***Abstract*—**

***Index Terms*—**

I. NOTATIONS

The Transmitter energy arrival instants are marked by t_i's with energy E_i^T while the receiver energy arrivals are marked by r_i's with energy E_i^R for $i \in \{0, 1..\}$. The receiver spends P amount of power to be *on* and no power when it is *off*. Hence each energy arrival E_i^R can be viewed as it adds $T_i^R = \dfrac{E_i^R}{P}$ amount of time for which the receiver can be *on*. The maximum amount of time for which the receiver (and hence the Transmitter) can be *on* till time '.' is given by function $T^R(.)$. It can be easily seen that $T^R(t) = \sum_{i=0}^{r_i<t} T_i^R$. Similarly the maximum energy harvested at the transmitter till time 't' is given by function $E^T(t) = \sum_{i=0}^{t_i<t} E_i^T$. The rate of bits transmition with power '.', given by function $g(.)$ is assumed to follow the following properties as proposed in [1]

$$P1) g(0) = 0 \text{ and } \lim_{x\to\infty} g(x) \to \infty. \quad (1)$$

$$P2) g(x) \text{ is concave in nature with } x. \quad (2)$$

$$P3) g(x) \text{ is increasing with } x. \quad (3)$$

$$P4) g(x)/(x) \text{ is monotonically decreasing with } x. \quad (4)$$

For convenience of presentation, we also follow the following convention : we use the notation $\overset{L1}{=}$ or $\overset{(1)}{=}$ or $\overset{P1}{=}$ or $\overset{T1}{=}$ to indicate that the equality "=" follows from Lemma 1 / Equation (1) / Property 1 / Theorem 1 respectively (same for inequalities).

II. OPTIMAL OFFLINE ALGORITHM

Before describing and proving the optimal algorithm we state the following lemmas which would be useful in later proofs

Lemma 1. *The transmitted power in an optimal solution in non-decreasing with time whenever the receiver is on.*

Proof. We prove this by contradiction. Assume that the transmit power is p_1 form time A to B and then p_2 for from B to C with $p_1 > p_2$. So this boils down to two cases-

$case - 1$: The receiver is on throughout time A to C as shown in figure. In this case suppose we transmit at a power $p' = \dfrac{p_1 t_1 + p_2 t_2}{t_1 + t_2}$ then the number of bits transmitted would be more over the same time duration due to concavity of $g(p)$ as shown below.

$$g(p_1)\frac{t_1}{t_1+t_2} + g(p_2)\frac{t_2}{t_1+t_2} \leq g\left(\frac{p_1 t_1 + p_2 t_2}{t_1+t_2}\right) \quad (5)$$

$$\implies g(p')(t_1+t_2) \geq g(p_1)t_1 + g(p_2)t_2 \quad (6)$$

As we can transmit more number of bits during duration $t_1 + t_2$ we could save total transmition time since we would have lesser number of bits left to transmit. Hence this case cannot be optimal.

$case - 2$: The receiver is *off* for certain duration (say t) of time during $t_1 + t_2$ as shown in figure. Now consider the case where keeping everything else intact we put the receiver *off* from instant A to $A + t$ and keep transmition from $A + t$ to $A + t_1 + t_2$. This would always be feasible from the receiver point as energy with the receiver can only be non-decreasing with time. This scenario now boils down to $case - 1$ from time $A + t$ to $A + t_1 + t_2$ and hence cannot be optimal. □

Lemma 2. *In an optimal solution once transmition has started the receiver is never off until transmition is complete.*

Proof. This is equivalent to saying there is no-breaks during transmition in optimal solution. We again prove this by contradiction. Keeping intact Lemma 1 the only case in which this can occur is the transmitter transmits with power p_1 from time A to B and then the receiver is *off* from B to C, again the transmitter is *on* with power p_2 from time C to D with $p_1 < p_2$ as shown in figure . Consider the case where we keep the receiver *on* for time $C - B$ from A. This makes the scenario as shown in figure . Now, a new energy arrival can occur at the transmitter anywhere between A to D.

$case - 1$:If the arrival is between A and B', then it can be easily seen that transmitting at a constant rate from B' to D would be better due to concavity of $g(p)$.

$case - 2$:If the arrival is between B' and C (say C'), then it can be easily seen that transmitting at a same rate p_1 from B' to C' and at a constant rate from C' to D would deliver more number of bits.(At worst case energy arrival occurring at C would make this scenario transmit equal number of bits as the original scenario).

$case - 3$:If there is an energy arrival from C to D (say D'), then transmitting at a constant power form B' to D' and at same rate p_2 would fetch more number of bits at the receiver.

Applying the above scenarios iteratively we could shift the receiver *off* duration $C - B$ to the beginning of transmition and still at worst case transmit equal number of bits in same time duration. Hence having a break in-between transmition is always discouraged. We can also see that the optimal solution may not be unique. □

Lemma 3. *In the optimal solution we consider transmit power can only change at energy arrival of transmitter once transmission has started.*

Proof. Keeping in mind Lemma 1 and 2 its proof becomes same as the one for Lemma 2, [1]. □

TABLE I
ON-LINE ALGORITHM FOR ENERGY HARVESTING TRANSMITTER AND RECEIVER

Input: Bits to transmit B_0; E_i^T, E_i^R for $t_i, r_i < t$ where t is the present time instant

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Initialize: $T_{start} = \min\limits_{T^R(t) g\left(\frac{E^T(t)}{T^R(t)}\right) \geq B_0} t$

$B_{left} = B_0$, $E_{left} = E^T(T_{start})$

While $B_{left} \geq 0$

Transmit at power $\frac{E_{left}}{T}$ s.t. $Tg\Big(\frac{E_{left}}{T}\Big) = B_{left}$

if $t = t_i$ for any i

$B_{left} = B_{left} - (t - \max(t_{i-1}, T_{start})) g\Big(\frac{E_{left}}{T}\Big)$

$E_{left} = E_{left} - (t - \max(t_{i-1}, T_{start})) \frac{E_{left}}{T}$

end

end

III. ONLINE ALGORITHM FOR ENERGY HARVESTING TRANSMITTER AND RECEIVER

Notation: The starting time of the transmission is denoted by T_{start} and the present time is denoted by t. The number of bits and energy left to transmit at any Transmitter energy epoch is represented by B_{left} and E_{left} receptively. The on-line algorithm that we propose is presented in table I. The following lemma can be easily concluded from the definition of the on-line algorithm and hence stated without proof.

Lemma 4. *The transmit power in the online algorithm is non-decreasing with time after T_{start}.*

Theorem 1. *The competitive ratio of the on-line algorithm presented in Table I is 2.*

Proof. This is equivalent to saying that the time taken by the on-line algorithm can at max be twice the time taken by optimal off-line algorithm. Let the time taken by the off-line version be T_{off} and the on-line version be T_{online}.

We now show that

$$T_{off} \geq T_{start} \tag{7}$$

This proof follows from contradiction. Let $T_{off} < T_{start}$ and the optimal off-line algorithm transmits with energy in sequence $\{e_1, e_2, .., e_k\}$ for time $\{l_1, l_2.., l_k\}$. Now the number of bits transmitted can be bounded as

$$\sum_{i=1}^{i=k} g\Big(\frac{e_i}{l_i}\Big) l_i \overset{P2}{\leq} g\Big(\frac{\sum_{i=1}^{i=k} e_i}{\sum_{j=1}^{j=k} l_j}\Big) \sum_{j=1}^{j=k} l_j \tag{8}$$

$$\overset{P3,P4}{\leq} g\Big(\frac{E^T(T_{off})}{T_{off}}\Big) T_{off} \tag{9}$$

$$\overset{P4}{\leq} \lim_{t \to T_{start}^-} g\Big(\frac{E^T(t)}{t}\Big) t < B_0 \tag{10}$$

where (10) follows form definition of T_{start}. But the off-line algorithm should transmit all B_0 bits and hence this concludes that $T_{off} \geq T_{start}$.

Next we estimate the maximum time taken to complete transmission after T_{start} in the on-line algorithm. Let the on-line version transmits at power sequence $\{p_1, p_2, .., p_k\}$ for time $\{l_1, l_2.., l_k\}$. Now,

$$\sum_{i=1}^{i=k} l_i g(p_i) = B_0 \tag{11}$$

$$\overset{L4}{\Longrightarrow} g(p_1) \sum_{i=1}^{i=k} l_i \leq B_0 \tag{12}$$

$$\Longrightarrow g\Big(\frac{E^T(T_{start})}{T}\Big) \sum_{i=1}^{i=k} l_i \leq B_0 \quad \text{s.t. } Tg\Big(\frac{E^T(T_{start})}{T}\Big) = B_0 \tag{13}$$

$$\Longrightarrow \sum_{i=1}^{i=k} l_i \leq T \tag{14}$$

But, $T \overset{P4}{\leq} T^R(T_{start})$ as $T_{start} g(\frac{E^T(T_{start})}{T_{start}}) \geq B_0 = Tg(\frac{E^T(T_{start})}{T})$ and from the definition of $T^R(T_{start})$ it follows that $T^R(T_{start}) \leq T_{start}$. So we can calculate the competitive ratio as

$$r = \max \frac{T_{online}}{T_{off}} = \frac{T_{start} + \sum_{i=1}^{i=k} l_i}{T_{off}} \leq \frac{T_{start} + T}{T_{off}} \tag{15}$$

$$\leq \frac{2T_{start}}{T_{off}} \overset{(7)}{\leq} 2 \tag{16}$$

$\square$

REFERENCES

[1] J. Yang and S. Ulukus, “Optimal packet scheduling in an energy harvesting communication system,” *Communications, IEEE Transactions on*, vol. 60, no. 1, pp. 220–230, January 2012.