答疑

1. 令 $P_n=\{n阶排列\}=\{(i_1,\cdots,i_n) \mid \{i_1,\cdots,i_n\}=\{1,\cdots,n\},\ i_j\neq i_k,\ j\neq k\}$.

P_n和S_n有多种对应方式

(1)
$$P_n \xrightarrow{f} S_n$$
$$(i_1,\cdots,i_n) \longmapsto \sigma=\begin{pmatrix}1 & 2 & \cdots & n\\ i_1 & i_2 & \cdots & i_n\end{pmatrix}$$

(2)
$$P_n \xrightarrow{g} S_n$$
$$(i_1,\cdots,i_n) \longmapsto (i_1\, i_2 \cdots i_n)$$

例如 $n=3$

$$P_3 \xrightarrow{f} S_3$$
$$(1,2,3) \longleftrightarrow \sigma = id = (1)$$

$$P_3 \xrightarrow{g} S_3$$
$$(1,2,3) \longmapsto (1\,2\,3)$$

现在考虑对换作用在P_n和S_n

$$\begin{tikzcd}
\{i_1,\cdots,i_s,\cdots,i_t,\cdots\} \arrow[d, mapsto] & P_n \arrow[r, "f"] \arrow[d, "交换 i_s,i_t"'] \arrow[dr, phantom, "交换"] & S_n \arrow[d] & \sigma=\begin{pmatrix}1\cdots s\cdots t\cdots n\\ i_1\cdots i_s\cdots i_t\cdots i_n\end{pmatrix} \arrow[d, mapsto, "左乘(s\,t)"] \\
\{i_1,\cdots,i_t\cdots i_s\cdots\} & P_n \arrow[r] & S_n & \sigma_0=\begin{pmatrix}1\cdots s\cdots t\cdots n\\ i_1\cdots i_t\cdots i_s\cdots i_n\end{pmatrix}=(s\,t)\sigma
\end{tikzcd}$$

$$\begin{tikzcd}
P_n \arrow[r, "g"] \arrow[d, "交换 i_s,i_t"'] & S_n \arrow[d] & \sigma \arrow[d, mapsto] \\
P_n \arrow[r] & S_n & \sigma_0
\end{tikzcd}$$

$$\sigma_0=(s\,t)\sigma(s\,t)$$

两图均交换.

2. $\mathbb{Z}_n$的子群和商群，$n\in\mathbb{N}$，$n\geq 2$

$\mathbb{Z}$的子群为某些 $d\mathbb{Z}$ 形式，$\mathbb{Z}_n$的子群形如 $d\mathbb{Z}/n\mathbb{Z}$，

$d|n$，但 $d\mathbb{Z}/n\mathbb{Z} \cong \mathbb{Z}_{n/d}$ 因此 $\mathbb{Z}_{n/d}$看作$\mathbb{Z}_n$的子群.

一般地，$\mathbb{Z}_n$的子群同构意义下是 $\mathbb{Z}_r$ $r|n$. $n=rd$.

$\mathbb{Z}_n$的商群：$\mathbb{Z}_n/\mathbb{Z}_r \cong \dfrac{\mathbb{Z}/n\mathbb{Z}}{d\mathbb{Z}/n\mathbb{Z}} \cong \mathbb{Z}_d$，$d|n$.

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$G\xrightarrow{\pi}H$ 满同态，$G\cong\mathbb{Z}_m$，$H\cong\mathbb{Z}_n$.

π满，$\mathbb{Z}_n$同构于$\mathbb{Z}_m$的商群 $= \mathbb{Z}_m/\ker\pi$，由以上

讨论 $n|m$.

~~易~~.