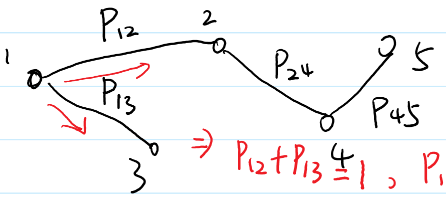
半正定规划问题

2020年3月18日 9:32

图上的最速混合 Markov chain.



$X(t) \in \{1, \cdots, 5\}$

$\Rightarrow P_{12}+P_{13} \le 1,\ P_{14}=0,\ P_{15}=0$

$$P_{ij} = \text{prob}\left(X(t+1) = i \mid X(t) = j \right)$$

① $P_{ij} = P_{ji},\ \forall\, i, j$

② $P_{ij} \ge 0$

③ $1^T P = 1^T$

$\Rightarrow P1 = 1$

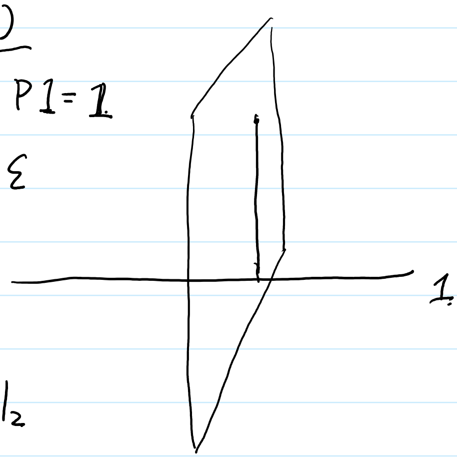
$(P_{ij}) = 0,\ \text{if } (i,j) \notin \mathcal{E}$

$\Rightarrow \quad \lambda(P) = \{1 = \lambda_1 \ge \lambda_2 \ge \cdots \ge \lambda_n\}$

$r = \max(\lambda_2, -\lambda_n) \quad \downarrow.$

$$\min \ \underline{r = \max(\lambda_2, -\lambda_n)}$$
$$\text{s.t.} \quad P \ge 0,\ P = P^T,\ P1 = 1$$
$$P_{ij} = 0 \quad (i,j) \notin \mathcal{E}$$

$$\underline{\|P - \tfrac{1}{n} 1 1^T\|}_2$$



$$\Leftrightarrow \quad \min_P \ \|P - \tfrac{1}{n} 1 1^T\|_2$$
$$\text{s.t.} \quad P \ge 0,\ P = P^T,\ P1 = 1,\ P_{ij} = 0,\ (i,j) \notin \mathcal{E}$$

$$\Leftrightarrow \quad \min_{P,t} \ t$$
$$\text{s.t.} \quad \underline{\|P - \tfrac{1}{n} 1 1^T\|_2 \le t} \quad \Leftrightarrow \quad -tI \preccurlyeq P - \tfrac{1}{n} 1 1^T \preccurlyeq tI$$
$$(\cdot\ - \cdot \cdot \cdot \cdot)$$

$$\min \ \mathrm{Tr}(C X)$$
$$\text{s.t.} \quad A(X) = b \ : \ A : S^n_+ \to R^m : (A(X))_i = \mathrm{Tr}(A_i X)$$
$$X \succcurlyeq 0$$

$$\min \ f(x) + g(Ax) \quad \Leftrightarrow \quad \min \ f(x) + g(y)$$

$$\underline{\min_{x} f(x) + g(Ax)} \iff \min_{x,y} f(x) + g(y)$$

$$\text{s.t. } Ax - y = 0 \leftarrow v$$

$$L(x,y;v) = f(x) + g(y) + \langle v, Ax-y \rangle$$

$$h(v) = \inf_{x,y} \{ f(x) + g(y) + \langle v, Ax-y \rangle \}$$

$$= \inf_{x} \{ f(x) + \langle A^T v, x \rangle \} + \inf \{ g(y) - \langle v, y \rangle \}$$

$$= - f^*(-A^T v) - g^*(v)$$