

Abstract

Index Terms

Lemma 1. *The transmitted power in a optimal solution in non-decreasing with time whenever the receiver is on.*

Proof. We prove this by contradiction. Assume that the transmit power is p_1 for some duration t_1 and then p_2 for some duration t_2 with $p_1 > p_2$. So this boils down to two cases-

case - 1 : The receiver is on throughout time t_1 to t_2 as shown in figure. In this case suppose we transmit at a power $p' = \frac{p_1 t_1 + p_2 t_2}{t_1 + t_2}$ then the number of bits transmitted would be more over the same time duration due to concavity of $g(p)$ as shown below.

$$g(p_1) \frac{t_1}{t_1 + t_2} + g(p_2) \frac{t_2}{t_1 + t_2} \leq g\left(\frac{p_1 t_1 + p_2 t_2}{t_1 + t_2}\right) \quad (1)$$

$$\implies g(p')(t_1 + t_2) \geq g(p_1)t_1 + g(p_2)t_2 \quad (2)$$

As we can transmit more number of bits during duration $t_1 + t_2$ we could save total transmission time since we would have lesser number of bits left to transmit. Hence this case cannot be optimal.

case - 2 : The receiver is *off* for certain duration (say t) of time during $t_1 + t_2$ as shown in figure. Now consider the case where keeping everything else intact we put the receiver *off* from instant A to $A + t$ and keep transmission from $A + t$ to $A + t_1 + t_2$. This would always be feasible from the receiver point as energy with the receiver can only be non-decreasing with time. This scenario now boils down to *case - 1* from time $A + t$ to $A + t_1 + t_2$ and hence cannot be optimal. \square

Lemma 2. *In a optimal solution once transmission has started the receiver is never off until transmission is complete.*

Proof. This is equivalent to saying there is no-breaks during transmission in optimal solution. We again prove this by contradiction. Keeping intact Lemma ?? the only case in which this can occur is the transmitter transmits with power p_1 from time A to B and then the receiver is *off* from B to C , again the transmitter is *on* with power p_2 from time C to D with $p_1 < p_2$ as shown in figure . Consider the case where we keep the receiver *on* for time $C - B$ from A . This makes the scenario as shown in figure . Now, a new energy arrival can occur at the transmitter anywhere between A to D .

case - 1 : If the arrival is between A and B' , then it can be easily seen that transmitting at a constant rate from B' to D would be better due to concavity of $g(p)$.

case - 2 : If the arrival is between B' and C (say C'), then it can be easily seen that transmitting at a same rate p_1 from B' to C' and at a constant rate from C' to D would deliver more number of bits. (At worst case energy arrival occurring at C would make this scenario transmit equal number of bits as the original scenario).

case - 3 : If there is an energy arrival from C to D (say D'), then transmitting at a constant power form B' to D' and at same rate p_2 would fetch more number of bits at the receiver.

Applying the above scenarios iteratively we could shift the receiver *off* duration $C - B$ to the beginning of transmission and still at worst case transmit equal number of bits in same time duration. Hence having a break in-between transmission is always discouraged. We can also see that the optimal solution may not be unique. \square

Lemma 3. *In the optimal solution we consider transmit power can only change at energy arrival of transmitter once transmission has started.*

Proof. Keeping in mind Lemma ?? and ?? its proof becomes same as the one for Lemma 2, []. \square