ADDITIONAL TOPICS (2): ZARISKI TOPOLOGY

Zariski topology is the most basic topological structure in algebraic geometry. Algebraic geometry is a subject which studies the zero locus of a system of polynomials and this gives us the motivation to define the Zariski topology.

Consider the set $\mathbb{C}^n$. A set $Y \subset \mathbb{C}^n$ is closed if Y is the common zero locus of some polynomials $f_1, \cdots, f_m \in \mathbb{C}[t_1, \cdots, t_1]$. Such Y is called an **affine algebraic variety**.

(1) Show that the above definition indeed defines a topology on $\mathbb{C}^n$. (Hint: You may need the fact that the ring $\mathbb{C}[t_1, \cdots, t_1]$ is noetherian which means that every ideal of $\mathbb{C}[t_1, \cdots, t_1]$ is finitely generated.)

A closed subset Y is called **irreducible** if it cannot be expresses as the union $Y = Y_1 \cup Y_2$ of two proper subsets, each one of which is closed in Y.

(1) Give an algebraic criterion for Y to be irreducible. (Hint: Let Y be the common zero locus of $f_1, \cdots, f_m \in \mathbb{C}[t_1, \cdots, t_1]$, consider the ideal generated by $f_1, \cdots, f_m$. What condition should be satisfied by this ideal?)