3(1) $\begin{cases} u_t = a^2 u_{xx} & (0<x<l,\ t>0) \\ u(t,0) = u(t,l) = 0 \\ u(0,x) = \delta(x-\xi) & (0<\xi<l). \end{cases}$

令 $u = T(t)X(x)$，则：

$$T'X = a^2 T X''$$

$$\Rightarrow \begin{cases} T' + \lambda a^2 T = 0 \\ X'' + \lambda X = 0 \end{cases}$$

边界条件为：$X(0) = X(l) = 0$

$$\Rightarrow \begin{cases} X'' + \lambda X = 0 \\ X(0) = X(l) = 0 \end{cases}$$

$\Rightarrow \lambda_n = \left(\frac{n\pi}{l}\right)^2,\ n = 1, 2, \cdots$

$X_n = C_n \sin \frac{n\pi}{l} x$

$\Rightarrow$ 将 λ_n 代入 $(T' + \lambda a^2 T = 0)$ 得：

$\Rightarrow T_n = D_n e^{-\left(\frac{n\pi}{l}\right)^2 a^2 t}$

$\therefore u = \sum_{n=1}^{+\infty} C_n e^{-\left(\frac{n\pi}{l}\right)^2 a^2 t} \sin \frac{n\pi}{l} x$

代入 $u(0,x) = \delta(x-\xi)$ 得：

$$\sum_{n=1}^{+\infty} C_n \sin \frac{n\pi}{l} x = \delta(x-\xi)$$

$$\therefore C_n = \frac{2}{l} \int_0^l \delta(x-\xi) \sin \frac{n\pi}{l} x \, dx = \frac{2}{l} \sin \frac{n\pi}{l} \xi$$

$$\therefore u(t,x) = \sum_{n=1}^{+\infty} \frac{2}{l} \sin\left(\frac{n\pi}{l}\xi\right) e^{-\left(\frac{n\pi}{l}\right)^2 a^2 t} \sin \frac{n\pi}{l} x$$

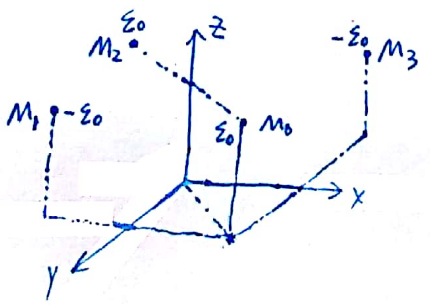
6(1) $\Delta G = -\delta(x-x_0,\ y-y_0,\ z-z_0)$

$G|_{x=0} = G|_{y=0} = 0$

分别在 $(-x_0, y_0, z_0)$ 设置 $-\varepsilon_0$，在 $(x_0, -y_0, z_0)$ 设置 $-\varepsilon_0$，在 $(-x_0, -y_0, z_0)$ 设置 ε_0 作为镜像.

$$\therefore G = \frac{1}{4\pi}\left[\frac{1}{r(M,M_0)} - \frac{1}{r(M,M_1)} + \frac{1}{r(M,M_2)} - \frac{1}{r(M,M_3)}\right]$$

$M_0(x_0, y_0, z_0)$，$M_1(-x_0, y_0, z_0)$，$M_2(-x_0, -y_0, z_0)$，$M_3(x_0, -y_0, z_0)$



8. $\begin{cases} u_t = a^2 u_{xx} + bu \\ u(0,x,y,z) = \delta(x,y,z) \end{cases}$

$u_t = Lu$ 型柯西问题基本解. P330

解：令 $\bar{U} = \int_{-\infty}^{+\infty} u(t,\xi,y,z) e^{i\xi} d\xi$

$\therefore \begin{cases} \frac{d\bar{U}}{dt} = a^2(-i\lambda)^2 \bar{U} + b\bar{U} \\ \bar{U}|_{t=0} = 1 \end{cases}$

$\therefore \bar{U} = \exp\{-a^2\lambda^2 t + bt\}$

$$\therefore u(t,x,y,z) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \exp\{-a^2\lambda^2 t + bt\}\exp\{-i\lambda x\} d\lambda.$$

$$= \frac{1}{2\pi}\int_{-\infty}^{+\infty} \exp\{bt\}\exp\{-a^2\lambda^2 t - i\lambda x\} d\lambda$$

$$= \frac{1}{2a\sqrt{\pi t}} \exp\left\{-\frac{x^2}{4a^2 t} + bt\right\}$$