《Financial Statistics》 Homework No.1

Deadline：April 16, 2019 Total score：**100**

Name:____________ Student ID:____________ Department:________________

1. (10') Generate a random sample of size 100 from the t-distribution with ν degrees of freedom for $\nu = 5, 15$ and ∞ (i.e., normal distribution). Apply the Jarque-Bera test to check for the normality and report the p-values.

2. (20') According to the efficient market hypothesis, is the return of a portfolio predictable? Is the volatility of a portfolio predictable? State the most appropriate mathematical form of the efficient market hypothesis.

3. (10') If the Ljung-Box test is employed to test the efficient market hypothesis, what null hypothesis is to be tested? If the autocorrelation for the first 4 lags of the monthly log-returns of the S&P 500 is

$$\widehat{\rho}_1 = 0.2, \quad \widehat{\rho}_2 = -0.15, \quad \widehat{\rho}_3 = 0.25, \quad \widehat{\rho}_4 = 0.12$$

based on past 5 years data, is the efficient market hypothesis reasonable?

4. (20') Suppose that a stock return follows the nonlinear dynamic

$$X_t = \varepsilon_t + \frac{0.8\varepsilon_{t-1}^2}{1+\varepsilon_{t-1}^2},$$

and that $\{\varepsilon_t\} \sim_{i.i.d.} \mathcal{N}(0, \sigma^2)$.

(a). Simulate the time series of length 1000 with $\sigma = 1$, and show the plots of ACF and PACF.

(b). Show that the ACF of $\{X_t\}$ is zero except at lag 0;

(c). Use (b) to show that the PACF of $\{X_t\}$ is zero.

This example shows that ACF and PACF are useful mainly for linear time series.

5. (40') Consider the AR(p) process:

$$X_t = c + \sum_{i=1}^{p} \phi_i X_{t-i} + \varepsilon_t,$$

where $\{\varepsilon_t\} \sim_{i.i.d.} \mathcal{N}(0, \sigma^2)$. We assume the process is stationary and the characteristic roots are not the same.

(a). What is the conditional distribution of

$$\mathbb{P}(X_t \le x | X_0 = x_0, X_{-1} = x_{-1}, ..., X_{-p+1} = x_{-p+1})?$$

(b). Use Monte-Carlo to verify your analytical result in (a), for

$$X_t = 0.7X_{t-1} - 0.01X_{t-2} + \varepsilon_t$$

and $\sigma = 1$, $x_0 = x_{-1} = 1$ and $x = 2$.

(c). Given a time series $\{X_1, ..., X_n\}$, explain how you will:

(1) determine p;

(2) estimate parameters;

(3) diagnose whether $\varepsilon_t \sim \mathcal{N}(0, \sigma^2)$ i.i.d. is a good assumption;

(4) evaluate whether this is a good model for the given time series.

The End !