$$\prod_{i=1}^{n} \frac{x_i^{\alpha-1} e^{-\frac{x_i}{\beta}}}{\Gamma(\alpha)\beta^{\alpha}} = \frac{\left(\prod x_i\right)^{\alpha-1} e^{-\frac{1}{\beta}\left(\sum x_i\right)}}{\Gamma(\alpha)^n \beta^{n\alpha}}$$

Homework 3

Oct. 7, 2019

NOTE: Homework 3 is due next Thursday (Oct. 20, 2019).

1. Let r.v.'s $X_1, \cdots, X_n$ *i.i.d.* $\sim N(\theta, \theta^2)$, is $\bar{X}$ a sufficient statistic of θ?
2. Let $X_1, \cdots, X_m$ *i.i.d.* $\sim N(a, \sigma^2)$, $Y_1, \cdots, Y_n$ *i.i.d.* $\sim N(b, \sigma^2)$ and X_i's and Y_j's are independent. Let $\bar{X} = \sum_{i=1}^m X_i/m$, $\bar{Y} = \sum_{j=1}^n Y_j/n$, and
$$S^2 = \frac{1}{n+m-2}\left[\sum_{i=1}^{m}(X_i - \bar{X})^2 + \sum_{j=1}^{n}(Y_j - \bar{Y})^2\right].$$
Show that $(\bar{X}, \bar{Y}, S^2)$ is a sufficient and complete statistic of (a, b, σ^2).
3. Let r.v.'s $X_1, \cdots, X_n$ be a random sample from a gamma(α, β) population. Find a two-dimentional sufficient statistic for (α, β)?
4. Let $X_1, \cdots, X_n$ be a random sample from the distribution with p.d.f.
$$f(x;\theta) = \frac{1}{2\theta}\exp\left\{-\frac{|x|}{\theta}\right\}, \quad -\infty < x < +\infty,\ \theta > 0.$$
Show that $T = \sum_{i=1}^n |X_i|$ is a sufficient and complete statistic of θ.
5. Let r.v.'s $X_1, \cdots, X_n$ *i.i.d.* $\sim U(\theta, 2\theta)$, $\theta > 0$, show that $(X_{(1)}, X_{(n)})$ is sufficient but not complete.
6. Let $X_1, \cdots, X_n$ be a random sample from two parameter exponential distribution with p.d.f.
$$f(x;\lambda,\mu) = \lambda^{-1}\exp\left\{-\frac{x-\mu}{\lambda}\right\} I_{\{x>\mu\}},$$
where $0 < \lambda < +\infty$, $-\infty < \mu < +\infty$ are two unknown parameters. Show that
   (i) $\left(X_{(1)}, \sum_{i=1}^n X_{(i)}\right)$ is sufficient for (λ, μ);
   (ii) $X_{(1)}$ is independent of $\sum_{i=1}^n (X_i - X_{(1)})$.
7. Let X be one observation from the p.d.f. $\quad \left(\frac{\theta}{2}\right)^{|x|}(1-\theta)^{1-|x|}$
$$f(x;\theta) = \left(\frac{1}{\theta}\right)^{|x|}(1-\theta)^{1-|x|}, x = -1, 0, 1; 0 \le \theta \le 1.$$
   (i) Is X a complete sufficient statistic?
   (ii) Is $|X|$ a complete sufficient statistic?
   (iii) Does $f(x;\theta)$ belong to the exponential class?

$E_\theta(\varphi(T(X))) = 0$

$\Rightarrow P(X=0)(1-\theta) + P(X\neq 0)\frac{\theta}{2}$

$\varphi(0) + \varphi(1) = 0$

$\frac{1}{\theta}^{\sum|x_i|}(1-\theta)^{n-\sum|x_i|}$

$(1-\theta)\ (\theta(1-\theta))^{-|x|}$

$(1-\theta)^n \cdot [\theta(1-\theta)]^{1-\sum|x_i|}$