Asset Pricing with Spatial Interaction

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Motivation

- Spatial interaction is important in the study of real estate market.
- Spatial econometrics use spatial models to study the effect of spatial correlation on economic variables.
- Spatial econometric models improve the estimation of house prices.
- Literature: Cressie (1993), Anselin (Spatial Econometrics, 1988), Kelley and Pace (Intro. to Spatial Econometrics, 2009) , etc.
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CSI Indices Futures

Capital Asset Pricing Model (CAPM)

- Mean-Variance Analysis
  - solve portfolio selection problem using only mean and covariance of returns (Markowitz, 1952).
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$$\tilde{r}_i - r_f = \alpha_i + \beta_i(\tilde{r}_M - r_f) + \tilde{\epsilon}_i, \; E[\tilde{\epsilon}_i] = 0$$

  - If investors hold mean-variance efficient portfolio, then in equilibrium, $\alpha_i = 0$ (Sharpe, 1963, Lintner, 1965).
  - Asset risk premium:

$$E[\tilde{r}_i] - r_f = \beta_i(E[\tilde{r}_M] - r_f)$$

  - Some empirical evidence suggest that CAPM may be misspecified, e.g., size effect (Banz, 1981), value effect (Statman, 1980).

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- No asymptotic arbitrage implies that $\alpha_i \approx 0$ (Ross, 1976).
- Asset risk premium:

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National Stock Indices of 11 Euro-zone Countries

Country	Austria	Belgium	Finland	France	Germany	Greece	Ireland	Italy	Netherlands	Portugal	Spain
Stock Index	ATX	BEL20	HEX	CAC	DAX	ASE	ISEQ	FTSEMIB	AEX	BVLX	IBEX

Table: The stock indices of the 11 Eurozone countries with developed stock markets.

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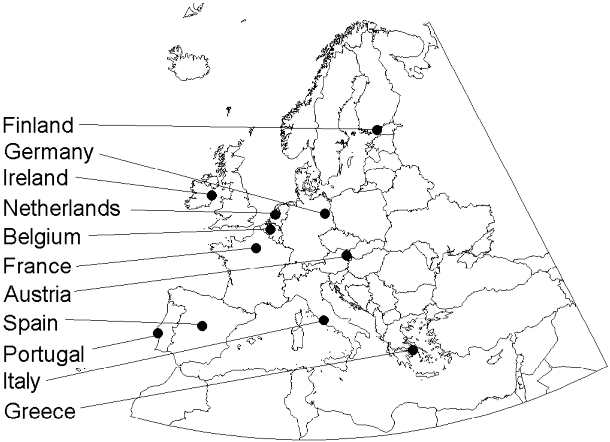

Data: monthly returns of national stock indices during 01/2001–10/2013.

Four Factors of Fama and French (2012, JFE)

- The four factors: the market factor (MKT), the size factor (SMB), the value factor (HML), and the momentum factor (MOM).
- The data of the four factors are downloaded from the website of Kenneth French.
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$$\tilde{r}_{it} - r_{ft} = \alpha_i + \sum_{k=1}^{4} \beta_{ik} \tilde{f}_{kt} + \tilde{\epsilon}_{it}, i = 1, \ldots, 11; t = 1, \ldots, T; \quad (1)$$

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The Spatial Correlation among Residuals of Stock Indices Returns

- To investigate potential spatial correlations among the 11 return residuals, we consider the following model for $\tilde{\epsilon}_t = (\epsilon_{1t}, \epsilon_{2t}, \ldots, \epsilon_{11t})'$:

$$\tilde{\epsilon}_t = \kappa W \tilde{\epsilon}_t + a + \tilde{\xi}_t, t = 1, 2, \ldots, T, \tag{2}$$

- We fit the model (2) to the residuals and found that κ is statistically positive with a 95% confidence interval of $[0.02, 0.21]$.
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Zero-intercept Constraint is Rejected

- We carry out the hypothesis test of the no asymptotic arbitrage constraint (i.e., zero-intercept constraint) of the APT model:

$$H_0 : \alpha_1 = \alpha_2 = \cdots = \alpha_{11} = 0; \; H_1 : \text{not all the } \alpha_i \text{ are zero.} \tag{3}$$

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- Economic rationale of asset pricing: optimal portfolio choice, equilibrium, and absence of arbitrage
- Existing asset pricing models have not taken into account spatial interaction.
- An unanswered question: what is the economic implication of spatial interaction on asset returns?

$$\text{spatial econometric model} + \text{equilibrium/no arbitrage} \Rightarrow ?$$

- We characterize how spatial interaction affects expected asset returns.
- We develop estimation and testing procedure for implementing the models.
- We test our asset pricing model by an empirical study of Euro-zone stock indices and S&P/Case-Shiller home price indices futures

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- If $1/\rho$ is not an eigenvalue of W, then the model can be written as

$$\tilde{r} = (I - \rho W)^{-1}\alpha + (I - \rho W)^{-1}\tilde{\epsilon}.$$

- Both the mean and covariance of $\tilde{r}$ depend on ρ and W:

$$\mu = E[\tilde{r}] = (I - \rho W)^{-1}\alpha,$$
$$\Sigma = \mathrm{Cov}(\tilde{r}) = (I - \rho W)^{-1} V (I - \rho W')^{-1}.$$

Outline

Mean-variance Analysis without a Risk Free Asset

$$\min_{\phi} \ \frac{1}{2}\phi'\Sigma\phi$$
$$s.t. \ \phi'\mu = e,$$
$$\phi'\mathbf{1} = 1.$$

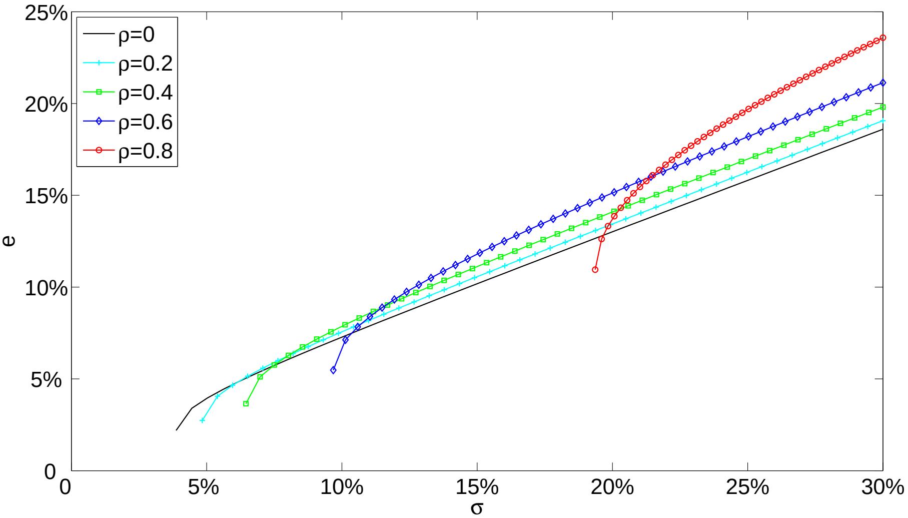
The optimal portfolio weights are

$$\phi^* = g + eh,$$

- g and h are functions of μ and Σ
- Mean-variance efficient frontier: $e = (A + \sqrt{D(F\sigma^2 - 1)})/F$
- ρ affects ϕ^* and efficient frontier.

Efficient Frontiers are Sensitive to Spatial Interaction

Efficient frontier with no risk free asset for $\rho = 0, 0.2, 0.4, 0.6,$ and 0.8



Joint Model of Traded Assets and Futures with Spatial Interaction

- Real estate indices futures are traded, but real estate indices are not traded.
- There are n_1 ordinary asset returns $(\tilde{r}_1, \ldots, \tilde{r}_{n_1})$.
- There are n_2 futures contracts. Define the return of the i-th futures contract as:

$$\tilde{r}_{n_1+i} := \frac{\tilde{F}_{i,1} - F_{i,0}}{F_{i,0}}, \ i = 1, \ldots, n_2$$

- Assume the $n = n_1 + n_2$ returns $\tilde{r} = (\tilde{r}_1, \ldots, \tilde{r}_{n_1}, \tilde{r}_{n_1+1}, \ldots, \tilde{r}_n)'$ satisfy the spatial interaction model

$$\tilde{r} = \rho W \tilde{r} + \alpha + \tilde{\epsilon}$$

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Spatial Capital Asset Pricing Model with Futures (S-CAPM with futures)

Theorem (Spatial Capital Asset Pricing Model)

Let $\tilde{r}_M$ be the return of the market portfolio. If each investor holds mean variance efficient portfolio, then in equilibrium, $\tilde{r}_M$ is mean-variance efficient, and
(i) for the ordinary assets,

$$E[\tilde{r}_i]-r_f = \frac{Cov(\tilde{r}_i,\tilde{r}_M)}{Var(\tilde{r}_M)}(E[\tilde{r}_M]-r_f) = \frac{\phi_M'\Sigma\eta_i}{\phi_M'\Sigma\phi_M}(E[\tilde{r}_M]-r_f),\ i=1,2,\dots,n_1.$$

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Implication of S-CAPM

- Let

$$1_{n_1,n_2} := (\underbrace{1,\ldots,1}_{n_1},\underbrace{0,\ldots,0}_{n_2})'.$$

- $\tilde{r} - r_f 1_{n_1,n_2}$ is the excess returns of the n assets
- Consider the spatial econometric model

$$\tilde{r} - r_f 1_{n_1,n_2} = \rho W(\tilde{r} - r_f 1_{n_1,n_2}) + \bar{\alpha} + \beta(\tilde{r}_M - r_f) + \tilde{\epsilon},$$
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Factor Pricing Model with Spatial Interaction

- $\tilde{r} = (\tilde{r}_1, \ldots, \tilde{r}_{n_1}, \tilde{r}_{n_1+1}, \ldots, \tilde{r}_n)'$, where $\tilde{r}_{n_1+1}, \ldots, \tilde{r}_n$ are returns of futures.
- $\tilde{f} = (\tilde{f}_1, \tilde{f}_2, \ldots, \tilde{f}_K)'$ are factors
- The factor model with spatial interaction:

$$\tilde{r} = \rho W \tilde{r} + \alpha + B\tilde{f} + \tilde{\epsilon}$$

- ρ is the spatial parameter.
- α are asset-specific constants.
- $B \in \mathbb{R}^{n \times K}$ is the factor loading matrix.
- $E[\tilde{\epsilon}] = 0$, $E[\tilde{\epsilon}\tilde{\epsilon}'] = V$.

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A Sequence of Economies

Consider a sequence of economies with increasing sets of risky assets

- In the n-th economy there are n risky assets and their returns satisfy:

$$\tilde{r}^{(n)} = \rho^{(n)} W^{(n)} \tilde{r}^{(n)} + \alpha^{(n)} + B^{(n)} \tilde{f} + \tilde{\epsilon}^{(n)}, \ n = 1, 2, \ldots$$

- The $(n+1)$-th economy includes all the assets in the n-th economy.
- The K factors $\tilde{f}$ are fixed.
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Asymptotic arbitrage (Huberman, 1982; Ingersoll, 1984)

- $\mathbf{1}_{n_1,n_2} = (\underbrace{1, \ldots, 1}_{n_1}, \underbrace{0, \ldots, 0}_{n_2})'$
- A zero-cost portfolio in the n-th economy has positions $h^{(n)}$ in the n risky assets and $-(h^{(n)})'\mathbf{1}_{n_1,n_2}$ in the risk-free asset
- The payoff of the portfolio is $(h^{(n)})'(\tilde{r}^{(n)} - r_f \mathbf{1}_{n_1,n_2})$.
- Asymptotic arbitrage is the existence of a subsequence of zero-cost portfolios $\{h^{(m_k)}, k = 1, 2, \ldots\}$ that satisfy

$$\lim_{k\to\infty} E[(h^{(m_k)})'(\tilde{r}^{(m_k)} - r_f \mathbf{1}_{n_1,n_2})] \geq \delta, \text{ for some } \delta > 0,$$

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Spatial Arbitrage Pricing Theory with Futures (S-APT with Futures)

Theorem (Spatial Arbitrage Pricing Theory)

Suppose that there is no asymptotic arbitrage. Then there exists a constant $0 < A < \infty$ *and for each n there exist* $\lambda^{(n)} = (\lambda_1^{(n)}, ..., \lambda_K^{(n)})'$*, such that*

$$(\bar{\alpha}^{(n)})'(V^{(n)})^{-1}\bar{\alpha}^{(n)} \leq A, \text{ where}$$

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- S-APT implies that

$$\bar{\alpha}^{(n)} \approx 0 \Leftrightarrow \alpha^{(n)} \approx (I - \rho^{(n)}W^{(n)})\mathbf{1}_{n_1,n_2}r_f + B^{(n)}\lambda^{(n)}.$$

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S-APT Implications on Asset Risk Premiums

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Identifying Factor Risk Premium λ

Assume that the factors $\tilde{f} = \tilde{g} - E[\tilde{g}]$, where $\tilde{g} = (\tilde{g}_1, \ldots, \tilde{g}_K)'$ are payoffs of zero-cost portfolios

- The factor model

$$\tilde{r} = \rho W \tilde{r} + \alpha + B(\tilde{g} - E[\tilde{g}]) + \tilde{\epsilon}$$

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$$\tilde{r} - r_f \mathbf{1}_{n_1,n_2} = \rho W(\tilde{r} - r_f \mathbf{1}_{n_1,n_2}) + \bar{\alpha} + B\tilde{g} + \tilde{\epsilon},$$

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- By forming zero-cost portfolios without systematic risk and assuming absence of asymptotic arbitrage, we get

$$\bar{\alpha} \approx 0, \text{ i.e., } \alpha \approx (I - \rho W)\mathbf{1}_{n_1,n_2} r_f + BE[\tilde{g}].$$

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Identifying Factor Risk Premium λ (Continued)

- Recall that S-APT states

$$\alpha \approx (I - \rho W)\mathbf{1}_{n_1,n_2} r_f + B\lambda$$

- λ can then be identified as

$$\lambda = E[\tilde{g}]$$

Outline

Econometric Model

Assume the factors $\tilde{f} = \tilde{g} - E[\tilde{g}]$. Then the factor model becomes

$$\tilde{r} - r_f \mathbf{1}_{n_1,n_2} = \rho W(\tilde{r} - r_f \mathbf{1}_{n_1,n_2}) + \bar{\alpha} + B\tilde{g} + \tilde{\epsilon}$$

A multi-period version of the model

$$\tilde{y}_t := \tilde{r}_t - r_{ft}\mathbf{1}_{n_1,n_2}$$
$$\tilde{y}_t = \rho W \tilde{y}_t + \bar{\alpha} + B\tilde{g}_t + \tilde{\epsilon}_t, t = 1, \ldots, T$$
$$(\tilde{y}_t, \tilde{g}_t), t = 1, \ldots, T \text{ are i.i.d.}$$
$$\tilde{\epsilon}_t \mid \tilde{g}_t \sim N(0, \sigma^2 I_n)$$

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Conditional Log Likelihood Function

The conditional log likelihood function of the model is

$$\sum_{t=1}^{T} l(\tilde{y}_t \mid \tilde{g}_t, \theta),$$

where

$$l(\tilde{y}_t \mid \tilde{g}_t, \theta) = -\frac{n}{2}\log(2\pi\sigma^2) + \frac{1}{2}\log(\det((I_n - \rho W')(I_n - \rho W))) \\ - \frac{1}{2\sigma^2}(\tilde{y}_t - \rho W\tilde{y}_t - B\tilde{g}_t - \bar{\alpha})'(\tilde{y}_t - \rho W\tilde{y}_t - B\tilde{g}_t - \bar{\alpha}).$$

Identifiability of Parameters

Definition (Neway and McFadden, 1994)

θ_0 is identifiable if for any $\theta \neq \theta_0$, it holds that

$$P(I(\tilde{y}_t \mid \tilde{g}_t, \theta) \neq I(\tilde{y}_t \mid \tilde{g}_t, \theta_0)) > 0.$$

Regularity of W

Definition

The spatial weight matrix W is not regular, if there exist $c_1 > 0$ and $c_2 \geq 0$, such that

$$\sum_{k=1}^{n} W_{ki}^2 = c_1, \ \forall i = 1, \ldots, n, \tag{4}$$

$$\sum_{k=1}^{n} W_{ki} W_{kj} = c_2 (W_{ij} + W_{ji}), \ \forall 1 \leq i < j \leq n. \tag{5}$$

- If W is not regular, then the elements of W satisfy $n(n+1)/2$ constraints.
- Unless W is very carefully constructed to satisfy these constraints, W is usually regular.

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$$\sum_{k=1}^{n} W_{ki} W_{kj} = c_2 (W_{ij} + W_{ji}), \ \forall 1 \leq i < j \leq n. \tag{5}$$

- If W is not regular, then the elements of W satisfy $n(n+1)/2$ constraints.
- Unless W is very carefully constructed to satisfy these constraints, W is usually regular.

Regularity of W

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Identifiability Condition

Proposition

Any θ_0 is identifiable if W is regular. More generally, a particular θ_0 is identifiable if and only if W satisfies one of the following conditions:

(i) *W is regular.*

(ii) *W is not regular and corresponds to the unique pair (c_1, c_2) in (4) and (5), and one of the following conditions hold:*

$$\rho_0 = -\frac{c_2}{c_1}; \tag{6}$$

$$\rho_0 \neq -\frac{c_2}{c_1} \text{ and } \frac{1 - c_2\rho_0}{c_2 + c_1\rho_0} = \rho_0; \tag{7}$$

$$\rho_0 \neq -\frac{c_2}{c_1} \text{ and } \frac{1 - c_2\rho_0}{c_2 + c_1\rho_0} \neq \rho_0 \text{ and } \theta_* := (\rho_*, \bar{\alpha}_*, B_*, \sigma_*^2) \notin \Theta. \tag{8}$$

Asymptotic Properties of MLE

The conditional MLE $\hat{\theta}$ is given by

$$\hat{\theta} = \arg\max_{\theta} \sum_{t=1}^{T} l(\tilde{y}_t \mid \tilde{g}_t, \theta).$$

Theorem

If θ_0 is identifiable, then the conditional MLE $\hat{\theta}$ has consistency and asymptotic normality.

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Hypothesis Testing of S-APT

- Assume $\tilde{g}$ are the payoff of tradable zero-cost portfolio.
- Hypothesis test of S-APT:

$$H_0: \ \bar{\alpha}_0 = 0, \ H_a: \ \bar{\alpha}_0 \neq 0$$

- Likelihood ratio test based on the MLE can be used to test the hypothesis.

Theorem

Under the null hypothesis, the LR test statistic

$$LR = 2\left[\sum_{t=1}^{T} l(\tilde{y}_t \mid \tilde{g}_t, \hat{\theta}) - \sum_{t=1}^{T} l(\tilde{y}_t \mid \tilde{g}_t, \theta^*)\right] \tag{9}$$

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Goodness of Fit

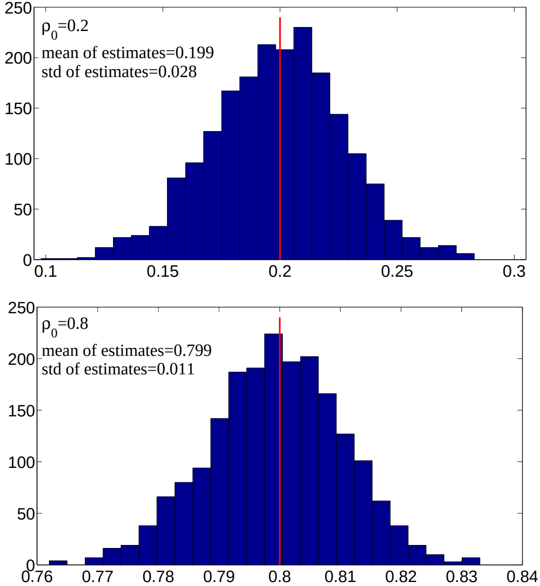
Goodness of fit is measured by adjusted R^2 for estimating the model under the S-APT constraint

$$R_i^2 = 1 - \frac{\mathrm{Var}(\tilde{\epsilon}_i)}{\mathrm{Var}(\tilde{y}_i)} \cdot \frac{T-1}{T-K+1}, i = 1, \cdots, n$$

Simulation Study

- We use the locations of the 20 major cities in the United States.
- An i.i.d. draw of 20 samples from $\mathcal{N}(0, 1)$ is fixed as B_0.
- Asset returns of these 20 assets for $T = 131$ are simulated, using
  - Factor $\tilde{g}$: an i.i.d. draw of $T = 131$ samples from $\mathcal{N}(0.5, 0.5)$.
  - Innovations $\tilde{\epsilon}$: uncorrelated, $\sigma_0^2 = 0.5$.
- Accuracy of M.L.E. is inspected by comparing $\hat{\rho}$ and ρ_0, via 2,000 simulated data sets
- Goodness of fit is inspected by comparing adjusted sample R^2 and adjusted theoretical R^2 for $\tilde{y}_i$

Simulation Study: Accuracy of M.L.E.



Simulation Study: R^2

	$\sigma_0^2 = 0.01$					
	$\tilde{y}_1$	$\tilde{y}_2$	$\tilde{y}_3$	$\tilde{y}_4$	$\tilde{y}_5$	$\tilde{y}_6$
theoretical adj. R^2	0.778	0.982	0.098	0.411	0.966	0.973
sample adj. R^2	0.777	0.982	0.092	0.407	0.966	0.973
	$\sigma_0^2 = 0.5$					
	$\tilde{y}_1$	$\tilde{y}_2$	$\tilde{y}_3$	$\tilde{y}_4$	$\tilde{y}_5$	$\tilde{y}_6$
theoretical adj. R^2	0.239	0.430	0.008	0.138	0.473	0.357
sample adj. R^2	0.235	0.427	0.004	0.134	0.471	0.354

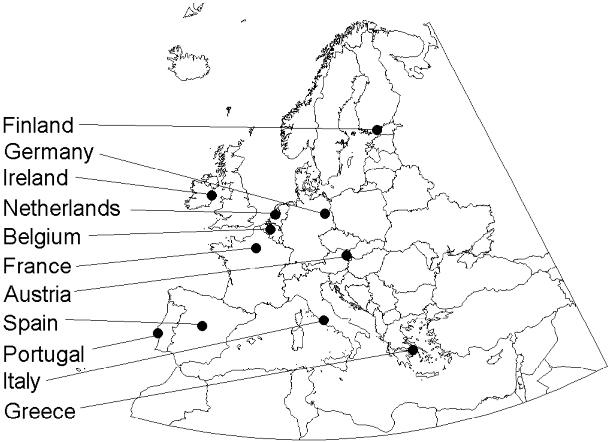
Simulation Study: LR Test

- Size of the LR test is examined at the level of 5% using 10,000 simulated data sets.

Number of data sets	10,000
Proportion of rejection	5.91%

Outline

National Stock Indices of 11 Euro-zone Countries



Data: monthly returns of national stock indices during 01/2001–10/2013.

Four Factors of Fama and French (2012, JFE)

- The four factors: the market factor (MKT), the size factor (SMB), the value factor (HML), and the momentum factor (MOM).
- The data of the four factors are downloaded from the website of Kenneth French.

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A New Credit Factor

	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013
Austria	AAA	AAA	AAA	AAA	AAA	AAA	AAA	AAA	AAA	AAA	AAA	AA+	AA+
Belgium	AA+	AA+	AA+	AA+	AA+	AA+	AA+	AA+	AA+	AA+	AA	AA	AA
Finland	AA+	AAA	AAA	AAA	AAA	AAA	AAA	AAA	AAA	AAA	AAA	AAA	AAA
France	AAA	AAA	AAA	AAA	AAA	AAA	AAA	AAA	AAA	AAA	AA+	AA+	AA+
Germany	AAA	AAA	AAA	AAA	AAA	AAA	AAA	AAA	AAA	AAA	AAA	AAA	AAA
Greece	A	A	A+	A+	A	A	A	A	A-	BBB+/BB+	BB/B/CCC/CC	SD/CCC	B-
Ireland	AAA	AAA	AAA	AAA	AAA	AAA	AAA	AAA	AAA	AA+/AA	AA-/A	A-/BBB+	BBB+
Italy	AA	AA	AA	AA	AA-	AA-	A+	A+	A+	A	BBB+	BBB+	
Netherland	AAA	AAA	AAA	AAA	AAA	AAA	AAA	AAA	AAA	AAA	AAA	AAA	AA+
Portugal	AA	AA	AA	AA	AA-	AA-	AA-	AA-	A+	A-	BBB-	BB	BB
Spain	AA+	AA+	AA+	AA+	AAA	AAA	AAA	AAA	AA+	AA	AA-	A/BBB+/BBB-	BBB-

Table: Credit Ratings of Euro-zone Countries (2001–2013)

- Germany is the only country that maintains top-notch AAA-rating
- Greece, Italy, Ireland, Portugal, and Spain (GIIPS) are the only countries which have breached A-rating

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	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013
Austria	AAA	AAA	AAA	AAA	AAA	AAA	AAA	AAA	AAA	AAA	AAA	AA+	AA+
Belgium	AA+	AA+	AA+	AA+	AA+	AA+	AA+	AA+	AA+	AA+	AA	AA	AA
Finland	AA+	AAA	AAA	AAA	AAA	AAA	AAA	AAA	AAA	AAA	AAA	AAA	AAA
France	AAA	AAA	AAA	AAA	AAA	AAA	AAA	AAA	AAA	AAA	AA+	AA+	AA+
Germany	AAA	AAA	AAA	AAA	AAA	AAA	AAA	AAA	AAA	AAA	AAA	AAA	AAA
Greece	A	A	A+	A+	A	A	A	A	A-	BBB+/BB+	BB/B/CCC/CC	SD/CCC	B-
Ireland	AAA	AAA	AAA	AAA	AAA	AAA	AAA	AAA	AAA	AA+/AA	AA-/A	A-/BBB+	BBB+
Italy	AA	AA	AA	AA	AA-	AA-	A+	A+	A+	A	BBB+	BBB+	
Netherland	AAA	AAA	AAA	AAA	AAA	AAA	AAA	AAA	AAA	AAA	AAA	AAA	AA+
Portugal	AA	AA	AA	AA	AA-	AA-	AA-	AA-	A+	A-	BBB-	BB	BB
Spain	AA+	AA+	AA+	AA+	AAA	AAA	AAA	AAA	AA+	AA	AA-	A/BBB+/BBB-	BBB-

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A New Credit Factor (Continued)

The credit factor is constructed as follows:

$$\tilde{g}_{Credit} = \tilde{r}_{Germany} - (\tilde{r}_{Greece} + \tilde{r}_{Italy} + \tilde{r}_{Ireland} + \tilde{r}_{Portugal} + \tilde{r}_{Spain})/5$$

Candidate Models of APT and S-APT

- Model 1: APT with MKV, SMB, HML and MoM (Fama and French, 2012, JFE)
- Model 2: APT with MKV, SMB, HML, MoM and Credit
- Model 3: APT with MKV, MoM and Credit
- Model 4: APT with MKV, SMB, MoM and Credit
- Model 5: APT with MKV, HML, MoM and Credit
- Model 6: APT with MKV, SMB, HML and Credit
- Model 1s: S-APT with MKV, SMB, HML and MoM
- Model 2s: S-APT with MKV, SMB, HML, MoM and Credit
- Model 3s: S-APT with MKV, MoM and Credit
- Model 4s: S-APT with MKV, SMB, MoM and Credit
- Model 5s: S-APT with MKV, HML, MoM and Credit
- Model 6s: S-APT with MKV, SMB, HML and Credit

Candidate Models of APT and S-APT (Continued)

- An “APT" model means the type of factor model considered in Fama and French (2012, JFE) in which heterogeneous variances of residuals for different returns are assumed.
- The S-APT model assumes homogeneous variances for residuals.
- In the S-APT models, the spatial weight matrix W is defined as $W_{ij} := (s_i d_{ij})^{-1}$ for $i \neq j$ and $W_{ii} = 0$, where d_{ij} is the driving distance between the capital of country i and that of country j and $s_i := \sum_j d_{ij}^{-1}$.

Model Fitting Results

Model	1	2	3	4	5	6
p-value	0.0	0.0	0.0002	0.0005	0.0	0.0
AIC	10150	9912	10048	9955	9939	9889
95% C.I. for κ in residuals	[0.02, 0.21]	[-0.01, 0.18]	[-0.01, 0.18]	[0.02, 0.20]	[-0.07, 0.13]	[-0.04, 0.15]
Model	**1s**	**2s**	**3s**	**4s**	**5s**	**6s**
p-value	0.0002	0.0099	**0.1854**	**0.2912**	0.0078	0.0002
AIC	8826	8625	**8793**	**8730**	8696	8668
95% C.I. of ρ_0	[0.065, 0.25]	[-0.012, 0.18]	[0.0065, 0.19]	[0.026, 0.21]	[-0.035, 0.16]	[0.0053, 0.19]
95% C.I. for κ in residuals	[-0.08, 0.12]	[-0.10, 0.10]	[-0.10, 0.11]	[-0.10, 0.11]	[-0.10, 0.10]	[-0.09, 0.11]

Table: The p-value for testing the zero-intercept hypothesis, the Akaike information criterion (AIC), and the 95% confidence interval (C.I.) of ρ_0 (only for the S-APT models) for different models of the Eurozone stock indices returns. Model 3s and Model 4s appear to perform better than the other models.

Model Fitting Results

- The only models that are not rejected in the test of zero-intercept hypothesis are model 3s and 4s, which incorporate spatial interaction.
- For all the S-APT models that are not rejected (Models 3s and 4s), ρ_0 are found to be significantly positive.
- The models 3s and 4s also have better performance than the APT models in terms of AIC.
- The S-APT models seem to eliminate the spatial interaction among regression residuals more effectively than the APT models, as indicated by κ.

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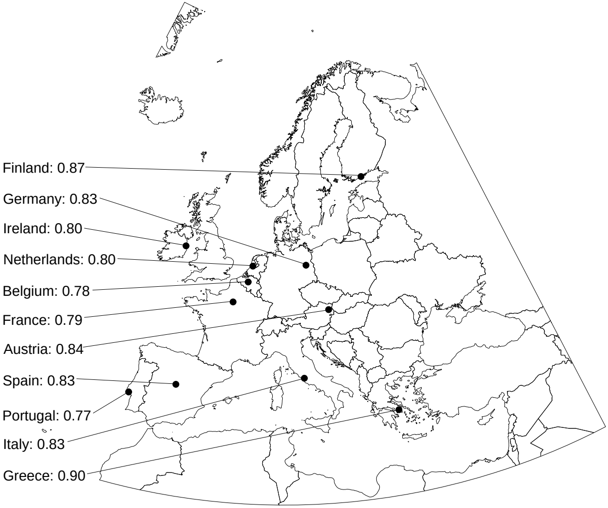
Adjusted R^2 of Fitting Model 4s with the Zero-Intercept Constraint



Robustness Check

The empirical results reported above seem to be robust with respect to different specifications of spatial matrix W.

W	p-value for testing $\bar{\alpha}_0 = 0$	C.I. of ρ_0	adjusted R^2				
			Austria	Belgium	Finland	France	Germany
Geographic distance	0.2866	[0.022, 0.20]	0.8404	0.7756	0.8676	0.7858	0.8304
Driving distance	0.2912	[0.026, 0.21]	0.8405	0.7757	0.8677	0.7859	0.8305

W	AIC	adjusted R^2					
		Greece	Ireland	Italy	Netherlands	Portugal	Spain
Geographic distance	8731	0.9006	0.7950	0.8345	0.8041	0.7698	0.8296
Driving distance	8730	0.9006	0.7951	0.8346	0.8042	0.7699	0.8296

Table: Robustness check: different spatial weight matrix W for the S-APT Model 4s.

Outline

S&P/Case-Shiller Home Price Indices (CSI Indices) Futures

- 10 CSI Indices futures contracts on the CSI indices of 10 major cities
- One futures contract on the composite 10-city CSI Index
- Data: monthly futures returns of the 11 CSI Indices futures during 06/2006–02/2014

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CSI Indices Futures

The Credit Factor for CSI Indices Futures

	2013	2012	2011	2010	2009	2008	2007	2006
Florida	AAA	AAA	AAA	AAA	AAA	AAA	AAA	AAA
Nevada	AA	AA	AA	AA+	AA+	AA+	AA+	AA+
Massachusetts	AA+	AA+	AA	AA	AA	AA	AA	AA
New York	AA	AA	AA	AA	AA	AA	AA	AA
Colorado	AA	AA	AA	AA	AA	AA	AA	AA-
Illinois	A-	A+	A+	A+	A+	A	A	A
California	A	A-	A-	A-	A	A+	A+	A+

Table: Credit Ratings by S&P

The credit factor:

$$\tilde{g}_{Credit} := \tilde{r}_{LosAngeles} + \tilde{r}_{SanDiego} + \tilde{r}_{SanFrancisco} - (\tilde{r}_{Miami} + \tilde{r}_{LasVegas}).$$

Three Factors for CSI Indices Futures

- *CS10f*: futures return on S&P/Case-Shiller composite 10-City Index
- *CS10fTr*: trend factor of CS10f, which is the difference between current value of CS10f and its previous 12-month average
- The credit factor

Empirical Test of APT and S-APT on CSI Indices Futures

Model	S-APT	APT (heterogeneous variance)	APT (homogeneous variances)
p-value	0.114	0.0612	0.0123
AIC	-5523	-4552	-4541
Number of Parameters	32	40	31
95% C.I. for ρ_0	[0.28, 0.44]	-	-
95% C.I. for κ in residuals	[-0.10, 0.11]	[0.29, 0.44]	[0.29, 0.44]

Table: The p-value for testing the zero-intercept hypothesis, Akaike information criterion (AIC), number of parameters, and 95% confidence interval (C.I.) of ρ_0 (only for S-APT) of the three models.

Empirical Test of APT and S-APT on CSI Indices Futures

- The S-APT model is not rejected.
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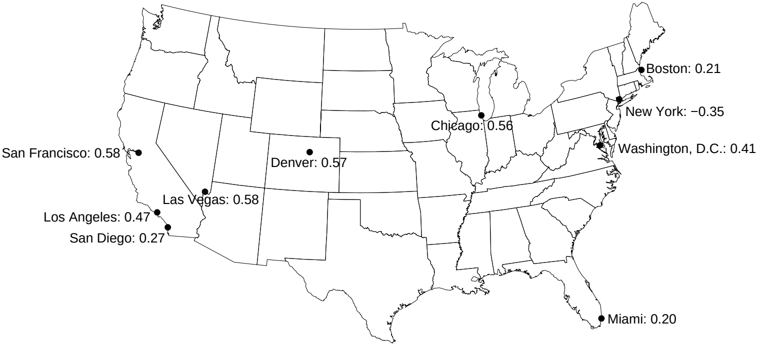
The adjusted R^2 of Fitting the S-APT Model with $\bar{\alpha}_0 = 0$



All adjusted R^2 are positive except that of New York.

Negative R^2 of New York

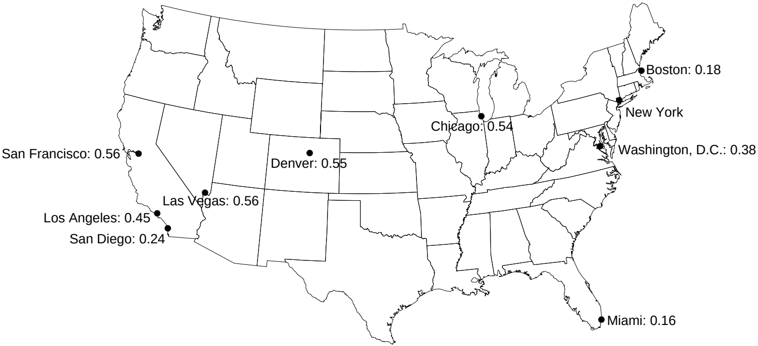
Possible explanation: the CSI Index of New York does not reflect the overall real estate market in the metropolitan area of New York.

- The CSI Index of New York account for only sales of single-family homes.
- The sales of co-ops and condominium account for 98% of Manhattan's non-rental properties.
- More factors may be needed to better explain the CSI futures return of New York.

To keep the analysis simple, we just exclude the CSI Indices futures of New York and test the S-APT again:

- The p-value is 0.11.
- 95% C.I. of ρ_0 is $[0.29, 0.45]$.

Adjusted R^2 of the Fitting (Excluding New York)



Summary

- We study the economic implication of spatial interaction on expected asset returns.
- We derive the S-CAPM and S-APT that characterize how spatial interaction affects the expected asset returns of both ordinary assets and futures contracts.
- We develop the statistical inference procedures for testing the S-APT.
- The S-APT is not rejected by the Euro-zone countries stock indices returns and the CSI Indices futures data.
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Thank you!

Difference between S-APT and Spatial Autoregression (SAR)

- Spatial Autoregression Model

$$\tilde{y}_i = \rho \sum_{j=1}^{n} w_{ij}\tilde{y}_j + \beta_0 + \sum_{k=1}^{K} \beta_k \tilde{x}_{ik} + \tilde{\epsilon}_i, i = 1, \ldots, n$$

- In S-APT, factors are common factors; while in SAR, factors are individual attributes.
- In S-APT, different assets have different factor loadings on the k-th factor; while in SAR, different assets have the same factor loading on the k-th factor.
- S-APT has a linear constraint on its parameters

$$\alpha = (I - \rho W)\mathbf{1}_{n_1,n_2} r + B\lambda,$$

while SAR does not.