Plackett-Burman (PB) and Model Robust Screening Designs

- We looked at 2^{k-p} designs, which give us designs that have 8, 16, 32, 64, 128, etc. number of runs
- However, there is a pretty big gap between 16 and 32, 32 to 64, etc. We sometimes need other alternative designs besides these with a different number of observations
- A class of designs that allows us to create experiments with some number between these fractional factorial designs are the Plackett-Burman designs, for

$$N = 12, [16], 20, 24, 28, [32], 36, 40, 44, 48, \ldots\ldots$$

  any number which is divisible by four

- These designs are similar to Resolution III designs, meaning you can estimate main effects clear of other main effects-often used for screening experiments (where the objective is to determine which factors from a list assembled by brainstorming are important enough to be studied in more detail in follow-up experiments)

- For run sizes that are powers of 2, they are the same as a 2^{k-p} fractional factorial design. For other run sizes, they retain the desirable orthogonality property of 2^{k-p} designs, but they do not have generators or a defining relation
- The designs for run sizes of 12, 20, and 24 can be created by cyclically rotating the factor levels for the first run

X Y Z
Z X Y
Y Z X

Run	A	B	C	D	E	F	G	H	J	K	L
1	+	+	−	+	+	+	−	−	−	+	−
2	−	+	+	−	+	+	+	−	−	−	+
3	+	−	+	+	−	+	+	+	−	−	−
4	−	+	−	+	+	−	+	+	+	−	−
5	−	−	+	−	+	+	−	+	+	+	−
6	−	−	−	+	−	+	+	−	+	+	+
7	+	−	−	−	+	−	+	+	−	+	+
8	+	+	−	−	−	+	−	+	+	−	+
9	+	+	+	−	−	−	+	−	+	+	−
10	−	+	+	+	−	−	−	+	−	+	+
11	+	−	+	+	+	−	−	−	+	−	+
12	−	−	−	−	−	−	−	−	−	−	−

12-Run PB Design

Run Size	Factor Levels
12	+ + − + + + − − − + −
20	+ + − − + + + + − + − + − − − − + + −
24	+ + + + + − + − + + − − + + − − + − + − − − −

Each combination of levels for any **pair** of factors appears the *same number of times*, throughout all the experimental runs

```
> library(FrF2)
> pb( nruns = 12, randomize=FALSE)#nfactors no more than nruns-1
```

```
> .libPaths()
```

Equivalent Design

- BP Design for $N = 12$ obtained by filling in by columns
- The cyclical pattern is a result of number theory properties that generate these orthogonal arrays

There is a lot of mathematical research behind these designs to achieve a matrix with orthogonal columns which is what we need

Run	*A*	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*I*	*J*	*K*
1	+	−	+	−	−	−	+	+	+	−	+
2	+	+	−	+	−	−	−	+	+	+	−
3	−	+	+	−	+	−	−	−	+	+	+
4	+	−	+	+	−	+	−	−	−	+	+
5	+	+	−	+	+	−	+	−	−	−	+
6	+	+	+	−	+	+	−	+	−	−	−
7	−	+	+	+	−	+	+	−	+	−	−
8	−	−	+	+	+	−	+	+	−	+	−
9	−	−	−	+	+	+	−	+	+	−	+
10	+	−	−	−	+	+	+	−	+	+	−
11	−	+	−	−	−	+	+	+	−	+	+
12	−	−	−	−	−	−	−	−	−	−	−

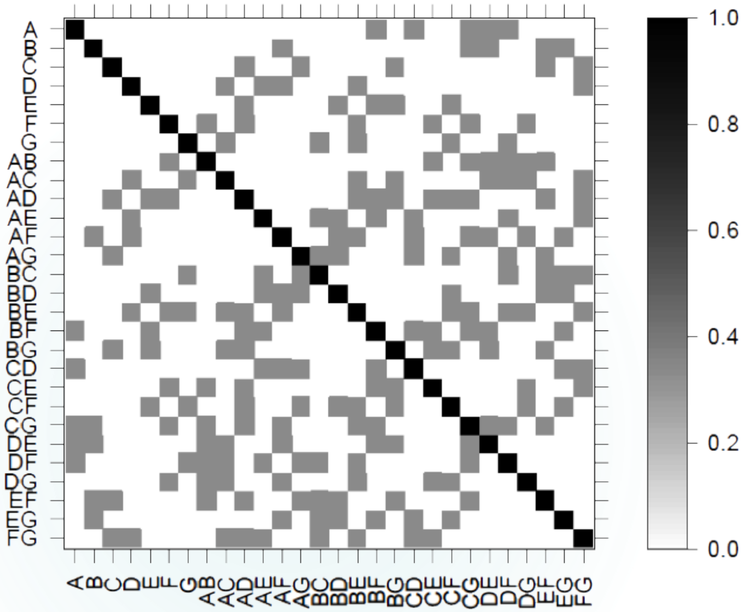
Partial Confounding

- Although some effects are orthogonal, they do not have the same structure allowing complete or orthogonal correlation with the other two way and higher order interactions
- For 11 factors and 12 runs: $[A] = A - \frac{1}{3}BC - \frac{1}{3}BD - \frac{1}{3}BE + \frac{1}{3}BF \ldots - \frac{1}{3}KL$
  - The correlation matrix for main effects is identity
  - Every main effect is partially aliased with every two-factor interaction not involving itself
- If you assume that interactions are not important, these are great designs, very efficient with small numbers of observations and useful
- If your assumption is wrong and there are interactions, it could show up as influencing one or the other main effects

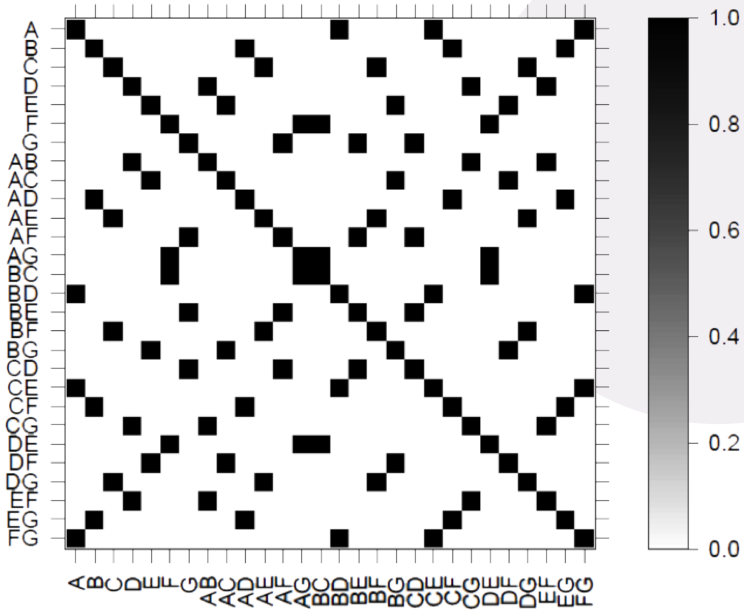
```
> design <- pb( nruns = 12)
> nmain <- apply(as.matrix(design), 2, as.numeric)
> cor(nmain)
> AB <- nmain[,1]*nmain[,2]
> BC <- nmain[,2]*nmain[,3]
> cor(AB, nmain[,1])  #0
> cor(AB, nmain[,3]) #-0.3333333
> cor(BC, nmain[,1])
```

Complex Alias Structure

- Color Map Comparison of Confounding between PB and FF Designs
- _____ factors? ____ runs?



(a) Plackett-Burman Design

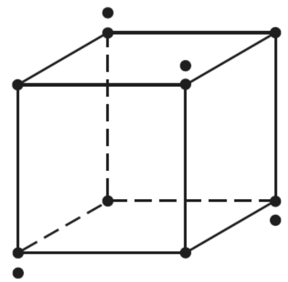


(b) 2^{7-4}_{III} design

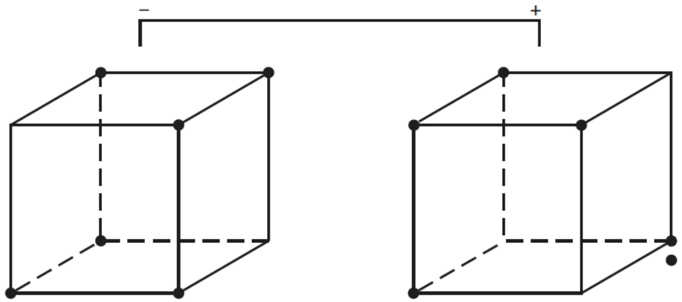
Nongeometric and Nonregular Designs

Plackett–Burman designs are examples of

- Nongeometric designs: these designs cannot be represented as cubes
- Nonregular designs
  - Basically, a regular design is one in which all effects can be estimated independently of the other effects
  - In the case of a fractional factorial, the effects that cannot be estimated are completely aliased with the other effects
- In nonregular designs, since the correlations between main effects and interactions are not ± 1, some interactions can be included in the model as long as the total number of terms in the model is less than the number of runs in the design



(*a*) Projection into three factors



(*b*) Projection into four factors

Projection of the 12-run PB design into three- and four-factor designs

Model Robust Screening Designs

- Resolution ___ fractional factorials allow estimation of all important main effects and all two-factor interactions
  - These designs may require too many runs to be practical for screening
- Using regression subset procedures, PB designs are efficient for fitting several possible models
  - They are called model robust
- By utilizing PB designs, the need for follow-up experiments to unconfound main effects from two-factor interactions or break confounding among strings of confounded two-factor interactions is reduced
  - They can be used as a one step screening and optimization experiment

清华大学统计学辅修课程

Design and Analysis of Experiments

Lecture 9 – 3-level and Mixed-level Factorials and Fractional Factorials

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Outline

- The 3^k Factorial Design
- The 3^{k-p} Fractional Factorial Designs
  - Connection with Latin Squares
  - Connection with Graeco-Latin Squares
- Mixed Factorials

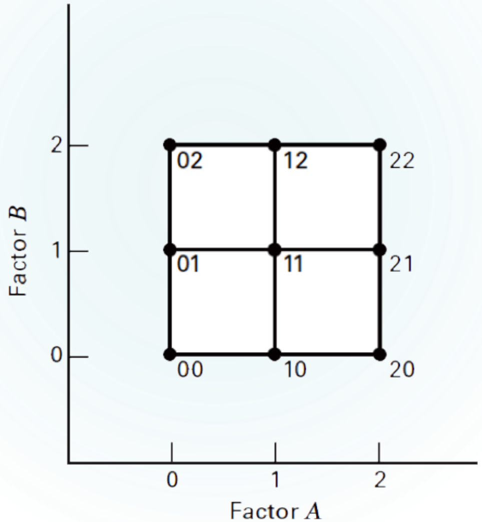
Introduction

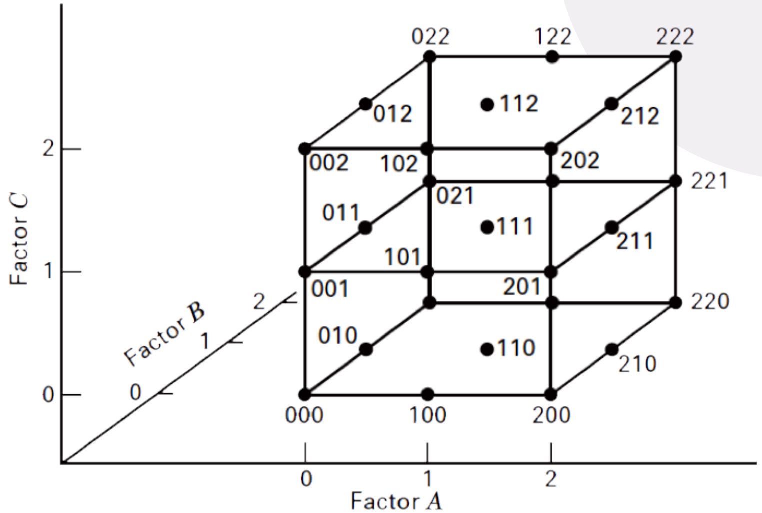
- 3^k designs are a generalization of the 2^k designs
- We will continue to talk about coded variables so we can describe designs in general terms, but in this case we assume that the factors are all quantitative
  - With 2^k designs we weren't as strict about this because we could have either qualitative or quantitative factors. Most 3^kdesigns are only useful where the factors are quantitative
- With 3^k designs we are moving from screening factors to analyzing them to understand what their actual response function looks like
- With 2 level designs, we had just two levels of each factor. This is fine for fitting a linear, straight line relationship. With three level of each factor we now have points at the middle so we will are able to fit curved response functions, i.e. quadratic response functions

The 3^k Factorial Design-Notation

- Factor levels are sometime denoted 1, 2, 3
- Qualitative or categorical factors are low, medium, high
- Individual runs are defined by k digits combinations, such as 000 to denote the test combination in a three-variable design where all three factors are at the low level
- -1, 0, +1 is the orthogonal coding for quantitative factors

Treatment combinations in a 3^2 design





Treatment combinations in a 3^3 design

Regression Model

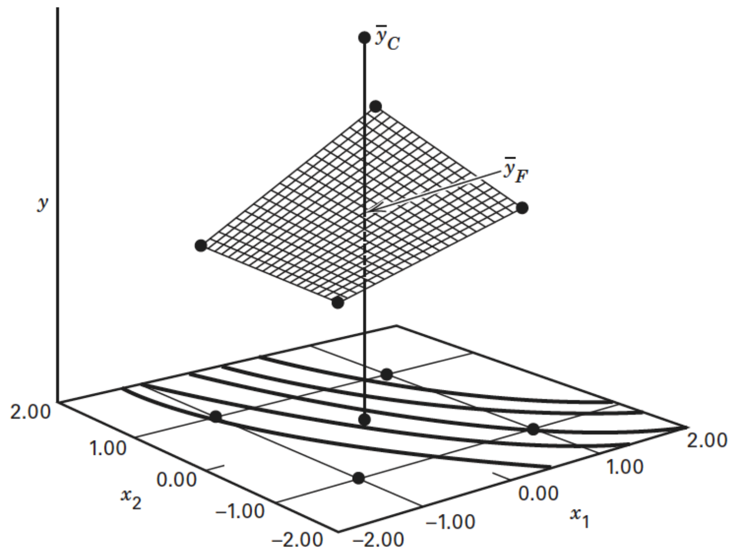
- For the 3^2 design, let x_1 represent factor A and x_2 represent factor B. A regression model relating the response y to x_1 and x_2 that is supported by this design is

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_{12} x_1 x_2 + \beta_{11} x_1^2 + \beta_{22} x_2^2 + \varepsilon$$

- Notice that the addition of a third factor level allows the relationship between the response and design factors to be modeled as a quadratic

Note on the Curvature Concern

- The 3^k design is certainly a possible choice by an experimenter who is concerned about curvature in the response function. However, two points need to be considered:
  - 1. It is not the most efficient way to model a quadratic relationship; the response surface designs are superior alternatives
  - 2. The 2^k design augmented with center points is an excellent way to obtain an indication of curvature, which allows one to keep the size and complexity of the design low and simultaneously obtain some protection against curvature. This sequential strategy of experimentation is far more efficient than running a 3^k factorial design with quantitative factors

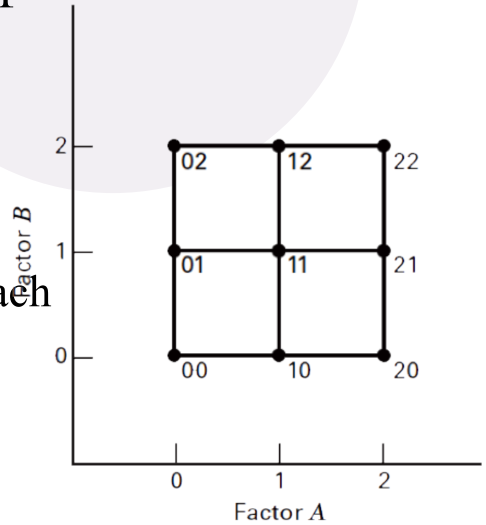


The 3^2 Design

- Two factors, each at three levels -quantitative
- $3^2 = 9$ treatment combinations, 8 degrees of freedom between them:
  - The main effects of A and B each have 2 df
  - The AB interaction has 4 df
- If there are n replicates, there will be $n3^2 - 1$ total df and $3^2(n-1)$ df for error
- For regression

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_{12} x_1 x_2 + \beta_{11} x_1^2 + \beta_{22} x_2^2 + \varepsilon$$

  - Each main effect can be represented by a linear and a quadratic component, each with one single df
  - What about the interaction?



Important Idea Used for Confounding and Taking Fractions

- How we consider three level designs will parallel what we did in two level designs, therefore we may confound the experiment in incomplete blocks or simply utilize a fraction of the design
- A p-way interaction has 2^p df- More complicated now!
  - To confound a main effect (2 df) with a 2-way interaction (4 df) we need to partition the interaction into 2 orthogonal pieces with 2 df each, then confound the main effect with one of the 2 pieces(There will be 2 choices)
  - To confound a main effect with a 3-way interaction? Break the interaction into 4 pieces with 2 df each
- Each piece of the interaction is represented by a psuedo-factor with 3 levels
- The method given using the Latin squares is quite simple(There is some clever modulus arithmetic, but the details are not important)
- The important idea is that just as with the 2^k designs, we can purposefully confound to achieve designs that are efficient either because they do not use the entire set of 3^k runs or because they can be run in blocks which do not disturb our ability to estimate the effects of most interest

Define the Pseudo Factors/Pseudo-Interaction Effects

- In 3^k designs we prefer coding {0,1,2}, a generalization of the {0,1} coding used in the 2^k designs
- For the AB interaction, we define two pseudo factors, called the AB component and the AB^2 component:

$$L_{AB} = X_1 + X_2 \ (\text{mod}3);\ L_{AB^2} = X_1 + 2X_2 \ (\text{mod}3)$$

- Each of these main effects or pseudo interaction components has 3 levels and therefore 2 df which allow us to confound a main effect or either component of the interaction AB
- Pseudo Factors have no actual meaning. However, this rather arbitrary partitioning of the AB interaction into two orthogonal two-df components is very useful in constructing more complex designs. Also, there is no connection between the AB and AB^2 components of interaction and the sums of squares for $AB_{L\times L}$, $AB_{L\times H}$, $AB_{H\times L}$ and $AB_{H\times H}$

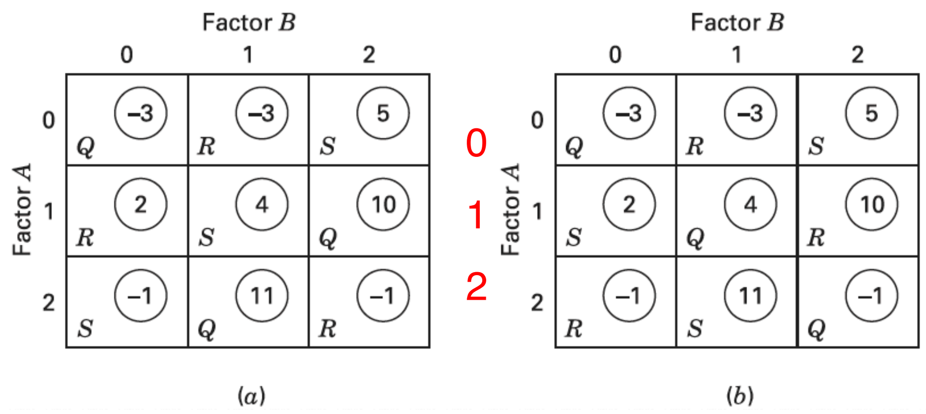
A	B	AB	AB^2
0	0	0	0
1	0	1	1
2	0	2	2
0	1	1	2
1	1	2	0
2	1	0	1
0	2	2	1
1	2	0	2
2	2	1	0

Partition of Two-Factor Interaction *AB*

- Based on orthogonal Latin squares
- Orthogonal means if one square is superimposed on the other, each letter in the first square will appear exactly once with each letter in the second square
- Tool Life Experiment, p209
- The sum of squares computed from square (a) is the AB component of interaction, and the sum of squares computed from square (b) is the AB^2 component of interaction. The components AB and AB^2 each have two degrees of freedom

Data for Tool Life Experiment

Total Angle (degrees)	Cutting Speed (in/min) 125		150		175		$y_{i..}$
15	−2, −1	−3	−3, 0	−3	2, 3	5	−1
20	0, 2	2	1, 3	4	4, 6	10	16
25	−1, 0	−1	5, 6	11	0, −1	−1	9
$y_{.j.}$	−2		12		14		$24 = y_{...}$



Another Partition of Two-Factor Interaction AB

- Another method: Subdividing AB into the four single-df components corresponding to $AB_{L\times L}$, $AB_{L\times H}$, $AB_{H\times L}$ and $AB_{H\times H}$. This can be done by fitting the terms $\beta_{12}x_1x_2$, $\beta_{122}x_1x_2^2$, $\beta_{112}x_1^2x_2$ and $\beta_{1122}x_1^2x_2^2$, respectively

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_{11}x_1^2 + \beta_{22}x_2^2 + \beta_{12}x_1x_2 + \beta_{122}x_1x_2^2 + \beta_{112}x_1^2x_2 + \beta_{1122}x_1^2x_2^2 + \varepsilon$$

- SS_{AB} = 8+2.667+42.667+8 =61.334
- This method partitions the interaction SS's into 1 df sums of squares associated with a polynomial, however this is just polynomial regression, does not seem to be readily applicable to creating interpretable confounding patterns

Response: life

	Df	Sum Sq	Mean Sq	F value	Pr(>F)	
angle	1	8.333	8.333	5.7692	0.0397723	*
speed	1	21.333	21.333	14.7692	0.0039479	**
I(angle^2)	1	16.000	16.000	11.0769	0.0088243	**
I(speed^2)	1	4.000	4.000	2.7692	0.1304507	
I(angle * speed)	1	8.000	8.000	5.5385	0.0430650	*
I(angle^2 * speed)	1	2.667	2.667	1.8462	0.2073056	
I(angle * speed^2)	1	42.667	42.667	29.5385	0.0004137	***
I(angle^2 * speed^2)	1	8.000	8.000	5.5385	0.0430650	*
Residuals	9	13.000	1.444			

Data for Tool Life Experiment

Total Angle (degrees)	Cutting Speed (in/min) 125		150		175		$y_{i..}$
15	−2, −1	−3	−3, 0	−3	2, 3	5	−1
20	0, 2	2	1, 3	4	4, 6	10	16
25	−1, 0	−1	5, 6	11	0, −1	−1	9
$y_{.j.}$	−2		12		14		$24 = y_{...}$

The 3^3 Design

- <u>Goals</u>:
  - To describe the AOV for it
  - To look at the connection between confounding in blocks and 3^{k-p} fractional factorials
- 3^3 treatments in 3 blocks with the *ABC* pseudo factor confounded with blocks, i.e., ->
- The three (color coded) blocks are determined by the levels of the *ABC* component of the three-way interaction which is confounded with blocks
- If replicate =1, 26 df
- Suppose this design is Rep 1 and add Reps 2, 3, 4, then it would result in a total of 3 × 4 = 12 blocks

L_{ABC}		
0	**1**	**2**
0, 0, 0	1, 0, 0	2, 0, 0
2, 1, 0	0, 1, 0	1, 1, 0
1, 2, 0	2, 2, 0	0, 2, 0
2, 0, 1	0, 0, 1	1, 0, 1
1, 1, 1	2, 1, 1	0, 1, 1
0, 2, 1	1, 2, 1	2, 2, 1
1, 0, 2	2, 0, 2	0, 0, 2
0, 1, 2	1, 1, 2	2, 1, 2
2, 2, 2	0, 2, 2	1, 2, 2

Partition of DF

Inter-block part of the analysis: Average the nine observations in each block and got a single number, then analyze those 12 numbers

- Blocks is 11 df
  - Reps is 3 df
  - Blocks nested in Reps with 2 df $\times$ 4 *Reps* = 8 df
    - ABC with 2 *df* and Rep $\times$ ABC with 6 *df*
- A $\times$ B $\times$ C would only have $2^3 - 1 \times 2 =$ 6 df because one component (ABC) is confounded with blocks
- All of other unconfounded effects remain the same df
- Total is $(3^3 \times 4) - 1 = 107$ df
- Error is then 72 df

AOV					DF
Blocks					11
	Reps			3	
	Blocks(Rep)			8	
		ABC	2		
		Rep × ABC	6		
A					2
B					2
C					2
A × B					4
A × C					4
B × C					4
A × B × C					6
Error					72
Total					107

An Alternate Design - Partial Confounding

- In thinking about how this design should be implemented a good idea would be to followed this first Rep with a second Rep that confounds L_{AB^2C}, confound L_{ABC^2} in Rep three, and finally confound $L_{AB^2C^2}$ in fourth Rep
- Now we could estimate all four components of the three-way interactions because in three of the Reps they would be unconfounded
- There is no information available in the way we had approached it previously. There is lots of information available using this partial confounding strategy of the three-way interactions

3^{k-p} designs - Fractional Factorial 3-level Designs

- The whole point of looking at this structure is because sometimes we want to only conduct a fractional factorial
- Construct a 3^{3-1} design, which is a 1/3 fraction of a 3^3 design. $N = 3^{3-1} = 3^2 = 9$, the total number of runs
- For the case where we use the L_{ABC} pseudo factor to create the design, we would use just one block of the design above, and the alias structure:

$$
\begin{aligned}
I &= ABC \\
A &= A \times ABC = (A^2BC) = AB^2C^2 \\
A &= A \times (ABC)^2 = A^3B^2C^2 = (B^2C^2)^2 = BC \\
B &= B \times ABC = AB^2C \\
B &= B \times (ABC)^2 = A^2B^3C^2 = AC \\
C &= C \times ABC = ABC^2 \\
C &= C \times (ABC)^2 = A^2B^2C^3 = (A^2B^2)^2 = AB
\end{aligned}
$$

A is confounded with part of the 3-way and part of the 2-way interaction, likewise for B and for C

3^{3-1} Fractional Factorial Design

- This design only has 9 observations, we can only include the main effects
- A, B and C main effects are estimable
- Partition the treatment combinations for one Rep of the experiment using the levels of L_{ABC}
- It is of interest to notice that a 3^{3-1} fractional factorial design is also a design we previously discussed:
- Look at the first column. Call A the row effect, B the column effect and C the Latin letters, or in this case 0, 1, 2. Use this procedure to assign the treatments to the square. This is how we get a 3 × 3 Latin square. So, a one third fraction of a 3^3 design is the same as a 3×3 Latin square design that we saw earlier

AOV	DF
A	2
B	2
C	2
Error	2
Total	8

L_{ABC}		
0	**1**	**2**
0, 0, 0	1, 0, 0	2, 0, 0
2, 1, 0	0, 1, 0	1, 1, 0
1, 2, 0	2, 2, 0	0, 2, 0
2, 0, 1	0, 0, 1	1, 0, 1
1, 1, 1	2, 1, 1	0, 1, 1
0, 2, 1	1, 2, 1	2, 2, 1
1, 0, 2	2, 0, 2	0, 0, 2
0, 1, 2	1, 1, 2	2, 1, 2
2, 2, 2	0, 2, 2	1, 2, 2

Note on Latin Squares

- It is important to see the connection here
- We have three factors: *A, B, C*, and before when we talked about Latin squares, two of these were blocking factors and the third was the treatment factor. We could estimate all three main effects and we could not estimate any of the interactions. And now you should be able to see why: <u>The interactions are all aliased with the main affects</u>

L_{ABC}

0	1	2
0, 0, 0	1, 0, 0	2, 0, 0
2, 1, 0	0, 1, 0	1, 1, 0
1, 2, 0	2, 2, 0	0, 2, 0
2, 0, 1	0, 0, 1	1, 0, 1
1, 1, 1	2, 1, 1	0, 1, 1
0, 2, 1	1, 2, 1	2, 2, 1
1, 0, 2	2, 0, 2	0, 0, 2
0, 1, 2	1, 1, 2	2, 1, 2
2, 2, 2	0, 2, 2	1, 2, 2

$$
\begin{array}{cc|ccc|}
 & C & \multicolumn{3}{c}{B} \\
 & & 0 & 1 & 2 \\
\hline
 & 0 & 0 & 2 & 1 \\
A & 1 & 2 & 1 & 0 \\
 & 2 & 1 & 0 & 2 \\
\hline
\end{array}
$$

Start

Another Partial- Another Latin Square

- Look at another component L_{AB^2C} of the three factor interaction $A \times B \times C$:

$$L_{AB^2C} = X_1 + 2X_2 + X_3 \pmod 3$$

- First, plug in the levels of X_1, X_2 and X_3 from the levels of A, B and C to generate the column L_{AB^2C}
- Then, assign treatments to the level of $L_{AB^2C} = 0$, you get an arrangement that follows (only the principle block filled in):
- This one third fraction is also a Latin square :

L_{AB^2C}		
0	**1**	**2**
0, 0, 0		
1, 1, 0		
2, 2, 0		
2, 0, 1		
0, 1, 1		
1, 2, 1		
1, 0, 2		
2, 1, 2		
0, 2, 2		

Since the ABC and the AB²C are orthogonal to each other - they partition the A×B×C interaction - the two Latin squares we constructed are orthogonal Latin Squares

			B	
	C	**0**	**1**	**2**
	0	0	1	2
A	**1**	2	0	1
	2	1	2	0

This is a Resolution III design

The Next Level Example - Four Factors

- Now let's take a look at the 3^{4-2} design. How do we create this design?
- In this case we would have to pick 2 generators. We have four factors, A, B, C and D. So, let's say we will begin (trial and error) by selecting

$$I = ABC = BCD$$

as our generators then we will also have the generalized interactions between those generators which are also included. Thus we will also confound:

$$\mathrm{ABC} \times \mathrm{BCD} = \mathrm{AB^2C^2D}$$
$$\mathrm{ABC} \times (\mathrm{BCD})^2 = \mathrm{AD^2}$$

- What is the resolution of this design?

3^{4-2} Design

- Let's try again, how about

$$I = ABC = BC^2D$$

as our generators? This confounds:

$$ABC \times BC^2D = AB^2D$$
$$ABC \times (BC^2D)^2 = AC^2D^2$$

- A Resolution III design
- Now, how do we generate the design? It is still a design with only nine observations, or a 1/9th fraction of a 3^4 design or 81 observations
- Write out the basic design with A and B(with nine observations), then use our generators to give C and D
- Use ABC such that:
  - $L_{ABC} = 0$: this principle fraction implies that $X_3 = 2X_1 + 2X_2$ (mod 3)
  - $L_{BC^2D} = 0$: this implies that $X_4 = 2X_2 + X_3$ (mod 3)

Explain the Use of Generators

- If we were confounding this in blocks we will want a principal block where these two defining relationships are both zero. You will see that by defining X_3 and X_4 in this way results in ABC being equal to zero
- Make sure that you understand how column C was generated by the function $X_3 = 2X_1 + 2X_2$ (mod 3) yet still preserves the principle implied where $L_{ABC} = 0$
- Also, by the same process column D was generated using the function $X_4 = 2X_2 + X_3$ (mod 3) in such a way that it preserves the principle implied where in $L_{BC^2D} = 0$

A	B	C	D
0	0	0	0
1	0	2	2
2	0	1	1
0	1	2	1
1	1	1	0
2	1	0	2
0	2	1	2
1	2	0	1
2	2	2	0

Rollover these values!

Graeco-Latin Square

- The 3^{4-2} design is equivalent to the Graeco-Latin square: There are two Latin squares, one for each component, C and D, superimposed

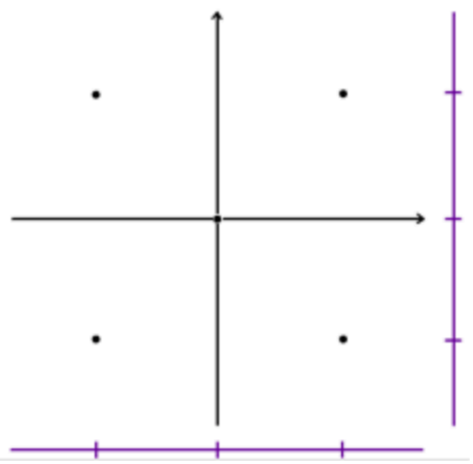
$$
\begin{array}{cc|ccc|}
C & & \multicolumn{3}{c}{B} \\
 & & 0 & 1 & 2 \\
\hline
 & 0 & 0 & 2 & 1 \\
A & 1 & 2 & 1 & 0 \\
 & 2 & 1 & 0 & 2 \\
\hline
\end{array}
\qquad
\begin{array}{cc|ccc|}
D & & \multicolumn{3}{c}{B} \\
 & & 0 & 1 & 2 \\
\hline
 & 0 & 0 & 1 & 2 \\
A & 1 & 2 & 0 & 1 \\
 & 2 & 1 & 2 & 0 \\
\hline
\end{array}
$$

Start

So we can see that the Graeco-Latin Square with three treatments is simply a fractional factorial of this 3^4 design!

Mixed Factorials

- We have been talking about 2-level designs and 3-level designs. 2 level designs for screening factors and 3 level designs analogous to the 2 level designs, but the beginning of our discussion of response surface designs
- If we take a 2 level design that has center points->
- Then, if you project into the A axis or the B axis, you have three distinct values, -1, 0, and +1
- In the main effect sense, <u>a two level design with center points gives you three levels</u>. This was our starting point towards moving to a three level design
- Three-level designs require a whole lot more observations. These designs grow very fast so we have to look for more efficient designs



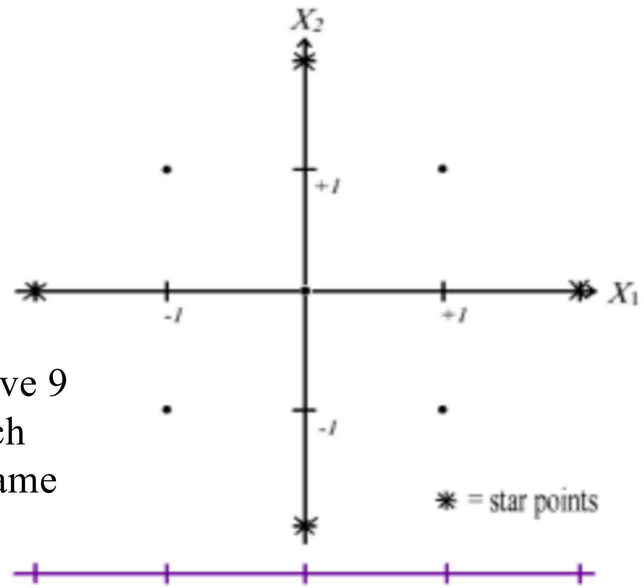
Next Level Designs

- We think of factors with 4 or 5 levels, or designs with combinations of 2, 3, 4, or 5 levels of factors
- In Analysis of Variance, it didn't distinguish between these factors. Instead, you looked at general machinery for factors with any numbers of level. What is new here is thinking about writing efficient designs
- Let's say you have a $2^3 \times 3^2$ - this would be a mixed level design with $8 \times 9 = 72$ observations in a single replicate. So this is growing pretty rapidly! As this gets even bigger we could trim the size of this by looking at fractions for instance, 2^{3-1}, a fractional factorial of the first part. And, as these numbers of observations get larger you could look at crossing fractions of factorial designs

Notes

- About factors with 4 levels – 2^2
  - This design is 2^2, nothing new here. By using the machinery of the 2^k designs you can always take a factor with four levels and call it the four combinations of 2^2

- About factors with 5 levels
  - Think quantitative - we should fit a polynomial regression function
  - This leads us to a whole new class of designs that we will look at next - Response Surface Designs



A 2^2 design with center points, then add star points. Instead of 25 points, we only have 9 points: It is a more efficient design but still in a projection we have five levels in each direction. What we want is enough points to estimate a response surface but at the same time keep the design as simple and with as few observations as possible

Introduce RSM

- The primary reason that we looked at the 3^k designs is to understand the confounding that occurs. When we have quantitative variables we will generally not use a 3 level design
- 3 level designs are not as practical as central composite designs(CCD, 中心组合设计)
- We will next consider response surface designs to address the goals of fitting a response surface model