

## ADDITIONAL TOPICS (2): ZARISKI TOPOLOGY

**Zariski topology** is the most basic topological structure in algebraic geometry. Algebraic geometry is a subject which studies the zero locus of a system of polynomials and this gives us the motivation to define the Zariski topology.

Consider the set  $\mathbb{C}^n$ . A set  $Y \subset \mathbb{C}^n$  is closed if  $Y$  is the common zero locus of some polynomials  $f_1, \dots, f_m \in \mathbb{C}[t_1, \dots, t_n]$ . Such  $Y$  is called an **affine algebraic variety**.

- (1) Show that the above definition indeed defines a topology on  $\mathbb{C}^n$ . (Hint: You may need the fact that the ring  $\mathbb{C}[t_1, \dots, t_n]$  is noetherian which means that every ideal of  $\mathbb{C}[t_1, \dots, t_n]$  is finitely generated.)

A closed subset  $Y$  is called **irreducible** if it cannot be expressed as the union  $Y = Y_1 \cup Y_2$  of two proper subsets, each one of which is closed in  $Y$ .

- (1) Give an algebraic criterion for  $Y$  to be irreducible. (Hint: Let  $Y$  be the common zero locus of  $f_1, \dots, f_m \in \mathbb{C}[t_1, \dots, t_n]$ , consider the ideal generated by  $f_1, \dots, f_m$ . What condition should be satisfied by this ideal?)