《Financial Statistics》 Homework No.2

Deadline: May 21, 2019 Total score: **100**

Name:____________ Student ID:____________ Department:________________

1. (30') Suppose that the volatilities of the daily log-return of the Coco-Cola company follow the GARCH(1,1) model

$$X_t = \sigma_t \varepsilon_t, \quad \sigma_t^2 = a_0 + a_1 X_{t-1}^2 + b_1 \sigma_{t-1}^2$$

with $a_1 + b_1 < 1$ and $\varepsilon_t \sim N(0,1)$.

(a) If $a_0 = 0.006$, $a_1 = 0.05$ and $b_1 = 0.55$, is the tail of the distribution lighter than that of t_4 in terms of kurtosis?

(b) What is the autocorrelation function of the series $\{X_t^2\}$?

(c) If a_0, a_1 and b_1 are estimated as 0.006, 0.1 and 0.4 respectively with associated covariance matrix

$$10^{-4} \begin{pmatrix} 15 & 5 & 0 \\ 5 & 4 & 0 \\ 0 & 0 & 30 \end{pmatrix},$$

what is the estimated long-run variance (unconditional variance)? What is the associated standard error?

(d) With the parameters in (a), if $X_T^2 = 0.02$ and $\sigma_T^2 = 0.03$, give the one-step and two-step forecast of the volatility.

(e) Now, suppose that we have observed the data and wish to fit the GARCH(p,q) with $p + q \leq 2$. Outline the key steps (including diagnostic) for fitting the data.

2. (30') Consider the log-monthly return of Intel from Janurary, 1990 to December 2013.

(a) Are the returns predictable?

(b) Using the PACF plot of the series to determine the order of the fit of the AR model for the return. Plot also the PACF for the squared return series.

(c) Fit a GARCH(1,1) model to the return series using the Gaussian innovation.

(d) Compute the mean return and long-run volatility (unconditional standard deviation).

(e) Use the Delta-method to get the SE of the mean return and long-run volatility.

(f) Provide necessary model diagnostics using graphs and test statistics.

3. (40') Consider the following portfolio optimization problem with a risk-free asset having return r_0:

$$\min \boldsymbol{\alpha}^T \boldsymbol{\Sigma} \boldsymbol{\alpha}, \qquad \text{s.t.} \quad \boldsymbol{\alpha}^T \boldsymbol{\mu} + (1 - \boldsymbol{\alpha}^T \mathbf{1}) r_0 = \mu.$$

That is, we minimize the variance of the portfolio consisting of allocation vector $\boldsymbol{\alpha}$ on risky assets with return vector $\boldsymbol{\mu}$ and allocation $(1 - \boldsymbol{\alpha}^T \mathbf{1})$ on the risk-free bond with return r_0, subject to the constraint that the portfolio's expected return is μ.

(a) The optimal solution is

$$\boldsymbol{\alpha} = P^{-1}(\mu - r_0)\boldsymbol{\Sigma}^{-1}\boldsymbol{\mu}_0,$$

where $P = \boldsymbol{\mu}_0^T \boldsymbol{\Sigma}^{-1}\boldsymbol{\mu}_0$ is the squared Sharpe ratio, and $\boldsymbol{\mu}_0 = \boldsymbol{\mu} - r_0\mathbf{1}$ is the vector of excess returns.

(b) The variance of this portfolio is $\sigma^2 = (\mu - r_0)^2/P$.

(c) When $r_0 < \mu$, show that $r_0 + P^{1/2}\sigma = \mu$, namely, the optimal allocation for the risky asset $\boldsymbol{\alpha}$ is the tangent portfolio.

The End !