清华大学本科生考试试题专用纸

考试课程：金融统计　　考试时间：2018年1月3日19:20-21:45

姓名:_____　学号:_____　院系名称:_____　成绩:_____

Notice: There are **8** questions and the full mark is **100**.

1 【10'】Suppose that P_t is the price of an asset at time t. Please give the formula for calculating the following returns:

(a). one-period simple return;

(b). k-period gross return;

(c). one period log return;

(d). k-period log return;

(e). Discuss the relationship among (a)-(d).

2 【5'】There are many stylized features of financial returns in financial market. Please state these stylized features briefly.

3 【15'】(a). (7') According to the efficient market hypothesis, is the return of a portfolio predictable? Is the volatility of a portfolio predictable? State the most approximate mathematical form of the efficient market hypothesis.

(b). (8') If the Ljung-Box test is employed to test the efficient market hypothesis, what null hypothesis is to be tested? If the autocorrelation for the first 4 lags of the monthly log-returns of the S&P 500 is

$$\widehat{\rho}_1 = 0.2, \quad \widehat{\rho}_2 = -0.15, \quad \widehat{\rho}_3 = 0.25, \quad \widehat{\rho}_4 = 0.12$$

based on past 5 years data, is the efficient market hypothesis reasonable?

4 【25'】Suppose that the volatilities of the daily log-returns of the Coco-Cola company follow the GARCH(1,1) model:

$$X_t = \sigma_t \varepsilon_t, \qquad \sigma_t^2 = a_0 + a_1 X_{t-1}^2 + b_1 \sigma_{t-1}^2$$

with $a_1 + b_1 < 1$ and $\varepsilon_t \sim \mathcal{N}(0,1)$.

(a)(5'). If $a_0 = 0.006$, $a_1 = 0.05$ and $b_1 = 0.55$, is the tail of the distribution lighter than that of t_4 in terms of kurtosis?

(b)(5'). What is the autocorrelation function of the series $\{X_t^2\}$?

(c)(5'). If a_0, a_1 and b_1 are estimated as 0.006, 0.1 and 0.4 respectively with associated covariance matrix

$$10^{-4} \begin{pmatrix} 15 & 5 & 0 \\ 5 & 4 & 0 \\ 0 & 0 & 30 \end{pmatrix},$$

what is the estimated long-run variance (unconditional variance)? What is the associated standard error?

(d)(5'). With the parameters in (a), if $X_T^2 = 0.02$ and $\sigma_T^2 = 0.03$, give the one-step and two-step forecast of the volatility.

(e)(5'). Now, suppose that we have observed the data and wish to fit the GARCH(p, q) with $p + q \leq 2$. Outline the key steps (including diagnostics) for fitting the data.

5 【15'】State the Capital Asset Pricing Model (CAPM) (Sharpe and Lintner version) and prove it.

6 【10'】(a). (5') What is the definition of the Sharpe ratio?

(b)(5'). Suppose that Jack holds a portfolio with expected return 10% and volatility 20% and that Jill has a portfolio with expected return 15% and volatility 30%. Furthermore, the risk-free interest rate is 5%. Whose investments are more efficient?

7 【5'】What is the multifactor pricing models?

8 【15'】Large/high dimensional covariance matrix plays a key role in portfolio allocation and risk assessment. Generally, the performance of the sample covariance matrix is not good. Some modifications are needed so that it has a good performance. Please give some modification methods you learned in this course.