HOMEWORK 3 (FOR THE 4-TH WEEK)

Armstrong's book Chapter 3, Exercise 21, 25, 26, 30, 32, 33, 34, 35, 37, 39, 40 together with the following exercise. Note that in exercise 25, a map means a continuous map. In exercise 26, an open (closed) map means it maps open (closed) sets to open (closed) sets.

Exercise 1. *In the class, we proved that a subset in $\mathbb{R}^n$ is compact if and only if it is closed and bounded.*

(1) *Give an example to show that a closed and bounded subset in a metric space is not necessarily compact in general.*
(2) *Let X be a metric space and let $\epsilon > 0$. A subset $E \subset X$ is called an $\epsilon-$**grid** if for any $x \in X$ there exists $e \in E$ such that $d(e,x) < \epsilon$. A metric space is called **totally bounded** if for any $\epsilon > 0$ there exists a finite $\epsilon-$grid. Prove that a subset in a complete metric space is compact if and only if it is closed and totally bounded.*