Abstract

Index Terms

Lemma 1. *The transmitted power in an optimal solution in non-decreasing with time whenever the receiver is on.*

Proof. We prove this by contradiction. Assume that the transmit power is p_1 for some duration t_1 and then p_2 for some duration t_2 with $p_1 > p_2$. So this boils down to two cases-

$case-1$: The receiver is on throughout time t_1 to t_2 as shown in figure. In this case suppose we transmit at a power $p' = \frac{p_1 t_1 + p_2 t_2}{t_1 + t_2}$ then the number of bits transmitted would be more over the same time duration due to concavity of $g(p)$ as shown below.

$$g(p_1)\frac{t_1}{t_1+t_2} + g(p_2)\frac{t_2}{t_1+t_2} \leq g(\frac{p_1 t_1 + p_2 t_2}{t_1+t_2}) \tag{1}$$

$$\implies g(p')(t_1+t_2) \geq g(p_1)t_1 + g(p_2)t_2 \tag{2}$$

As we can transmit more number of bits during duration $t_1 + t_2$ we could save total transmition time since we would have lesser number of bits left to transmit. Hence this case cannot be optimal.

$case-2$: The receiver is *off* for certain duration (say t) of time during $t_1 + t_2$ as shown in figure. Now consider the case where keeping everything else intact we put the receiver *off* from instant A to $A+t$ and keep transmition from $A+t$ to $A+t_1+t_2$. This would always be feasible from the receiver point as energy with the receiver can only be non-decreasing with time. This scenario now boils down to $case-1$ from time $A+t$ to $A+t_1+t_2$ and hence cannot be optimal. □

Lemma 2. *In an optimal solution once transmition has started the receiver is never off until transmition is complete.*

Proof. This is equivalent to saying there is no-breaks during transmition in optimal solution. We again prove this by contradiction. Keeping intact Lemma 1 the only case in which this can occur is the transmitter transmits with power p_1 from time A to B and then the receiver is *off* from B to C, again the transmitter is *on* with power p_2 from time C to D with $p_1 < p_2$ as shown in figure . Consider the case where we keep the receiver *on* for time $C-B$ from A. This makes the scenario as shown in figure . Now, a new energy arrival can occur at the transmitter anywhere between A to D.

$case-1$:If the arrival is between A and B', then it can be easily seen that transmitting at a constant rate from B' to D would be better due to concavity of $g(p)$.

$case-2$:If the arrival is between B' and C (say C'), then it can be easily seen that transmitting at a same rate p_1 from B' to C' and at a constant rate from C' to D would deliver more number of bits.(At worst case energy arrival occurring at C would make this scenario transmit equal number of bits as the original scenario).

$case-3$:If there is an energy arrival from C to D (say D'), then transmitting at a constant power form B' to D' and at same rate p_2 would fetch more number of bits at the receiver.

Applying the above scenarios iteratively we could shift the receiver *off* duration $C-B$ to the beginning of transmition and still at worst case transmit equal number of bits in same time duration. Hence having a break in-between transmition is always discouraged. We can also see that the optimal solution may not be unique. □

Lemma 3. *In the optimal solution we consider transmit power can only change at energy arrival of transmitter once transmission has started.*

Proof. Keeping in mind Lemma 1 and 2 its proof becomes same as the one for Lemma 2, [1]. □

REFERENCES

[1] J. Yang and S. Ulukus, "Optimal packet scheduling in an energy harvesting communication system," *Communications, IEEE Transactions on*, vol. 60, no. 1, pp. 220–230, January 2012.