then at same rate p_2 from D' to D would fetch more number of bits at the receiver.

Applying the above scenarios iteratively we could shift the receiver *off* duration $C - B$ to the beginning of transmission and still at worst case transmit equal number of bits in same time duration. Hence having a break in-between transmission is always discouraged. We can also see that the optimal solution may not be unique. \square

Lemma 3. *In the optimal solution we consider transmit power can only change at energy arrival of transmitter once transmission has started.*

Proof. Keeping in mind Lemma 1 and 2 its proof becomes same as the one for Lemma 2, [1]. \square

III. ONLINE ALGORITHM FOR ENERGY HARVESTING TRANSMITTER AND RECEIVER

Notation: The starting time of the transmission is denoted by T_{start} and the present time is denoted by t . The number of bits and energy left to transmit at any Transmitter energy epoch is represented by B_{left} and E_{left} receptively. The on-line algorithm that we propose is presented in table I. The following lemma can be easily concluded from the definition of the on-line algorithm and hence stated without proof.

Lemma 4. *The transmit power in the online algorithm is non-decreasing with time after T_{start} .*

Theorem 1. *The competitive ratio of the on-line algorithm presented in Table I is 2.*

Proof. This is equivalent to saying that the time taken by the on-line algorithm can at max be twice the time taken by optimal off-line algorithm. Let the time taken by the off-line version be T_{off} and the on-line version be T_{online} .

We now show that

$$T_{off} \geq T_{start} \quad (7)$$

This proof follows from contradiction. Let $T_{off} < T_{start}$ and the optimal off-line algorithm transmits with energy in sequence $\{e_1, e_2, \dots, e_k\}$ for time $\{l_1, l_2, \dots, l_k\}$. Now the number of bits transmitted can be bounded as

$$\sum_{i=1}^{i=k} g\left(\frac{e_i}{l_i}\right) l_i \stackrel{P2}{\leq} g\left(\frac{\sum_{i=1}^{i=k} e_i}{\sum_{j=1}^{j=k} l_j}\right) \sum_{j=1}^{j=k} l_j \quad (8)$$

$$\stackrel{P3, P4}{\leq} g\left(\frac{E^T(T_{off})}{T_{off}}\right) T_{off} \quad (9)$$

$$\stackrel{P4}{\leq} \lim_{t \rightarrow T_{start}^-} g\left(\frac{E^T(t)}{t}\right) t < B_0 \quad (10)$$

where (10) follows from definition of T_{start} . But the off-line algorithm should transmit all B_0 bits and hence this concludes that $T_{off} \geq T_{start}$.

Next we estimate the maximum time taken to complete transmission after T_{start} in the on-line algorithm. Let the on-line version transmits at power sequence $\{p_1, p_2, \dots, p_k\}$ for time $\{l_1, l_2, \dots, l_k\}$. Now,

$$\sum_{i=1}^{i=k} l_i g(p_i) = B_0 \quad (11)$$

$$\stackrel{L4}{\Rightarrow} g(p_1) \sum_{i=1}^{i=k} l_i \leq B_0 \quad (12)$$

$$\Rightarrow g\left(\frac{E^T(T_{start})}{T}\right) \sum_{i=1}^{i=k} l_i \leq B_0 \quad \text{s.t. } Tg\left(\frac{E^T(T_{start})}{T}\right) = B_0 \quad (13)$$

$$\Rightarrow \sum_{i=1}^{i=k} l_i \leq T \quad (14)$$

But, $T \stackrel{P4}{\leq} T^R(T_{start})$ as $T_{start} g\left(\frac{E^T(T_{start})}{T_{start}}\right) \geq B_0 = Tg\left(\frac{E^T(T_{start})}{T}\right)$ and from the definition of $T^R(T_{start})$ it follows that $T^R(T_{start}) \leq T_{start}$. So we can calculate the competitive ratio as

$$r = \max \frac{T_{online}}{T_{off}} = \frac{T_{start} + \sum_{i=1}^{i=k} l_i}{T_{off}} \leq \frac{T_{start} + T}{T_{off}} \quad (15)$$

$$\leq \frac{2T_{start}}{T_{off}} \stackrel{(7)}{\leq} 2 \quad (16)$$

\square

REFERENCES

- [1] J. Yang and S. Ulukus, "Optimal packet scheduling in an energy harvesting communication system," *Communications, IEEE Transactions on*, vol. 60, no. 1, pp. 220–230, January 2012.