Homework 9

Dec. 20, 2018

NOTE: Homework 9 is due Next Thursday (Dec. 27, 2018).

1. Let $\boldsymbol{X} = (X_1, \cdots, X_n)$ be a random sample from Normal distribution $N(\mu, \sigma^2)$, where μ is unknown and σ is known.
   (i) Derive the UMP test for testing the hypothesis $H_0 : \mu = \mu_0 \longleftrightarrow H_1 : \mu > \mu_0$ at level of significance α.
   (ii) Carry out the testing hypothesis for $n = 100, \sigma^2 = 4, \mu_0 = 3, \bar{x} = 3.2, \alpha = 0.01$ and compute the power for $\mu = 3.5$.

2. Let $\boldsymbol{X} = (X_1, \cdots, X_n)$ be a random sample from Gamma distribution Gamma(α_0, β) with α_0 known and β unknown.
   (i) Construct the MP test for testing the hypothesis $H_0 : \beta = \beta_1 \longleftrightarrow H_1 : \beta = \beta_2$ $(\beta_2 > \beta_1)$ at level of significance α.
   (ii) Show that $X_1 + \cdots + X_n \sim$ Gamma$(n\alpha_0, \beta)$.
   (iii) Use the CLT to carry out the test when $n = 30, \alpha_0 = 10, \beta_1 = 2.5, \beta_2 = 3, \alpha = 0.05$ and compute the power.

3. Let X be a r.v. distributed as $B(n, \theta)$, $\theta \in \Theta = (0, 1)$.
   (i) Derive the UMP test for testing the hypothesis $H_0 : \theta \leq \theta_0 \longleftrightarrow H_1 : \theta > \theta_0$ at level of significance α.
   (ii) Specify the test in part (i) for $n = 10, \theta_0 = 0.25$, and $\alpha = 0.05$.
   (iii) Compute the power of the test for $\theta = 0.375, 0.500$.
   (iv) Use the CLT in order to determine the sample size n if $\theta_0 = 0.125, \alpha = 0.1$ and $\pi(0.25) = 0.9$.

4. Let X be a r.v. with p.d.f. $f(x; \theta) = \frac{1}{\theta} e^{-x/\theta}, x > 0, \theta \in \Theta = (0, +\infty)$
   (i) Show that $f(\cdot; \theta)$ is of the exponential family.
   (ii) Derive the UMP test for testing the hypothesis $H_0 : \theta \geq \theta_0 \longleftrightarrow H_1 : \theta < \theta_0$ at level of significance α, on the basis of the random sample $X_1, \cdots, X_n$ from the above p.d.f..
   (iii) Show that the r.v. $Y = 2 \times (\sum_{i=1}^n X_i)/\theta$ is distributed as χ^2_{2n}.
   (iv) Use parts (ii) and (iii) in order to find an expression for the critical value C and the power function of the test.
   (v) If $\theta_0 = 1000$ and $\alpha = 0.05$, determine the sample size n, so that the power of the test at $\theta = 500$ is at least 0.95.

5. Consider the data set "data.csv" in the previous homework. For simplicity, suppose that the dependent variable SalePrice (denoted as Y) and the independent variables Gr_Liv_Area (denoted as X_1), Central_Air (denoted as X_2) follow a linear regression model with intercept, i.e.,

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \epsilon.$$

   Testing the following hypotheses:

(i) $H_0 : \beta_1 = 0 \longleftrightarrow H_1 : \beta_1 \neq 0$

(ii) $H_0 : \beta_2 = 0 \longleftrightarrow H_1 : \beta_2 \neq 0$

(iii) Let σ^2 be the variance of ϵ,

$$H_0 : \sigma = 53000 \longleftrightarrow H_1 : \sigma \neq 53000$$