Game T&A

Game Theory and its Applications

Part VII: Behavioral Game Theory

行为博弈论简介

清 华 大 学 数 学 科 学 系 谢金星

办公室：理科楼A202# 电话：62787812

E-mail: xiejx@tsinghua.edu.cn

Game T&A

Behavioral Game Theory (BGT) and Game Practice

- Game theory: how rational individuals should behave
- Who are these rational individuals?
- BGT: looks at how people actually behave
  - experiment by setting up real economic situations
  - account for people's economic decisions
  - don't break game theory when it works
- Fit a model to observations, not "rationality"

Camerer, Colin F., *Behavioral Game Theory: Experiments on Strategic Interaction* (Princeton, NJ: Princeton University Press, 2003).

Game T&A

Outline

- Motivation
- Mutual Consistency: CH Model
- Noisy Best-Response: QRE Model
- Instant Convergence: EWA Learning

Acknowledgement:

This note is basically based on the notes from Prof. Tech H. Ho (UC, Berkeley).

Game T&A

Dual Pillars of Economic Analysis & Game Theory

- Specification of Utility
  - Only final allocation matters
  - "Self-interest"
  - Exponential discounting
- Solution Method
  - Nash equilibrium and its refinements (instant equilibration)

Game T&A

Motivation: Utility Specification

- Reference point matters: People care both about the final allocation as well as the changes with respect to a target level
- Fairness: John cares about Mary's payoff. In addition, the marginal utility of John with respect to an increase in Mary's income increases when Mary is kind to John and decreases when Mary is unkind
- Hyperbolic discounting: People are impatient and prefer instant gratification

Game T&A

Motivation: Solution Method

- Nash equilibrium and its refinements: Dominant theories in marketing for predicting behaviors in non-cooperative games.
- Subjects do not play Nash in many one-shot games.
- Behaviors do not converge to Nash with repeated interactions in some games.
- Multiplicity problem (e.g., coordination games).
- Anything go in infinitely repeated games.
- Modeling subject **heterogeneity** really matters in games.

Game T&A

Bounded Rationality in Markets: Revised Utility Function

Behavioral Regularities	Standard Assumption	New Model Specification Reference Example	Marketing Application Example
1. Revised Utility Function			
- Reference point and loss aversion	- Expected Utility Theory	- Prospect Theory Kahneman and Tversky (1979)	- Nonlinear pricing-double marginalization problem
- Fairness	- Self-interested	- Inequality aversion Fehr and Schmidt (1999)	- Price discrimination
- Impatience	- Exponential discounting	- Hyperbolic Discounting Ainslie (1975)	- Price promotion and packaging size design

Ho, Lim, and Camerer (JMR, 2006)

7

Game T&A

Bounded Rationality in Markets: Alternative Solution Methods

Behavioral Regularities	Standard Assumption	New Model Specification Example	Marketing Application Example
2. Bounded Computation Ability			
- Nosiy Best Response	- Best Response	- Quantal Best Response McKelvey and Palfrey (1995)	- NEIO
- Limited Thinking Steps	- Rational expectation	- Cognitive hierarchy Camerer, Ho, Chong (2004)	- Market entry competition
- Myopic and learn	- Instant equilibration	- Experience weighted attraction Camerer and Ho (1999)	- Lowest price guarantee competition

8

Game T&A

Standard Assumptions in Equilibrium Analysis

Assumptions	Nash Equilibrum	Cognitive Hierarchy	QRE	EWA Learning
Solution Method				
Strategic Thinking	X	X	X	X
Best Response	X	X		X
Mutual Consistency	X		X	
Instant Convergence	X	X	X	

9

Game T&A

Modeling Philosophy

Simple	(Economics)
General	(Economics)
Precise	(Economics)
Empirically disciplined	(Psychology)

"the empirical background of economic science is definitely inadequate...it would have been absurd in physics to expect Kepler and Newton without Tycho Brahe" (von Neumann & Morgenstern '44)

"Without having a broad set of facts on which to theorize, there is a certain danger of spending too much time on models that are mathematically elegant, yet have little connection to actual behavior. At present our empirical knowledge is inadequate..." (Eric Van Damme '95)

10

Game T&A

Example: ultimatum games

Feeling in ultimatum games: How much do you offer out of $10?

- Proposer has $10
- Offers x to Responder (keeps $10-x)
- What should the Responder do?
  - Self-interest: Take any x>0
  - Empirical: Reject x=$2 half the time

11

Game T&A

How People Ultimatum-Bargain

Thousands of games have been played in experiments…

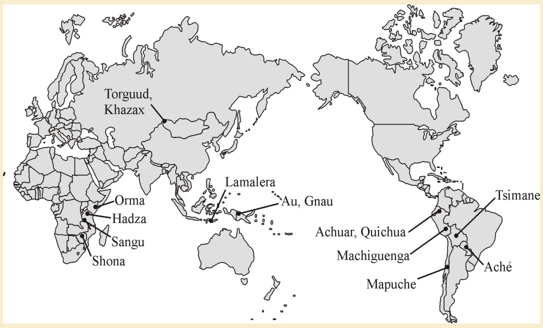
- In different cultures around the world
- With different stakes
- With different mixes of men and women
- By students of different majors
- Etc. etc. etc.

Pretty much always, two things prove true:

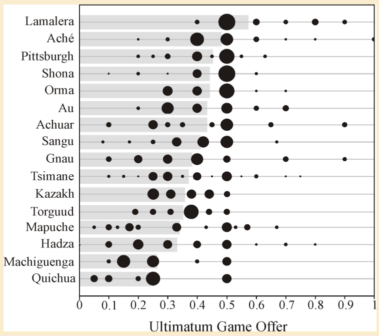
1. Player 1 offers close to, but less than, half (40% or so)
2. Player 2 rejects low offers (20% or less)

12

Ultimatum offer experimental sites



Ultimatum offers across societies (mean shaded, mode is largest circle…)



Outline

- Motivation
- Mutual Consistency: CH Model
- Noisy Best-Response: QRE Model
- Instant Convergence: EWA Learning

Standard Assumptions in Equilibrium Analysis

Assumptions	Nash Equilibrum	Cognitive Hierarchy	QRE	EWA Learning
Solution Method				
Strategic Thinking	X	X	X	X
Best Response	X	X		X
Mutual Consistency	X		X	
Instant Convergence	X	X	X	

Example A: Exercise

❑ Consider matching pennies games in which the row player chooses between *Top* and *Bottom* and the column player simultaneously chooses between *Left* and *Right*, as shown below:

$p_T=1/2$

G0

	Left	Right
Top	80,40	40,80
Bottom	40,80	80,40

G1

	Left	Right
Top	320,40	40,80
Bottom	40,80	80,40

G2

	Left	Right
Top	44,40	40,80
Bottom	40,80	80,40

$a>40$ $q_L=40/a$

Example A: Data

Table 1. Three One-Shot Matching Pennies Games (with choice percentages)

Game		*Left* (48%)	*Right* (52%)
Symmetric Matching Pennies	*Top* **(48%)**	80, 40	40, 80
	Bottom **(52%)**	40, 80	80, 40

Game		*Left* (16%)	*Right* (84%)
Asymmetric Matching Pennies	*Top* **(96%)**	320, 40	40, 80
	Bottom **(4%)**	40, 80	80, 40

Game		*Left* (80%)	*Right* (20%)
Reversed Asymmetry	*Top* **(8%)**	44, 40	40, 80
	Bottom **(92%)**	40, 80	80, 40

Example B: Exercise

- The two players choose "effort" levels simultaneously, and the payoff of each player is given by $\pi_i = min\ (e_1, e_2) - ce_i$
- Efforts are integer from 110 to 170.
- c = 0.1 or 0.9.
- NE: $e_1 = e_2 = any\ value\ from\ 110\ to\ 170$

Example B: Data

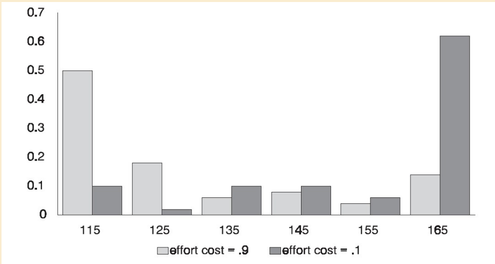


Figure 2. Effort Choice Frequencies for a Minimum Effort Coordination Game With High Effort Cost (Light Bars) and Low Effort Cost (Dark Bars)

Motivation: CH - Cognitive Hierarchy

- Model heterogeneity explicitly (people are not equally smart)
- Introduce the word surprise into the game theory's dictionary
- Generate new predictions (reconcile various treatment effects in lab data not predicted by standard theory)

Camerer, Ho, and Chong (QJE, 2004)

Example 1: "zero-sum game"

			COLUMN	
		L	C	R
	T	0,0	10,-10	-5,5
ROW	M	-15,15	15,-15	25,-25
	B	5,-5	-10,10	0,0

Messick(1965), Behavioral Science

Nash Prediction: "zero-sum game"

			COLUMN		Nash Equilibrium
		L	C	R	
	T	0,0	10,-10	-5,5	0.40
ROW	M	-15,15	15,-15	25,-25	0.11
	B	5,-5	-10,10	0,0	0.49
Nash Equilibrium		0.56	0.20	0.24	

CH Prediction: "zero-sum game"

			COLUMN		Nash Equilibrium	CH Model (τ = 1.55)
		L	C	R		
	T	0,0	10,-10	-5,5	0.40	0.07
ROW	M	-15,15	15,-15	25,-25	0.11	0.40
	B	5,-5	-10,10	0,0	0.49	0.53
Nash Equilibrium		0.56	0.20	0.24		
CH Model (τ = 1.55)		0.86	0.07	0.07		

Empirical Frequency: “zero-sum game”

		L	C	R	Nash Equilibrium	CH Model (τ = 1.55)	Empirical Frequency
			COLUMN				
	T	0,0	10,-10	-5,5	0.40	0.07	0.13
ROW	M	-15,15	15,-15	25,-25	0.11	0.40	0.33
	B	5,-5	-10,10	0,0	0.49	0.53	0.54
Nash Equilibrium		0.56	0.20	0.24			
CH Model (τ = 1.55)		0.86	0.07	0.07			
Empirical Frequency		0.88	0.08	0.04			

The Cognitive Hierarchy (CH) Model

- People are different and have different decision rules.
- Modeling heterogeneity (i.e., distribution of types of players). Types of players are denoted by levels k = 0, 1, 2, 3,… (k-step players / thinkers)
  - Distribution: frequency (probability) $f(k)$, k = 0, 1, 2, 3,…
- Modeling decision rule of each type.

Assumptions

- Denote a k-step player’s belief about the proportion of h-step players by $g_k(h)$. (But the “real” frequency is $f(k)$).
- Overconfidence: $g_k(h) = 0$ for $h \geq k \geq 1$, *i.e.*, k-step players do not realize that there are k *or more* steps players
- k-step players have an accurate guess about the relative proportions of players doing less thinking $g_k(h) = \frac{f(h)}{\sum_{h'=0}^{k-1} f(h')}$ **(h<k)**
- “Increasingly rational expectations”: As k increases, the total absolute deviation between the actual frequencies $f(h)$ and the beliefs $g_k(h)$ shrinks.

Modeling Decision Rule for player *i*

- Pure strategy space: $S_i=\{s_i^j: j=1,2,\ldots,m_i\}$ (finite)
- Step 0 choose randomly: $|S_i|=m_i$, $P_0(s_i^j) = 1/m_i$
- k-step thinkers know proportions $f(0),\ldots f(k\text{-}1)$, $k\geq 1$
- Form beliefs $g_k(h) = \frac{f(h)}{\sum_{h'=0}^{k-1} f(h')}$ (h<k) and best-respond based on beliefs
- Iterative and no need to solve a fixed point

Modeling Decision Rule for player *i*

Given the k-step thinker’s beliefs, the expected payoff to a k-step thinker from choosing strategy s_i^j is

$$E_k(\pi_i(s_i^j)) = \sum_{j'=1}^{m_{-i}} \pi_i(s_i^j, s_{-i}^{j'})\{\sum_{h=0}^{k-1} g_k(h)\cdot P_h(s_{-i}^{j'})\}$$

For simplicity, we assume that players best-respond ($P_k(s_i^*)$ = 1 iff $s_i^* = argmax\ E_k(\pi_i(s_i^j))$), and randomize equally if two or more strategies have identical expected payoffs.

• Starting with 0-step player behavior and iterating to compute $P_1(s_i^j)$, $P_2(s_i^j)$,. . . .
• In practice, we truncate the recursion at a k large enough that the remaining frequencies, $f(k')$ for $k' > k$, are tiny.

Poission-CH: mean=1.55

		L	C	R
			COLUMN	
	T	0,0	10,-10	-5,5
ROW	M	-15,15	15,-15	25,-25
	B	5,-5	-10,10	0,0

K	Proportion, f(k)
0	0.212
1	0.329
2	0.255
3	0.132
>3	0.072

Level (K)	K's Proportion	K+1's Belief	ROW T	ROW M	ROW B	COL L	COL C	COL R
0	0.212	1.00	0.33	0.33	0.33	0.33	0.33	0.33
Aggregate		1.00	0.33	0.33	0.33	0.33	0.33	0.33
0	0.212	0.39	0.33	0.33	0.33	0.33	0.33	0.33
1	0.329	0.61	0	1	0	1	0	0
Aggregate		1.00	0.13	0.74	0.13	0.74	0.13	0.13
0	0.212	0.27	0.33	0.33	0.33	0.33	0.33	0.33
1	0.329	0.41	0	1	0	1	0	0
2	0.255	0.32	0	0	1	1	0	0
Aggregate		1.00	0.09	0.50	0.41	0.82	0.09	0.09

Game T&A

Theoretical Implications

- Exhibits "increasingly rational expectations"
  - Normalized $g_k(h)$ approximates $f(h)$ more closely as $k \rightarrow \infty$ (i.e., highest level types are "sophisticated" (or "worldly") and earn the most.
- Highest level type <u>actions</u> converge as $k \rightarrow \infty$
  - → marginal benefit of thinking harder → 0

31

Game T&A

Alternative Specifications

- Overconfidence: k-steps think others are all one step lower (k-1) (Stahl, GEB, 1995; Nagel, AER, 1995; Ho, Camerer and Weigelt, AER, 1998)
  - "Increasingly <u>ir</u>rational expectations" as $k \rightarrow \infty$
  - Has some odd properties (e.g., cycles in entry games)
- Self-conscious: k-steps think there are other k-step thinkers ($g_k(k)>0$).
  - Similar to Quantal Response Equilibrium/Nash
  - Fits worse

32

Game T&A

Plausibility of Poisson Distribution

We focus on the one-parameter Poisson distribution for $f(k)$ because

• The simple one-parameter Poisson form fits almost as well as a seven-parameter model (with frequencies $f(k)$ up to k =7)—allowing each $f(k)$ to be independent results in less than a 1 percent decrease in log likelihood—in four of the five data sets we examined.

•The Poisson model is also easier to compute and estimate, and easier to work with theoretically.

33

Game T&A

Modeling Heterogeneity, *f(k)*

- A1: $\frac{f(k)}{f(k-1)} = \frac{\tau}{k}$
  - $f(k)/f(k-1)$ declines (proportionally to $1/k$)
  - sharp drop-off due to increasing difficulty in simulating others' behaviors
- A2: $f(0) + f(1) = 2f(2)$

34

Game T&A

Implications

- A1→ Poisson distribution with mean and variance = τ

$$f(k) = e^{-\tau} \cdot \frac{\tau^k}{k!}$$

- A1,A2 → Poisson, τ=1.618... (golden ratio Φ)

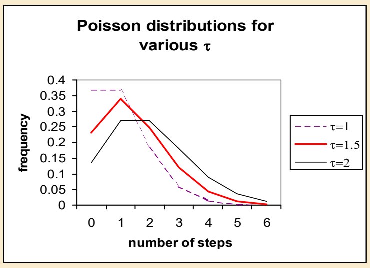
$$\tau = (\sqrt{5} + 1)/2 \approx 1.618$$

35

Game T&A

Poisson Distribution

- $f(k)$ with mean step of thinking τ: $f(k) = e^{-\tau} \cdot \frac{\tau^k}{k!}$



36

Existence and Uniqueness: CH Solution

- Existence: There is always a CH solution in any game
- Uniqueness: It is always unique

Theoretical Properties of CH Model

- *Dominance-Solvable Games*
  - When $f(k)$ is Poisson-distributed, the relative proportions of types one step below and two steps below a k-step thinker, $f(k\text{-}1)/f(k\text{-}2) = \tau/(k\text{-}1)$, puts overwhelming weight on the k-1 types if τ is very large (i.e., $k<<\tau$). In that case, a k-step thinker acts as if almost all others are using k -1 steps.
  - This property of the Poisson distribution provides a simple way to link thinking steps to iterated deletion of dominated strategies.
    - 1-step thinkers will never choose weakly dominated strategies, because those strategies are never best responses to the random strategies of 0-step types
    - τ is large: 2-step thinkers, 3-step thinkers, …

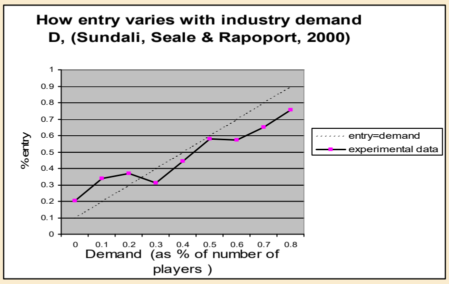
Theoretical Properties of CH Model

- Advantages over Nash equilibrium
  - Can "solve" multiplicity problem (picks one statistical distribution)
  - Sensible interpretation of mixed strategies (de facto purification)
- Theory:
  - $\tau \rightarrow \infty$ converges to Nash equilibrium in (weakly) dominance solvable games

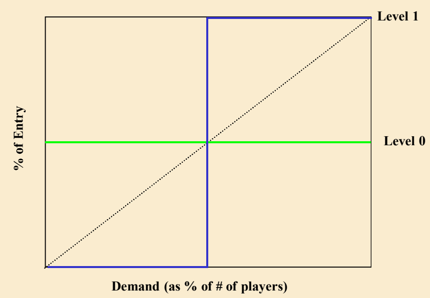
Example 2: Entry games

- Market entry with many entrants:
  - Industry demand D (as % of # of players) is announced
  - Prefer to enter if expected *%(entrants)* < *D*;
  - Stay out if expected *%(entrants)* > *D*
  - All choose simultaneously
- Experimental regularity in the 1st period:
  - Consistent with Nash prediction, *%(entrants)* increases with *D*
  - "To a psychologist, it looks like magic"-- D. Kahneman '88

Example 2: Entry games (data)

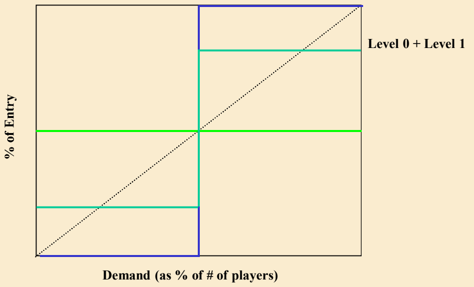


Behaviors of Level 0 and 1 Players (τ =1.25)



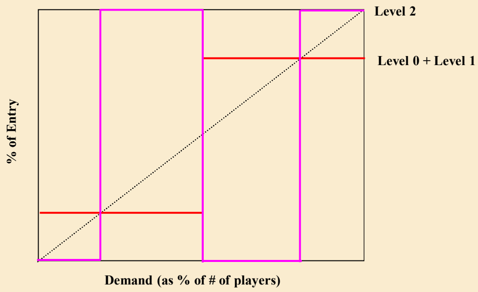
Game T&A

Behaviors of Level 0 and 1 Players (τ =1.25)



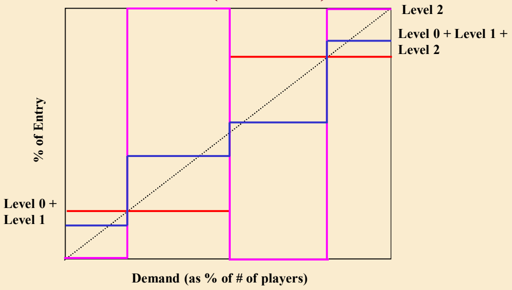
Game T&A

Behaviors of Level 2 Players (τ =1.25)



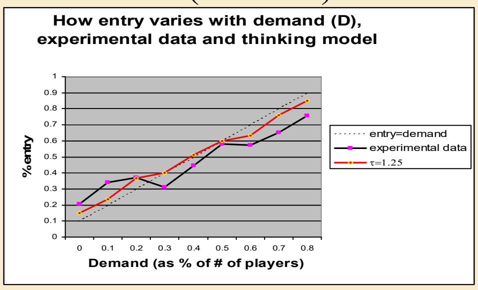
Game T&A

Behaviors of Level 0, 1, and 2 Players (τ =1.25)



Game T&A

CH Predictions in Entry Games (τ = 1.25)



Game T&A

Homework

- What value of τ can help to explain the data in Example A?
- How does CH model explain the data in Example B?

Game T&A

Empirical Frequency: "zero-sum game"

			COLUMN		Nash Equilibrium	CH Model (τ = 1.55)	Empirical Frequency
		L	C	R			
	T	0,0	10,-10	-5,5	0.40	0.07	0.13
ROW	M	-15,15	15,-15	25,-25	0.11	0.40	0.33
	B	5,-5	-10,10	0,0	0.49	0.53	0.54
Nash Equilibrium		0.56	0.20	0.24			
CH Model (τ = 1.55)		0.86	0.07	0.07			
Empirical Frequency		0.88	0.08	0.04			

MLE Estimation

	Count	Label
T	13	N1
M	33	N2
B	54	N3
L	88	M1
C	8	M2
R	4	M3

$$LL(\tau) = p_1(\tau)^{N_1} \cdot p_2(\tau)^{N_2} \cdot (1 - p_1(\tau) - p_2(\tau))^{N_3} \cdot q_1(\tau)^{M_1} \cdot q_2(\tau)^{M_2} \cdot (1 - q_1(\tau) - q_2(\tau))^{M_3}$$

这是似然函数，取对数才是对数似然函数LL (Log Likelihood)

Estimates of Mean Thinking Step τ

Table 1: Parameter Estimate τ for Cognitive Hierarchy Models

Data set	Stahl & Wilson (1995)	Cooper & Van Huyck	Costa-Gomes et al.	Mixed	Entry
Game-specific τ					
Game 1	2.93	16.02	2.16	0.98	0.69
Game 2	0.00	1.04	2.05	1.71	0.83
Game 3	1.35	0.18	2.29	0.86	-
Game 4	2.34	1.22	1.31	3.85	0.73
Game 5	2.01	0.50	1.71	1.08	0.69
Game 6	0.00	0.78	1.52	1.13	
Game 7	5.37	0.98	0.85	3.29	
Game 8	0.00	1.42	1.99	1.84	
Game 9	1.35		1.91	1.06	
Game 10	11.33		2.30	2.26	
Game 11	6.48		1.23	0.87	
Game 12	1.71		0.98	2.06	
Game 13			2.40	1.88	
Game 14				9.07	
Game 15				3.49	
Game 16				2.07	
Game 17				1.14	
Game 18				1.14	
Game 19				1.55	
Game 20				1.95	
Game 21				1.68	
Game 22				3.06	
Median τ	1.86	1.01	1.91	1.77	0.71
Common τ	1.54	0.80	1.69	1.48	0.73

CH Model: CI of Parameter Estimates

Table A1: 95% Confidence Interval for the Parameter Estimate τ of Cognitive Hierarchy Models

Data set	Stahl & Wilson (1995)		Cooper & Van Huyck		Costa-Gomes et al.		Mixed		Entry	
	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
Game-specific τ										
Game 1	2.40	3.65	15.40	16.71	1.58	3.04	0.67	1.22	0.21	1.43
Game 2	0.00	0.00	0.83	1.27	1.44	2.80	0.98	2.37	0.73	0.88
Game 3	0.75	1.73	0.11	0.30	1.66	3.18	0.57	1.37	-	-
Game 4	2.34	2.45	1.01	1.48	0.91	1.84	2.65	4.26	0.56	1.09
Game 5	1.61	2.45	0.36	0.67	1.22	2.30	0.70	1.62	0.26	1.58
Game 6	0.00	0.00	0.64	0.94	0.89	2.26	0.87	1.77		
Game 7	5.20	5.62	0.75	1.23	0.40	1.41	2.45	3.85		
Game 8	0.00	0.00	1.16	1.72	1.48	2.67	1.21	2.09		
Game 9	1.06	1.69			1.28	2.68	0.62	1.64		
Game 10	11.29	11.37			1.67	3.06	1.34	3.58		
Game 11	5.81	7.56			0.75	1.85	0.64	1.23		
Game 12	1.49	2.02			0.55	1.46	1.40	2.35		
Game 13					1.75	3.16	1.64	2.15		
Game 14							6.61	10.84		
Game 15							2.46	5.25		
Game 16							1.45	2.64		
Game 17							0.82	1.52		
Game 18							0.78	1.60		
Game 19							1.00	2.15		
Game 20							1.28	2.59		
Game 21							0.95	2.21		
Game 22							1.70	3.63		
Common τ	1.39	1.67	0.74	0.87	1.53	2.13	1.30	1.78	0.42	1.07

Nash versus CH Model: LL and MSD

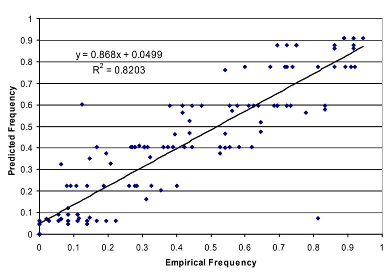
Table 2: Model Fit (Log Likelihood LL and Mean-squared Deviation MSD)

Data set	Stahl & Wilson (1995)	Cooper & Van Huyck	Costa-Gomes et al.	Mixed	Entry
Within-dataset Forecasting					
Cognitive Hierarchy (Game-specific τ)					
LL	-721	-1690	-540	-824	-150
MSD	0.0074	0.0079	0.0034	0.0097	0.0004
Cognitive Hierarchy (Common τ)					
LL	-918	-1743	-560	-872	-150
MSD	0.0327	0.0136	0.0100	0.0179	0.0005
Cross-dataset Forecasting					
Cognitive Hierarchy (Common τ)					
LL	-941	-1929	-599	-884	-153
MSD	0.0425	0.0328	0.0257	0.0216	0.0034
Nash Equilibrium 1					
LL	-3657	-10921	-3684	-1641	-154
MSD	0.0882	0.2040	0.1367	0.0521	0.0049

Note 1: The Nash Equilibrium result is derived by allowing a non-zero mass of 0.0001 on non-equilibrium strategies.

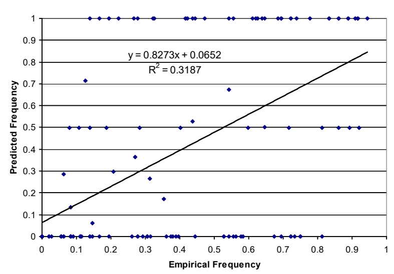
CH Model: Theory vs. Data (Mixed Games)

Figure 2a: Predicted Frequencies of Cognitive Hierarchy Models for Matrix Games (common τ)



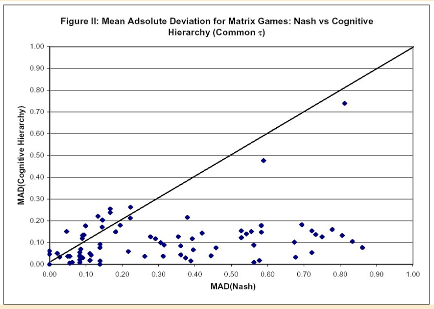
Nash: Theory vs. Data (Mixed Games)

Figure 3a: Predicted Frequencies of Nash Equilibrium for Matrix Games



Game T&A

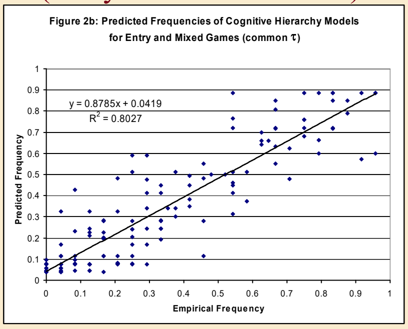
Nash vs. CH (Mixed Games)



55

Game T&A

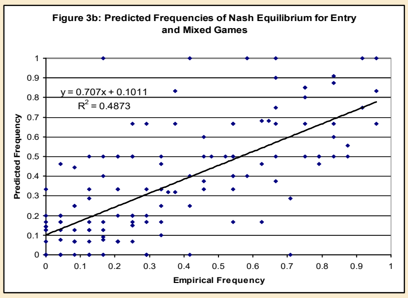
CH Model: Theory vs. Data (Entry and Mixed Games)



56

Game T&A

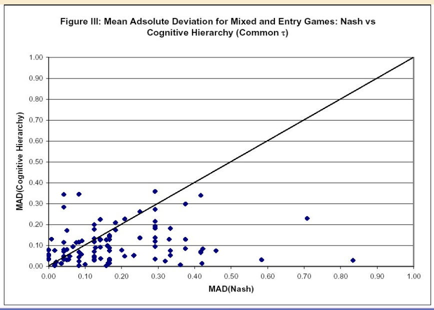
Nash: Theory vs. Data (Entry and Mixed Games)



57

Game T&A

CH vs. Nash (Entry and Mixed Games)



58

Game T&A

Economic Value

- Evaluate models based on their value-added rather than statistical fit (Camerer and Ho, 2000)
- Treat models like consultants
- If players were to hire Mr. Nash and Ms. CH as consultants and listen to their advice (i.e., use the model to forecast what others will do and best-respond), would they have made a higher payoff?
- A measure of disequilibrium

59

Game T&A

Nash versus CH Model: Economic Value

Table VIII: Economic Value of Various Theories

Data set	Stahl & Wilson	Cooper & Van Huyck	Costa-Gomes et al.	Mixed	Entry
Observed Payoff	195	586	264	328	118
Clairvoyance Payoff	243	664	306	708	176
Economic Value					
Clairvoyance	48	78	42	380	58
Cognitive Hierarchy (Common τ)	13	55	22	132	10
Nash Equilibrium	5	30	15	-17	2
% Maximum Economic Value Achieved					
Cognitive Hierarchy (Common τ)	26%	71%	52%	35%	17%
Nash Equilibrium	10%	39%	35%	-4%	3%

Note 1: The economic value is the total value (in USD) of all rounds that a "hypothetical" subject will earn using the respective model to predict other's behavior and best responds with the strategy that yields the highest expected payoff in each round.

60

Game T&A

Example 3: *P*-Beauty Contest

- *n* players
- Every player simultaneously chooses a number from 0 to 100
- Compute the group average
- Define Target Number to be 0.7 times the group average (P = 0.7 ≈ 2/3)
- The winner is the player whose number is the closet to the Target Number
- The prize to the winner is US$20

Game T&A

Results in various *p*-BC games

DATA AND CH ESTIMATES OF τ IN VARIOUS *p*-BEAUTY CONTEST GAMES

Subject pool or game	Source[a]	Group size	Sample size	Nash equil'm	Pred'n error	Data: Mean	Data: Std dev	Data: Mode	Fit of CH model: τ	Fit of CH model: Mean	Fit of CH model: Error	Fit of CH model: Std dev	Fit of CH model: Mode	Bootstrapped 90% c.i.
p = 1.1	HCW (98)	7	69	200	47.9	152.1	23.7	150	**0.10**	151.6	−0.5	28.0	165	(0.0,0.5)
p = 1.3	HCW (98)	7	71	200	50.0	150.0	25.9	150	**0.00**	150.4	0.5	29.4	195	(0.0,0.1)
High $	CHW	7	14	72	11.0	61.0	8.4	55	**4.90**	59.4	−1.6	3.8	61	(3.4,4.9)
Male	CHW	7	17	72	14.4	57.6	9.7	54	**3.70**	57.6	0.1	5.5	58	(1.0,4.3)
Female	CHW	7	46	72	16.3	55.7	12.1	56	**2.40**	55.7	0.0	9.3	58	(1.6,4.9)
Low $	CHW	7	49	72	17.2	54.8	11.9	54	**2.00**	54.7	−0.1	11.1	56	(0.7,3.8)
.7(Mean+18)	Nagel (98)	7	34	42	−7.5	49.5	7.7	48	**0.20**	49.4	−0.1	26.4	48	(0.0,1.0)
PCC	CHC (new)	2	24	0	−54.2	54.2	29.2	50	**0.00**	49.5	−4.7	29.5	0	(0.0,0.1)
p = 0.9	HCW (98)	7	67	0	−49.4	49.4	24.3	50	**0.10**	49.5	0.0	27.7	45	(0.1,1.5)
PCC	CHC (new)	3	24	0	−47.5	47.5	29.0	50	**0.10**	47.5	0.0	28.6	26	(0.1,0.8)
Caltech board	Camerer	73	73	0	−42.6	42.6	23.4	33	**0.50**	43.1	0.4	24.3	34	(0.1,0.9)
p = 0.7	HCW (98)	7	69	0	−38.9	38.9	24.7	35	**1.00**	38.8	−0.2	19.8	35	(0.5,1.6)
CEOs	Camerer	20	20	0	−37.9	37.9	18.8	33	**1.00**	37.7	−0.1	20.2	34	(0.3,1.8)
German students	Nagel (95)	14–16	66	0	−37.2	37.2	20.0	25	**1.10**	36.9	−0.2	19.4	34	(0.7,1.5)
80 yr olds	Kovalchik	33	33	0	−37.0	37.0	17.5	27	**1.10**	36.9	−0.1	19.4	34	(0.6,1.7)
U. S. high school	Camerer	20–32	52	0	−32.5	32.5	18.6	33	**1.60**	32.7	0.2	16.3	34	(1.1,2.2)
Econ PhDs	Camerer	16	16	0	−27.4	27.4	18.7	N/A	**2.30**	27.5	0.0	13.1	21	(1.4,3.5)
1/2 mean	Nagel (98)	15–17	48	0	−26.7	26.7	19.9	25	**1.50**	26.5	−0.2	19.1	25	(1.1,1.9)
Portfolio mgrs	Camerer	26	26	0	−24.3	24.3	16.1	22	**2.80**	24.4	0.1	11.4	26	(2.0,3.7)
Caltech students	Camerer	17–25	42	0	−23.0	23.0	11.1	35	**3.00**	23.0	0.1	10.6	24	(2.7,3.8)
Newspaper	Nagel (98)	3696, 1460, 2728	7884	0	−23.0	23.0	20.2	1	**3.00**	23.0	0.0	10.6	24	(3.0,3.1)
Caltech	CHC (new)	2	24	0	−21.7	21.7	29.9	0	**0.80**	22.2	0.6	31.6	0	(4.0,1.4)
Caltech	CHC (new)	3	24	0	−21.5	21.5	25.7	0	**1.80**	21.5	0.1	18.6	26	(1.1,3.1)
Game theorists	Nagel (98)	27–54	136	0	−19.1	19.1	21.8	0	**3.70**	19.1	0.0	9.2	16	(2.8,4.7)
Mean									**1.30**					
Median									**1.61**					

a. HCW (98) is Ho, Camerer, Weigelt *AER* 98; CHC are new data from Camerer, Ho, and Chong; CHW is Camerer, Ho, Weigelt (unpublished); Kovalchik is data reported by Kovalchik et al. [in press].

Game T&A

Results in various *p*-BC games

Subject Pool	Group Size	Sample Size	Mean	Error (Nash)	Error (CH)	τ
CEOs	20	20	37.9	-37.9	-0.1	1.0
80 year olds	33	33	37.0	-37.0	-0.1	1.1
Economics PhDs	16	16	27.4	-27.4	0.0	2.3
Portfolio Managers	26	26	24.3	-24.3	0.1	2.8
Game Theorists	27-54	136	19.1	-19.1	0.0	3.7

63

Game T&A

Summary

- CH Model:
  - Discrete thinking steps
  - Frequency *Poisson* distributed
- One-shot games
  - Fits better than Nash and adds more economic value
  - Explains "magic" of entry games
  - Sensible interpretation of mixed strategies
  - Can "solve" multiplicity problem
- Initial conditions for learning

64

Game T&A

Outline

- Motivation
- Mutual Consistency: CH Model
- Noisy Best-Response: QRE Model
- Instant Convergence: EWA Learning

Game T&A

Standard Assumptions in Equilibrium Analysis

Assumptions	Nash Equilbrium	Cognitive Hierarchy	QRE	EWA Learning
Solution Method				
Strategic Thinking	X	X	X	X
Best Response	X	X		X
Mutual Consistency	X		X	
Instant Convergence	X	X	X	

66

Example C: Exercise

- ❑ Consider the game in which two players independently and simultaneously choose integer numbers between and including 180 and 300. Both players are paid the lower of the two numbers, and, in addition, an amount $R > 1$ is transferred from the player with the higher number to the player with the lower number.
- ❑ Traveler's Dilemma

Traveler's Dilemma

- ➢ Two travelers purchased identical antiques on a recent trip
- ➢ Each put the antiques in their luggage and flew home. But the luggage was lost and never found
- ➢ Realizing the prospects for excessive claims, the airline devised a policy for claims

Airline Policy

- ➢ The antiques are known to be worth from \$80-\$200 and are the same for both travelers
- ➢ Airline will pay the lower claim amount
- ➢ The higher claim amount must pay a penalty of $R > 1$ dollars to the lower claimant
- ➢ If both claims are identical, each traveler gets paid his or her claim

Example C: Exercise

- ❑ Consider the game in which two players independently and simultaneously choose integer numbers between and including 180 and 300. Both players are paid the lower of the two numbers, and, in addition, an amount $R > 1$ is transferred from the player with the higher number to the player with the lower number.
- ❑ $R = 5$ versus $R = 180$

Equilibrium

- ➢ Suppose traveler 2 claims x.
  - ➢ Traveler 1 can also claim x and get x
  - ➢ Traveler 1 can claim more than x and get $x - R$
  - ➢ Traveler 1 can claim x-1 and get $x - 1 + R$
- ➢ Undercutting is the best response; hence the equilibrium is for both travelers to claim \$80
  - ➢ Notice that this is independent of the penalty, R (>1).

Example C: Data

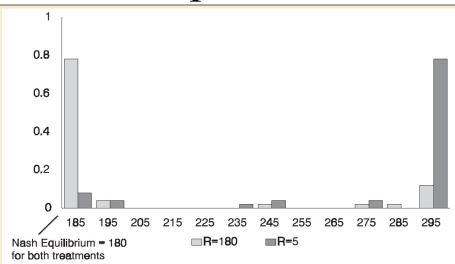


Figure 1. Claim Frequencies in a Traveler's Dilemma for R = 180 (light bars) and R = 5 (dark bars)

Ex 1: Bertrand Price Competition

- *2* firms with identical marginal cost, *c*, compete for a consumer with unit demand.
- Firms simultaneously choose prices from the (bounded) set $[0, P]$.
- Firm choosing the lower price obtains that price while high price firm earns nothing.
- Prediction: Pricing $p = c$
- Same prediction with $n > 2$ firms

Bertrand Price Competition: Experiment

- Dufwenberg and Gneezy (2000) conducted an experiment to test this.
- 12 Dutch subjects randomly matched in pairs
- Wrote down (integer) numbers from 2 to 100
- Low bidder received payment in guilders
- High bidder received nothing.
- Tie splits 50-50
- Unique equilibrium: $p = 2$.

Bertrand Price Competition: Data

- Two player case:
  - Mean price = 33.33
  - Inter-quartile range = 19-49
- Four player case:
  - Mean price = 25.75
  - Inter-quartile range = 5-33.5
- No convergence to marginal cost over time
- Number of players matter!

What to Do with This Data?

- Tough Test
  - Nash prediction is everyone chooses 2
  - This didn't happen
  - Reject theory
- "Regression" test
  - $Bid = \beta_0 + \beta_1 D_N + \varepsilon$
  - Test $\beta_0 = 2 \quad \beta_1 = 0$
  - But where did the error term come from?

Motivation: QRE

- Provide a formal methodology to quantify deviations from standard theory predictions in the lab or field
- Allow for errors but restrict so that the probability of errors is inversely proportional to the cost of errors
- Generate new predictions (reconcile various treatment effects in lab data not predicted by standard theory)

Example 2: Prisoner's Dilemma

	C	D
C	3,3	0,4
D	4,0	1,1

Example 2: Data

- An experiment by Frank *et al.* comparing the generosity of economics majors to non-economics majors
- 60.4% of economics majors defected in a one-shot prisoner's dilemma
- Less than half of non-economics majors defected

Discrete (Logistic) Choice Model

- Use the discrete choice model to estimate:

$$\text{Prob.}(i) = \frac{e^{\beta \cdot \pi_i}}{\sum_j e^{\beta \cdot \pi_j}}$$

- Where π_i is the (expected) payoff from choice i
- β is a parameter of the model.
  - If $\beta = 0$, then choices are purely random
  - If $\beta \to \infty$, then decision error vanishes

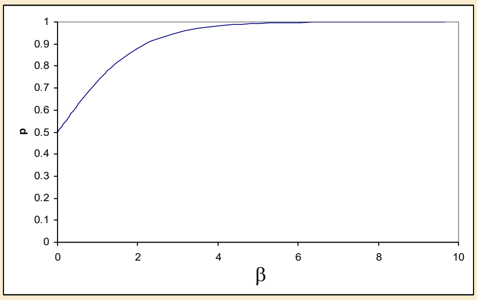
Games versus Decisions

- But what is π_i? Clearly this depends on the choices of others playing the game

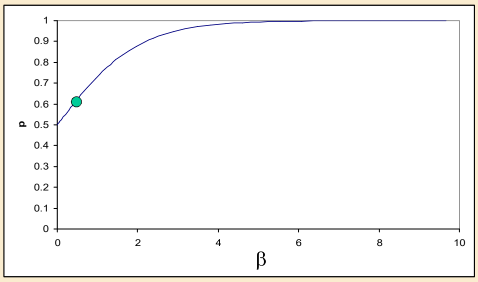
	C	D
C	3,3	0,4
D	4,0	1,1

- Let Prob. $(D) = p$
  - $\pi_D = 4\,(1-p) + 1\,p$
  - $\pi_C = 3\,(1-p)$
  - $p = \exp(\beta\,\pi_D) / [\exp(\beta\,\pi_D) + \exp(\beta\,\pi_C)]$
- Let $p(\beta)$ denote the solution to this system as a function of the parameter to be estimated, β.

A Graph of $p(\beta)$



Best Fitted β



β = 0.42 to fit the aggregate choice best

Summary

- Can think of choices in games as a function of a "rationality parameter" β
  - Rationality increasing in β
- Solve for the choice probabilities as a fixed point of the logistic equation with the expected payoffs from choices as a function of the choice probabilities
- Use the locus of fixed points to estimate β
- This approach is a version of **quantal response equilibrium (QRE)**
  - McKelvey and Palfrey (GEB, 1995)

General QRE

- What are the general conditions required for a QRE?
- Notation
  - σ be a (possibly mixed) strategy profile
  - $\pi_{ij}(\sigma)$ be the expected payoff to player i under σ when playing pure strategy j

General QRE – Cont'd

- More notation
  - f_i the density of a payoff disturbance term for each of the pure strategies available to player i
  - $\pi^*_{ij}(\sigma) = \beta_i\, \pi_{ij}(\sigma) + \varepsilon_{ij}$ is the "disturbed payoff" to i from playing strategy j
- Best response: Play the strategy that has the highest disturbed payoff.
  - B_{ij} = the set of realizations of payoff disturbances such that j is optimal for i.
- The probability that i will play strategy j is
  - $P_{ij} = \int_B f(\varepsilon)\, d\varepsilon$

General QRE – Main Result

- A QRE is a strategy σ^* where

  $\sigma^* = P(\pi(\sigma^*))$
- Proposition: For any f that is continuous, independent across players, and unbiased, a QRE exists
- Key implication: Any familiar error term structure falls in this class
  - Logit, probit, power functions, etc.

Bertrand Price Competition Revisited

- Can we use QRE to understand the Bertrand results?
- Need a specification for a continuous strategy space
- Continuous analog to the earlier formulation:
  - $Prob.\ (price < p) = \int P(\pi(t, \sigma_{-i}^*))\, dt$ for all p

QRE Under Bertrand Competition

- Power function is more tractable than Logit
- Differentiate with respect to p to obtain a differential equation
- Closed form solution:

$$F_i^Q(p) = 1 - \left(\frac{g(p^M) - g(p)}{g(p^M) - g(c)}\right)^{\frac{1}{1+\lambda-n\lambda}} \qquad (4)$$

where $g(p) \equiv \int \pi(p)^\lambda dp + K.$

where λ is the rationality parameter

Source: Baye and Morgan (2004)

Key Predictions

- Prices are far from marginal cost
- Prices are dispersed
- Number of firms (i.e., competition) reduces the average price charged

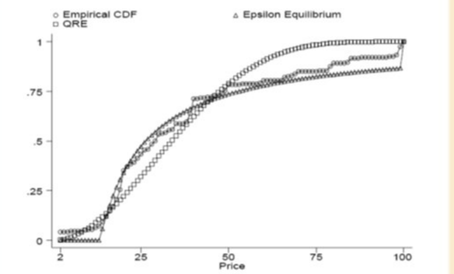


Figure 4. Cumulative Distributions of Prices under QRE and Epsilon Equilibrium Compared to Actual Empirical Distribution of Prices: Duopoly Experiments with Dutch Subjects, All Rounds. *Data source: Dufwenberg and Gneezy (2000).*

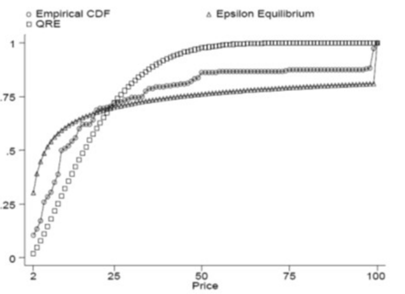


Figure 5. Cumulative Distributions of Prices under QRE and Epsilon Equilibrium Compared to Actual Empirical Distribution of Prices: Quadopoly Experiments with Dutch Subjects, All Rounds. *Data source: Dufwenberg and Gneezy (2000).*

Why is the QRE So Far from Nash?

- The logic of the undercutting argument hinges on two things:
  - I know the price my rival will charge
  - I'll undercut with certainty even if the benefit is very small
- QRE
  - Eliminates certainty about rival's action
  - By eliminating certainty, means that the value to undercutting—even in expectation—is much less
  - In the fixed point construction, these two effects reinforce one another

Example 3: Matching Pennies

- Consider a version of matching pennies
- This game has a mixed strategy equilibrium:
  - Row: *Prob. (Up) = ½*
  - Column: *Prob. (Left) = 1/(x+1)*

	Left	Right
Up	X, 0	0, 1
Down	0, 1	1, 0

Quantal Response Equilibrium

In this case, we solve the system of the two logit equations:

$$\text{Prob. }(Up) = \frac{e^{\beta\cdot\pi_{Up}}}{e^{\beta\cdot\pi_{Up}} + e^{\beta\cdot\pi_{Down}}}$$

$$\text{Prob. }(Left) = \frac{e^{\beta\cdot\pi_{Left}}}{e^{\beta\cdot\pi_{Left}} + e^{\beta\cdot\pi_{Right}}}$$

QRE, X= 4

Pr[up], Pr[left]

β

Observations

1. Player 1's choice in a QRE varies with his own payoff

2. Probability of playing "up" is more than 50-50 when $X > 1$.

Conclusions

- QRE can be thought of as the game-theoretic extension of the familiar discrete choice framework
- Can use ML methods to estimate a rationality parameter in the QRE model
- QRE also provides striking non-Nash predictions
  - Cooperation in the prisoner's dilemma depends on cooperation payoffs
  - Bertrand competition does not lead to marginal cost pricing
  - Mixed strategy QRE depend on own payoffs

Outline

- Motivation
- Mutual Consistency: CH Model
- Noisy Best-Response: QRE Model
- Instant Convergence: EWA Learning

Standard Assumptions in Equilibrium Analysis

Assumptions	Nash Equilibrum	Cognitive Hierarchy	QRE	EWA Learning
Solution Method				
Strategic Thinking	X	X	X	X
Best Response	X	X		X
Mutual Consistency	X		X	
Instant Convergence	X	X	X	

Motivation: EWA Learning

- Provide a formal model of dynamic adjustment based on information feedback (e.g., a model of equilibration)
- Allow players who have varying levels of expertise to behave differently (adaptation vs. sophistication)

Outline

- Research Question
- Criteria of a Good Model
- Experience Weighted Attraction (EWA 1.0) Learning Model
- Sophisticated EWA Learning (EWA 2.0)

Game T&A

Median-Effort Game

THE MEDIAN EFFORT GAME

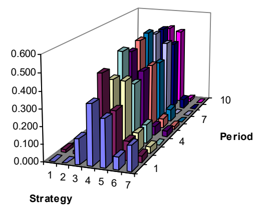
		Median {*Xi*}						
		7	6	5	4	3	2	1
Xi	7	1.30	1.15	0.90	0.55	0.10	−0.45	−1.10
	6	1.25	1.20	1.05	0.80	0.45	0.00	−0.55
	5	1.10	1.15	1.10	0.95	0.70	0.35	−0.10
	4	0.85	1.00	1.05	1.00	0.85	0.60	0.25
	3	0.50	0.75	0.90	0.95	0.90	0.75	0.50
	2	0.05	0.40	0.65	0.80	0.85	0.80	0.65
	1	−0.50	−0.05	0.30	0.55	0.70	0.75	0.70

Payoff = $0.1M-0.05(M-x_i)^2+.6$

Game T&A

Median-Effort Game

Figure 1a: Actual choice frequencies



Van Huyck, Battalio, and Beil (1990): 6 sessions, each with 9 players played for 10 periods

Game T&A

The learning setting

- In games where each player is aware of the payoff table
- Player i's strategy space consists of discrete choices indexed by j (e.g., 1, 2,…, 7)
- The game is repeated for several rounds (periods)
- At each round, all players observed:
  - Strategy or action history of all other players
  - Own payoff history

Game T&A

Research question

- To develop a <u>good</u> descriptive model of adaptive learning to predict the probability of player i (i=1,…,n) choosing strategy j at round t in any game

$$P_{ij}(t) \quad \text{or} \quad P_i^j(t)$$

Game T&A

Criteria of a "good" model

- Use (potentially) all available information subjects receive in a sensible way
- Satisfies plausible principles of behavior (i.e., conformity with other sciences such as psychology)
- Fits and predicts choice behavior well
- Ability to generate new insights
- As simple as the data allow

Game T&A

Models of Learning

- Introspection (P^j): Requires too much human cognition
  - Nash equilibrium (Nash, 1950)
  - Quantal response equilibrium (Nash-λ) (McKelvey and Palfrey, 1995)
- Evolution ($P^j(t)$): Players are pre-programmed
  - Replicator dynamics (Friedman, 1991)
  - Genetic algorithm (Ho, 1996)
- Learning ($P_i^j(t)$): Uses about the right level of cognition
  - Experience-weighted attraction learning (Camerer and Ho, 1999)
    - Reinforcement (Roth and Erev, 1995)
    - Belief-based learning
      - Cournot best-response dynamics (Cournot, 1838)
      - Simple Fictitious Play (Brown, 1951)
      - Weighted Fictitious Play (Fudenberg and Levine, 1998)
  - Directional learning (Selten, 1991)

Game T&A

Information Usage in Learning

- Choice reinforcement learning (Thorndike, 1911; Bush and Mosteller, 1955; Herrnstein, 1970; Arthur, 1991; Erev and Roth, 1998): *successful strategies played again*
- Belief-based Learning (Cournot, 1838; Brown, 1951; Fudenberg and Levine 1998): *form beliefs based on opponents' action history and choose according to expected payoffs*
- The information used by reinforcement learning is own payoff history and by belief-based models is opponents' action history
- EWA uses both kinds of information

Game T&A

"Laws" of Effects in Learning

	L	R
T	8	8
M	5	9
B	4	10

Row player's payoff table

Colin chose B and received 4
Teck chose M and received 5
Their opponents chose L

- Law of actual effect: successes increase the probability of chosen strategies
  - Teck is more likely than Colin to "stick" to his previous choice (other things being equal)
- Law of simulated effect: strategies with simulated successes will be chosen more often
  - Colin is more likely to switch to T than M
- Law of diminishing effect: Incremental effect of reinforcement diminishes over time
  - \$1 has more impact in round 2 than in round 7.

Game T&A

Assumptions of Reinforcement and Belief Learning

- Reinforcement learning ignores simulated effect
- Belief learning predicts actual and simulated effects are equally strong
- EWA learning allows for a positive (and smaller than actual) simulated effect

Game T&A

Notation

We start with notation. We study n-person normal-form games. Players are indexed by $i\,(i=1,\ldots,n)$, and the strategy space of player i, S_i consists of m_i discrete choices, that is, $S_i=\{s_i^1,s_i^2,\ldots,s_i^j,\ldots,s_i^{m_i-1},s_i^{m_i}\}$. $S=S_1\times\ldots\times S_n$ is the Cartesian product of the individual strategy spaces and is the strategy space of the game. $s_i\in S_i$ denotes a strategy of player i, and is therefore an element of S_i. $s=(s_1,\ldots,s_n)\in S$ is a strategy combination, and it consists of n strategies, one for each player. $s_{-i}=(s_1,\ldots,s_{i-1},s_{i+1},\ldots,s_n)$ is a strategy combination of all players except i. S_{-i} has a cardinality of $m_{-i}=\Pi_{k=1,k\neq i}^{n}m_k$. The scalar-valued payoff function of player i is $\pi_i(s_i,s_{-i})$. Denote the actual strategy chosen by player i in period t by $s_i(t)$, and the strategy (vector) chosen by all other players by $s_{-i}(t)$. Denote player i's payoff in a period t by $\pi_i(s_i(t),s_{-i}(t))$.

112

Game T&A

Two basic variables

The core of the EWA model is two variables which are updated after each round. The first variable is $N(t)$, which we interpret as the number of 'observation-equivalents' of past experience. The second variable is $A_i^j(t)$, player i's attraction of strategy s_i^j *after* period t has taken place.

The variables $N(t)$ and $A_i^j(t)$ begin with some prior values, $N(0)$ and $A_i^j(0)$.These prior values can be thought of as reflecting pregame experience, either due to learning transferred from different games or due to introspection. (Then $N(0)$ can be interpreted as the number of periods of actual experience, which is equivalent in attraction impact to the pregame thinking.)

113

Game T&A

The EWA Model

- Initial attractions and experience (i.e., $A_{ij}(0), N(0)$)
- Updating rules

$$A_{ij}(t)=\begin{cases}\dfrac{\phi\cdot N(t-1)\cdot A_{ij}(t-1)+\pi_i(s_{ij},s_{-i}(t))}{N(t)} & s_{ij}=s_i(t)\\[2ex] \dfrac{\phi\cdot N(t-1)\cdot A_{ij}(t-1)+\delta\cdot\pi_i(s_{ij},s_{-i}(t))}{N(t)} & s_{ij}\neq s_i(t)\end{cases}$$

$$N(t)=\rho\cdot N(t-1)+1$$

- Choice probabilities $P_{ij}(t+1)=\dfrac{e^{\lambda\cdot A_{ij}(t)}}{\sum_{k=1}^{m_i}e^{\lambda\cdot A_{ik}(t)}}$

Camerer and Ho (Econometrica, 1999)

EWA Model and Laws of Effects

$$A_{ij}(t) = \begin{cases} \dfrac{\phi \cdot N(t-1) \cdot A_{ij}(t-1) + \pi_i(s_{ij}, s_{-i}(t))}{N(t)} & s_{ij} = s_i(t) \\ \dfrac{\phi \cdot N(t-1) \cdot A_{ij}(t-1) + \delta \cdot \pi_i(s_{ij}, s_{-i}(t))}{N(t)} & s_{ij} \neq s_i(t) \end{cases}$$

$$N(t) = \rho \cdot N(t-1) + 1$$

- Law of actual effect: successes increase the probability of chosen strategies (positive incremental reinforcement increases attraction and hence probability)
- Law of simulated effect: strategies with simulated successes will be chosen more often ($\delta > 0$)
- Law of diminishing effect: Incremental effect of reinforcement diminishes over time ($N(t) >= N(t-1)$)

The EWA model: An Example

	L	R
T	8	8
B	$\boxed{4}$	10

History: Period 1 = (B,L)

Row player's payoff table

- Period 0: $A^T(0), A^B(0)$
- Period 1:

$$A^T(1) = \frac{\phi \cdot A^T(0) \cdot N(0) + \delta \cdot 8}{\rho \cdot N(0) + 1}$$

$$A^B(1) = \frac{\phi \cdot A^B(0) \cdot N(0) + 4}{\rho \cdot N(0) + 1}$$

Reinforcement Model: An Example

	L	R
T	8	8
B	$\boxed{4}$	10

History: Period 1 = (B,L)

Row player's payoff table

- Period 0: $R^T(0), R^B(0)$
- Period 1: $R^T(1) = \phi \cdot R^T(0)$ $\quad A^T(1) = \dfrac{\phi \cdot A^T(0) \cdot N(0) + \delta \cdot 8}{\rho \cdot N(0) + 1}$

$R^B(1) = \phi \cdot R^B(0) + 4$ $\quad A^B(1) = \dfrac{\phi \cdot A^B(0) \cdot N(0) + 4}{\rho \cdot N(0) + 1}$

If $\delta = 0, \rho = 0, N(0) = 1,$
$EWA \Rightarrow$ RF

Belief-based(BB) model: An Example

	L	R
T	8	8
B	$\boxed{4}$	10

- Period 0:

$N(0) = N^L(0) + N^R(0)$

$$B^L(0) = \frac{N^L(0)}{N(0)} \qquad B^R(0) = \frac{N^R(0)}{N(0)}$$

$$E^T(0) = 8 \cdot B^L(0) + 8 \cdot B^R(0) = \frac{N^L(0) \cdot 8 + N^R(0) \cdot 8}{N(0)}$$

$$E^B(0) = 4 \cdot B^L(0) + 10 \cdot B^R(0) = \frac{N^L(0) \cdot 4 + N^R(0) \cdot 10}{N(0)}$$

- Period 1:

$$B^L(1) = \frac{\rho \cdot N^L(0) + 1}{\rho \cdot N(0) + 1} \qquad B^R(1) = \frac{\rho \cdot N^R(0) + 0}{\rho \cdot N(0) + 1}$$

$$E^T(1) = 8 \cdot B^L(1) + 8 \cdot B^R(1) = \frac{\rho \cdot E^T(0) \cdot N(0) + 8}{\rho \cdot N(0) + 1}$$

$$E^B(1) = 4 \cdot B^L(1) + 10 \cdot B^R(1) = \frac{\rho \cdot E^B(0) \cdot N(0) + 4}{\rho \cdot N(0) + 1}$$

Bayesian Learning with Dirichlet priors

Relationship between Belief-based (BB) and EWA Learning Models

BB **EWA**

$$B^L(1) = \frac{\rho \cdot N^L(0) + 1}{\rho \cdot N(0) + 1} \qquad B^R(1) = \frac{\rho \cdot N^R(0) + 0}{\rho \cdot N(0) + 1}$$

$$E^T(1) = 8 \cdot B^L(1) + 8 \cdot B^R(1) = \frac{\rho \cdot E^T(0) \cdot N(0) + 8}{\rho \cdot N(0) + 1} \qquad A^T(1) = \frac{\phi \cdot A^T(0) \cdot N(0) + \delta \cdot 8}{\rho \cdot N(0) + 1}$$

$$E^B(1) = 4 \cdot B^L(1) + 10 \cdot B^R(1) = \frac{\rho \cdot E^B(0) \cdot N(0) + 4}{\rho \cdot N(0) + 1} \qquad A^B(1) = \frac{\phi \cdot A^B(0) \cdot N(0) + 4}{\rho \cdot N(0) + 1}$$

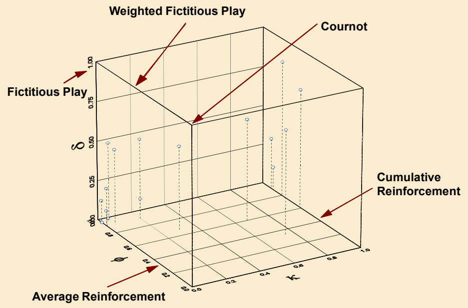
If $\delta = 1, \rho = \phi$
$EWA \Rightarrow$ BB

Model Interpretation

- Simulation or attention parameter (δ) : measures the degree of sensitivity to foregone payoffs
- Exploitation parameter ($\kappa = \dfrac{\phi - \rho}{\phi}$): measures how rapidly players lock-in to a strategy (average versus cumulative)
- Stationarity or motion parameter (ϕ): measures players' perception of the degree of stationarity of the environment

Game T&A

Model Interpretation



Game T&A

New Insight

- Reinforcement and belief learning were thought to be fundamental different for 50 years.
- For instance, "….in rote [reinforcement] learning success and failure directly influence the choice probabilities. … Belief learning is *very different*. Here experiences strengthen or weaken beliefs. Belief learning has only an indirect influence on behavior." (Selten, 1991)
- *EWA shows that belief and reinforcement learning are related and special kinds of EWA learning*

Game T&A

Actual versus Belief-Based Model Frequencies

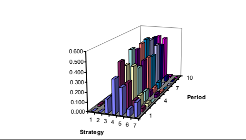


Figure 1a: Actual choice frequencies

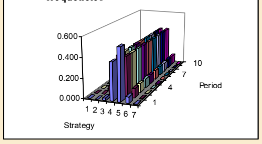


Figure 1b: Belief-based Model frequencies

Game T&A

Actual versus Reinforcement Model Frequencies

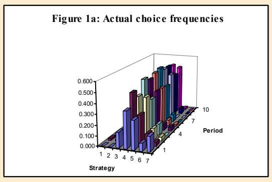


Figure 1a: Actual choice frequencies

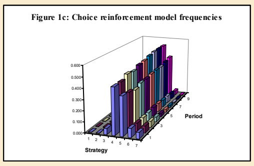


Figure 1c: Choice reinforcement model frequencies

Game T&A

Actual versus EWA Model Frequencies

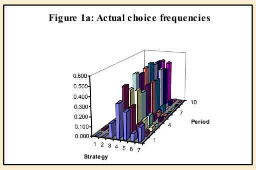


Figure 1a: Actual choice frequencies

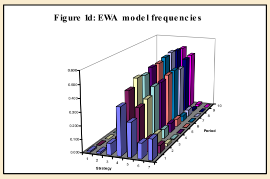


Figure 1d: EWA model frequencies

Game T&A

Estimation and Results

Game Model	No. of Parameters	Calibration LL	Calibration AIC	Calibration BIC	Calibration $\rho 2$	Validation LL	Validation MSD
Median Action(M=378)							
1-segment							
Random Choice	0	-677.29	-677.29	-677.29	0.0000	-315.24	0.1217
Choice Reinforcement	8	-341.70	-349.70	-365.44	0.4837	-80.27	0.0301
Belief-based	9	-438.74	-447.74	-465.45	0.3389	-113.90	0.0519
EWA	11	-309.30	-320.30	*-341.94**	0.5271	-41.05	0.0185
2-Segment							
Random	0	-677.29	-677.29	-677.29	0.0000	-315.24	0.1217
Choice Reinforcement	17	-331.25	-348.25	-381.70	0.4858	-66.32	0.0245
Belief-based	19	-379.24	-398.24	-435.62	0.4120	-70.31	0.0250
EWA	23	*-290.25*	*-313.25**	-358.51	*0.5374**	*-34.79**	*0.0139**

Table 1a: A summary of EWA parameter estimates and forecast accuracy (games estimated by us)

		EWA estimates (standard error)			Model accuracy					Comments
CITATION	GAME	δ	ϕ	$\rho=(1-\kappa)\phi$	EWA	Choice reinforcement – EWA	Belief – EWA	In / Out of sample	Fit technique	
Camerer, Ho and Hua (2000)	Sealed bid mechanism+	n.a.	1.00	0.91	1102.0	30.8	65.5	IN	-LL	ψ, α & κ replace δ & ρ
Camerer, Ho and Wang (1999)	"Continental divide" coordination	0.75	0.61	0.00	348.9	88.1	235.8	OUT	-LL	
Camerer and Ho (1998)	Weak-link coordination	0.65	0.58	0.20	358.1	29.1	438.6	IN	-LL	
Anderson and Camerer (in press)	Signaling games (game 3) 95% Confidence Interval	0.69 (0.47,1.00)	1.020 (0.99,1.04)	1.00 (0.98,1.00)	72.2	6.5	10.1	OUT	-LL	
	Signaling games (game 5) 95% Confidence Interval	0.54 (0.45,0.63)	0.65 (0.59,0.71)	0.46 (0.39, 0.54)	139.5	14.1	23.7	OUT	-LL	
Camerer and Ho (1999b)	Median-action coordination	0.85 (0.01)	0.80 (0.02)	0.00 (0.00)	41.1	39.2	72.8	OUT	-LL	
	4x4 Mixed-strategy games	0.00 (0.04)	1.04 (0.01)	0.96 (0.01)	326.4	9.1	-40.8	OUT	-LL	Payoff = 5 rupees
		0.73 (0.10)	1.01 (0.01)	0.95 (0.01)	341.7	18.0	8.4	OUT	-LL	Payoff =10 rupees
	6x6 Mixed-strategy games	0.41 (0.08)	0.99 (0.01)	0.94 (0.01	301.7	6.8	-5.4	OUT	-LL	Payoff =5 rupees
		0.55 (0.05)	0.99 (0.01)	0.93 (0.02)	302.3	13.7	8.9	OUT	-LL	Payoff =10 rupees
	p-beauty contests*	0.95 (0.01)	0.11 (0.00)	0.00 (0.00)	1917.0	647.0	35.0	OUT	-LL	Experienced and Inexperienced Combined
Camerer, Ho and Wang (1999)	Normal form centipede (odd player)	0.32 (0.32)	0.91 (0.14)	0.00 (0.00)	1016.8	57.6	536.3	OUT	-LL	Clairvoyance full update, κ
	Normal form centipede (even player)	0.24 (0.32)	0.90 (0.14)	0.95 (0.03)	951.3	48.4	604.7	OUT	-LL	Clairvoyance full update, κ

+ In Figure 1, we did not include this study.
* Unlike the previous estimates, these new estimates assume that subjects do not know the winning numbers.

Table 1b: A summary of EWA parameter estimates and forecast accuracy (games estimated by others)

		EWA estimates (standard error)			Model accuracy					Comments
CITATION	GAME	δ	ϕ	$\rho=(1-\kappa)\phi$	EWA	Choice reinforcement – EWA	Belief – EWA	In / Out of sample	Fit technique	
Chen and Khoroshilov (1999)	Cost allocation+	.80–1.0	1 (fixed)	.1–.3	.73–.88	-.01–.07	n.a.	IN	MSD	
Morgan and Sefton (1999)	"Unprofitable" games (baseline games)	0.08 (0.07)	0.93 (0.01)	0.92 (0.03)	1729.5	0.8	n.a.	IN	-LL	
	"Unprofitable" games (upside games)	0.14 (0.06)	0.89 (0.01)	0.00 (0.00)	1906.5	16.2	n.a.	IN	-LL	
Stahl (1999)	5x5 matrix games	.66 (0.02)	.34 (0.04)	.086 (0.08)	4803.7	64.7	n.a.	OUT	-LL	
Hsu (1999)	Call markets	0.47 (0.32)	0.97 (0.01)	0.74 (0.06)	1915.0	0.0	403.0	IN	-LL	
Amaldoss (1998)	Same function alliance – equal profit sharing	0.00	0.95	0.89	886.3	1.6	529.6	IN	-LL	High reward
		0.00	0.99	0.99	767.5	30.1	390.4	IN	-LL	Med. Reward
		0.07	0.89	0.87	1399.7	9.4	541.5	IN	-LL	Low reward
	Same function alliance – proportional sharing	0.00	0.99	1.00	910.8	36.4	813.0	IN	-LL	High reward
		0.00	0.92	0.96	1055.0	18.3	615.9	IN	-LL	Med. Reward
		0.00	0.97	0.96	1013.7	13.3	1095.6	IN	-LL	Low reward
	Parallel development of product – equal sharing	0.00	0.91	0.59	1194.2	0.1	566.3	IN	-LL	High reward
		0.17	0.90	0.93	1321.5	9.5	497.2	IN	-LL	Med. Reward
		0.21	0.88	0.66	1297.7	4.5	484.1	IN	-LL	Low reward
Rapoport and Amaldoss (2000)	Patent race game – symmetric players	0.00	0.94	0.93	3331.7	12.1	1097.7	IN	-LL	Low reward
		0.00	0.97	0.98	2908.1	20.2	725.9	IN	-LL	High reward
	Patent race game – asymmetric players	0.48	0.90	0.86	3031.5	89.1	706.8	IN	-LL	Strong player
		0.14	0.96	0.97	2835.5	15.7	611.0	IN	-LL	Weak player

+ In Figure 1, we did not include this study.

Extensions

- Heterogeneity (JMP, Camerer and Ho, 1999)
- Payoff learning (EJ, Ho, Wang, and Camerer 2006)
- Sophistication and strategic teaching
  - Sophisticated learning (JET, Camerer, Ho, and Chong, 2002)
  - Reputation building (GEB, Chong, Camerer, and Ho, 2006)
- EWA Lite (Self-tuning EWA learning) (JET, Ho, Camerer, and Chong, 2007)
- Applications:
  - Product Choice at Supermarkets (JMR, Ho and Chong, 2004)

Homework

- Provide a general proof that Bayesian learning (i.e., weighted fictitious play) is a special case of EWA learning.
- If players are faced with a stationary environment (i.e., decision problems), will EWA learning lead to EU maximization in the long-run?

Outline

- Research Question
- Criteria of a Good Model
- Experience Weighted Attraction (EWA 1.0) Learning Model
- Sophisticated EWA Learning (EWA 2.0)

Three User Complaints of EWA 1.0

- Experience matters.
- EWA 1.0 prediction is not sensitive to the structure of the learning setting (e.g., matching protocol).
- EWA 1.0 model does not use opponents' payoff matrix to predict behavior.

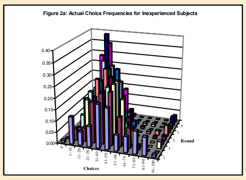
Example 3: *p*-Beauty Contest

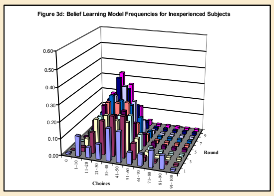
- *n* players
- Every player simultaneously chooses a number from 0 to 100
- Compute the group average
- Define Target Number to be 0.7 times the group average
- The winner is the player whose number is the closet to the Target Number
- The prize to the winner is US$20

Ho, Camerer, and Weigelt (AER, 1998)

Game T&A

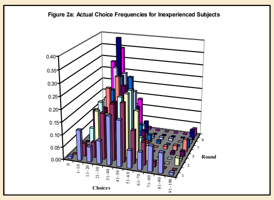
Actual versus Belief-Based Model Frequencies: *pBC* (inexperienced subjects)

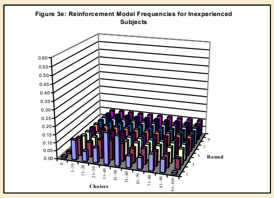




Game T&A

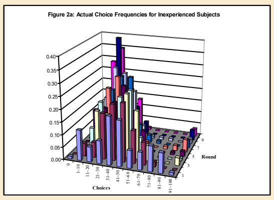
Actual versus Reinforcement Model Frequencies: *pBC* (inexperienced subjects)

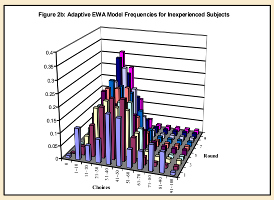




Game T&A

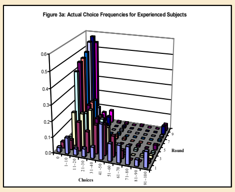
Actual versus EWA Model Frequencies: *pBC* (inexperienced subjects)

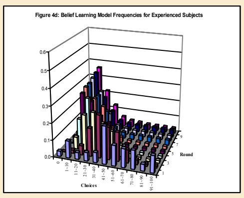




Game T&A

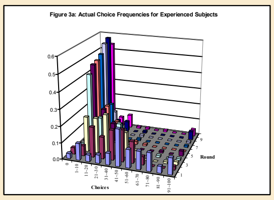
Actual versus Belief-Based Model Frequencies: pBC (experienced subjects)

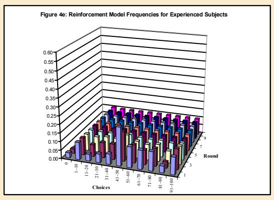




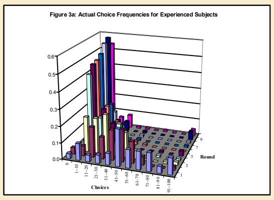
Game T&A

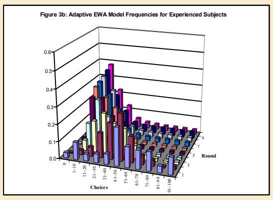
Actual versus Reinforcement Model Frequencies: pBC (experienced subjects)





Actual versus EWA Model Frequencies: pBC (experienced subjects)





Sophisticated EWA Learning (EWA 2.0)

- The population consists of both adaptive and sophisticated players.
- The proportion of sophisticated players is denoted by α. Each sophisticated player however believes the proportion of sophisticated players to be α'.
- Use latent class to estimate parameters.

The EWA 2.0 Model: Adaptive players

- Adaptive $(1-\alpha)$ + sophisticated (α)
- Adaptive players

$$A_{ij}(a,t)=\begin{cases}\dfrac{\phi\cdot N(t-1)\cdot A_{ij}(a,t-1)+\pi_i(s_{ij},s_{-i}(t))}{N(t)} & s_{ij}=s_i(t)\\[2ex] \dfrac{\phi\cdot N(t-1)\cdot A_{ij}(a,t-1)+\delta\cdot\pi_i(s_{ij},s_{-i}(t))}{N(t)} & s_{ij}\neq s_i(t)\end{cases}$$

$$P_{ij}(a,t+1)=\frac{e^{\lambda\cdot A_{ij}(t)}}{\sum_{k=1}^{m_i} e^{\lambda\cdot A_{ik}(t)}}$$

The EWA 2.0 Model: Sophisticated Players

- Adaptive $(1-\alpha)$ + sophisticated (α)
- Sophisticated players believe $(1-\alpha')$ proportion of the players are adaptive and best respond based on that belief:

$$A_{ij}(s,t)=\sum_{k=1}^{m_{-i}}[(1-\alpha')\cdot P_{-i,k}(a,t+1)+\alpha'\cdot P_{-i,k}(s,t+1)]\cdot\pi_{\iota}(s_{ij},s_{-i,k})$$

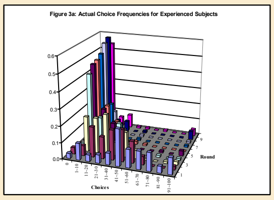
$$P_{ij}(s,t+1)=\frac{e^{\lambda\cdot A_{ij}(s,t)}}{\sum_{k=1}^{m_i} e^{\lambda\cdot A_{ik}(s,t)}}$$

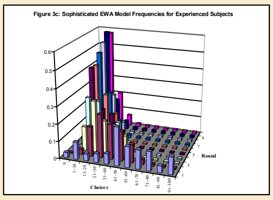
- Better-than-others $(\alpha'<\alpha)$; false consensus $(\alpha'>\alpha)$

Well-known Special Cases

- Nash equilibrium: α = α' = 1 and λ = infinity
- Quanta response equilibrium: α = α' = 1
- Rational expectation model: α = α'
- Better-than-others model: α > α'

Results





Game T&A

MLE Estimates

	INEXPERIENCED SUBJECTS			EXPERIENCED SUBJECTS		
	Sophisticated EWA	Adaptive EWA	QRE[1]	Sophisticated EWA	Adaptive EWA	QRE
ϕ	**0.44**	0.00	-	**0.29**	0.22	-
	(0.05)[2]	(0.00)	-	(0.03)	(0.03)	-
δ	**0.78**	0.90	-	**0.67**	0.99	-
	(0.08)	(0.05)	-	(0.05)	(0.02)	-
ρ	**0.00**	0.00	-	**0.01**	0.00	-
	(0.00)	(0.00)	-	(0.00)	(0.00)	-
α	**0.24**	0.00	1.00	**0.77**	0.00	1.00
	(0.04)	(0.00)	(0.00)	(0.02)	(0.00)	(0.00)
α'	**0.00**	0.00	-	**0.41**	0.00	-
	(0.00)	(0.00)	-	(0.03)	(0.00)	-
d	**0.16**	0.13	0.04	**0.15**	0.11	0.04
	(0.02)	(0.01)	(0.01)	(0.01)	(0.01)	(0.00)
LL						
(in sample)	**-2095.32**	-2155.09	-2471.50	**-1908.48**	-2128.88	- 2141.45
(out of sample)	**-968.24**	-992.47	-1129.25	**-710.28**	-925.09	- 851.31
Avg. Prob.						
(in sample)	6%	5%	3%	7%	5%	5%
(out of sample)	7%	7%	4%	13%	9%	9%

Game T&A

Summary

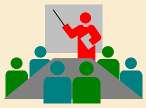
- ❑ EWA cube provides a simple but useful framework for studying learning in games.
- ❑ EWA 1.0 model fits and predicts better than reinforcement and belief learning in many classes of games because it allows for a positive (and smaller than actual) simulated effect.
- ❑ EWA 2.0 model allows us to study equilibrium and adaptation simultaneously (and nests most familiar cases including QRE)

Game T&A

Key References for This Talk

- ➢ Refer to Ho, Lim, and Camerer (JMR, 2006) for important references
- ➢ CH Model: Camerer, Ho, and Chong (QJE, 2004)
- ➢ QRE Model: McKelvey and Palfrey (GEB, 1995); Baye and Morgan (RAND, 2004)
- ➢ EWA Learning: Camerer and Ho (Econometrica, 1999), Camerer, Ho, and Chong (JET, 2002)

Game T&A



附：

fairness concerns

(inequality aversion)

Game T&A

背景与问题

背景 **最后通牒博弈(Ultimatum Game)**

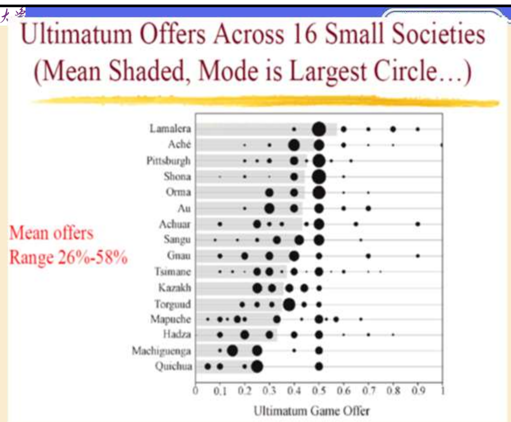
- 甲乙两人就分配 1 笔钱（如100元）进行博弈.
- 甲首先提出分配方案 (分给乙的钱: s).
- 如果乙接受，则按此分配；否则双方什么也得不到.
- 完全信息动态博弈：
  均衡结果是(s=0,乙接受);
  如果要求严格均衡，则s= 1 分钱.
- 现实中的情况果真如此吗？

Game T&A

实验

Henrich et. al (2001; 2005)

结果



Game T&A

背景与问题

- 现实(实验)中的情况:
  - 多数s=总额的40~50%
  - s越小，越容易被乙拒绝

自私:	公平:
理性 /	利他 /
非理性?	互惠?

"不患寡而患不均"！

Game T&A

模型假设与建立

财富总额为1

接受提议：甲乙所得x_1=1-s, x_2= s；否则：x_1=x_2=0

1. 每个参与者都喜欢对所有参与者公平的结果；
2. 每个参与者自己受到不公平对待时的"愤怒"，胜过其他参与者受到不公平对待时的"愧疚".

效用函数 $U_i(x_1,x_2)=x_i-\alpha_i\max\{x_j-x_i,0\}-\beta_i\max\{x_i-x_j,0\}$

$i=1,2,\ j=3-i \qquad \alpha_i\ge\beta_i\ge 0$

$\beta_i<1/2$ 否则，x_i>x_j=1-x_i时，

$U_i(x)=x_i-\beta_i(x_i-x_j)=\beta_i-(2\beta_i-1)x_i$

关于x_i的系数非正（过分"愧疚"）

Game T&A

求解 $U_i(x_1,x_2)=x_i-\alpha_i\max\{x_j-x_i,0\}-\beta_i\max\{x_i-x_j,0\}$

乙的最优反应（给定s）

如果不接受，则x_1=x_2=0; $U_1(s)$=$U_2(s)$=0 .

如果接受，则x_1=1-s, x_2=s.

- 若$s\ge 1/2$,则$x_2\ge x_1$ $\quad U_2(s)=s-\beta_2(2s-1)\ \ge 1/2\ge 0$ （$\beta_2<1/2$）
- 若$s\le 1/2$,则$x_2\le x_1$ $\quad U_2(s)=s-\alpha_2(1-2s)=(1+2\alpha_2)s-\alpha_2$

$U_2(s)\ge 0 \Leftrightarrow s\ge \bar{s}(\alpha_2)=\alpha_2/(1+2\alpha_2)$

(s =1/2, 两者一致) 易知 $0\le\bar{s}(\alpha_2)<1/2$

乙的最优反应 当 $s\ge\bar{s}(\alpha_2)$ 接受; 否则，不接受

Game T&A

模型求解：甲的决策(只需考虑乙接受情形)

Case 1: 甲知道乙的α_2

- 若$s\ge 1/2$,则$x_2\ge x_1$ $\quad U_1(s)=1-s-\alpha_1(2s-1)$

  ➔s=1/2时达到最大值1/2

- 若$s\le 1/2$,则$x_2\le x_1$ $\quad U_1(s)=1-s-\beta_1(1-2s)=1-\beta_1+(2\beta_1-1)s$

  但 $s\ge\bar{s}(\alpha_2)$ $\qquad \beta_1<1/2$

甲的决策 $s^*=\bar{s}(\alpha_2)=\alpha_2/(1+2\alpha_2)$

均衡: (s^*,接受) s^*严格小于50%;
是乙的"愤怒"系数α_2的增函数.

Game T&A

模型求解：甲的决策

Case 2: 甲不知道乙的α_2,
但知道α_2的分布$F(\alpha_2)$

$\underline{\alpha}=\max\{\alpha|F(\alpha)=0\}$

$\overline{\alpha}=\min\{\alpha|F(\alpha)=1\}$

- 若$s\ge 1/2$,则$x_2\ge x_1$ $\quad U_1(s)=1-s-\alpha_1(2s-1)$ ➔同前
- 若$s\le 1/2$,则$x_2\le x_1$

乙接受概率

$$p=\begin{cases}0, & s\le\bar{s}(\underline{\alpha})\\ F(s/(1-2s)), & \bar{s}(\underline{\alpha})<s<\bar{s}(\overline{\alpha})\\ 1, & s\ge\bar{s}(\overline{\alpha})\end{cases}$$

期望效用

$$EU_1(s)=\begin{cases}0, & s\le\bar{s}(\underline{\alpha})\\ [1-\beta_1+(2\beta_1-1)s]F(s/(1-2s)), & \bar{s}(\underline{\alpha})<s<\bar{s}(\overline{\alpha})\\ 1-\beta_1+(2\beta_1-1)s, & s\ge\bar{s}(\overline{\alpha})\end{cases}$$

甲的决策 $\max\limits_{\bar{s}(\underline{\alpha})\le s\le\bar{s}(\overline{\alpha})}[1-\beta_1+(2\beta_1-1)s]F(s/(1-2s))$ ➔s^*

清华大学 Game T&A

模型解释

- 甲永远不会提出大于1/2的方案s
- 乙拒绝过小的方案s
- 乙接受概率随s增加不减

很好地解释了实际中的最后通牒博弈.

主要参考文献

A Theory of Fairness, Competition, and Cooperation
Author(s): Ernst Fehr and Klaus M. Schmidt
Source: *The Quarterly Journal of Economics*, Vol. 114, No. 3 (Aug., 1999), pp. 817-868
Published by: The MIT Press
Stable URL: http://www.jstor.org/stable/2586885

清华大学 Game T&A

简要评述

- 对于多人参与的博弈，如何建立考虑公平性的博弈模型？
- 在模型中考虑人的行为因素，是一个很有挑战性的研究方向。

（行为博弈论、行为经济学、行为营销学、行为金融学、行为运作管理，……)

清华大学 Game T&A

作业

内容 阅读有关材料（自选）

GAME NEVER OVER

159