

Lec 11 p12

# Fisher's Approach

## Result 11.3

The linear combination  $\hat{y} = \hat{a}'x = (\bar{x}_1 - \bar{x}_2)'S_{pooled}^{-1}x$  maximizes the ratio

$$\max_a \frac{(\bar{y}_1 - \bar{y}_2)^2}{s_y^2} = \max_a \frac{(a'\bar{x}_1 - a'\bar{x}_2)^2}{a'S_{pooled}a} = \max_a \frac{(a'd)^2}{a'S_{pooled}a}$$

over all possible coefficient vectors  $a$ , where  $d = \bar{x}_1 - \bar{x}_2$ .

The maximum of this ratio is  $D^2 = (\bar{x}_1 - \bar{x}_2)'S_{pooled}^{-1}(\bar{x}_1 - \bar{x}_2)$ .

Linear discriminant  
analysis (LDA)

Q: What is the intuition of separation?  
Does the  $D^2$  looks familiar?

Lec 12 p70

# Model Selection

- Any model selection criterion (AIC, likelihood ratio, BIC) can be used to select the best fitting model.

- Mclust uses the **Bayesian Information Criterion** (BIC) to choose the best model

$$BIC = -2\log(L) + \log(n)m$$

where L is the likelihood function, m is the number of free parameters to be estimated and n is the number of observations. A model with low BIC fits the data better than one with high BIC.

- **Akaike information criterion** (AIC)

$$AIC = -2\log(L) + 2m$$

- Note that the formulas in textbook and Mclust use reversed sign. So the higher the better.