

Game T&A

对策论及其应用

Game Theory and its Applications

引言

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Introduction: What, Why & How?

- 什么是对策论 (Game Theory, 博弈论)?
- 为什么要学习对策论?
- 怎样学习对策论: 本课程基本要求

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博弈论书籍: 大行其道、雅俗共赏

算计?

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What is Game Theory all about?

对策论 / 博弈论 (大陆); “赛局” 理论 (港台)

- Game theory:
 - Theory of games / gaming (playing games)
- What is a **game**? (游戏? 体育比赛? 斗智斗勇?)

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Example: “田忌赛马”之真假

	第一场	第二场	第三场
上等马			
中等马			
下等马			

- 规则: 决策什么? → 决策顺序? → 信息透明?
- 决策者: 理性假设? → 何为理性? → 目标, 能力?

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Example: combinatorial game

- Two players compete (win/lose): play **alternately** with **no hidden information** and **no chance elements**.
 - e.g. Nim, Go and Chess




- Nim: Some (e.g. 40) tokens on the table.
 - Alternately, remove 1 or 2 tokens.
 - Whoever removes the last token wins.

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Example: combinatorial game

- Nim: Three heaps of tokens (e.g. 29, 21, 11).
 - Each turn, remove **any number of** tokens from **any one** heap.
 - Whoever removes the last token wins.






29 tokens 21 tokens 11 tokens

Not trivial! (A beautiful theorem by **Bouton 1902**)

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What is Game Theory all about?



- Game theory** is a branch of **applied mathematics** that is often used in the context of **economics**. It studies strategic **interactions** between agents. ...

- Glossary of game theory:**

Game theory is the branch of **mathematics** in which games are studied: that is, **models** describing human **behaviour**.

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What is Game Theory all about?

The theory of rational choices
 Objective: to understand the behavior
 of **rational interacting** decision-makers

- 博弈论是关于多个**相互影响**的决策主体的**决策理论** (决策模型: 假设, 建立, 求解, 检验, 应用, ...)

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Decision Sciences: Brief Introduction

- Decision in practice
 - Experience-based (leadership, authority)
 - Experimentation-based (try and error, big data, etc.)
- Decision theory (mathematical models)
 - Single decision maker: optimization, cybernetics
 - Deterministic models: LP, NLP, DP, MCDM/MADM, ...
 - Stochastic models: MDP (SDP), ...
 - Uncertainty models (B. Liu: Uncertainty theory)
 - Multiple (interactive) decision makers**
 - Mathematics: **game theory**
 - Engineering: Multi-agent system (based on ML/AI etc.)

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History: In the beginning

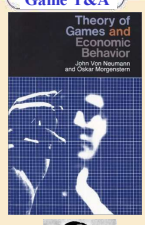


- Some game-theoretic **ideas** traced back to the 18-th century or even earlier (even to the **ancient times**).
 - e.g., “田忌赛马” (春秋战国时期)
- Emile Borel** (1871~1956) and **John von Neumann** (1903~1957) began the major development of game theory.

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The first book


- Game theory became a field since the book published in **1944**.
- John von Neumann** (1903~1957)
 - Born to an aristocratic family in Budapest
 - In 1932, appointed one of the original and youngest permanent members of the IAS at Princeton (along with A. Einstein)
 - Made fundamental contributions to quantum physics, functional analysis, economics, computer science, etc.
- Oskar Morgenstern** (1902~1977)
 - His mother was the daughter of the German emperor
 - In 1938, dismissed from Univ. Vienna by the Nazis
 - Then became a prof. of econ. at Princeton Univ.

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John Nash (1928-2015)

- Born in Bluefield, West Virginia
- Received his Ph.D. from Princeton Univ. with a **28-page** thesis on his **22nd** birthday
 - Invented the notion of **Nash equilibrium (NE)**
 - 1978: von Neumann Theory Prize
 - 1994: Nobel Prize in Econ.
 - 2010: Double Helix Medal (biomedical)
- Wrote a seminal paper on **bargain theory**
- Published only about 20 papers (1945-1996)
- As mathematician, contributed to PDE, differential geometry, global analysis
 - 1999: Leroy P. Steele Prize (by AMS)
 - 2015: Abel Prize



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Applications of game theory

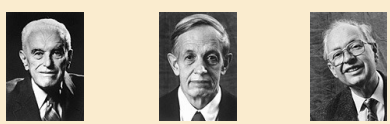
- Economics**
 - Management sciences (also marketing, finance, etc.)
 - Operations management; **Supply Chain Management**
 - Political science; Public policy
 - Psychological study
 - Evolutionary biology (1970-)
 - Computer science
 - Yao's principle (or lemma, 1977)
 -
- 自然科学 vs. 社会科学: 力学 vs. 博弈论**



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Nobel Prize in Economic Sciences 1994



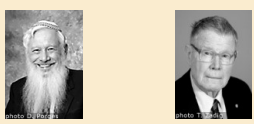
John C. Harsanyi John F. Nash Jr. Reinhard Selten

"for their pioneering analysis of **equilibria** in the theory of **non-cooperative games**"

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Nobel Prize in Economic Sciences 2005




Robert J. Aumann Thomas C. Schelling

"for having enhanced our understanding of **conflict and cooperation** through game-theory analysis"

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Nobel Prize in Economic Sciences 2007



Leonid Hurwicz Eric S. Maskin Roger B. Myerson

"for having laid the foundations of **mechanism design theory**"

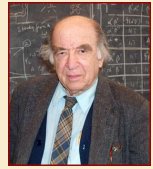
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Nobel Prize in Economic Sciences 2007

Leonid Hurwicz

Born: August 21, 1917
 Place of birth: Moscow, Russia
 Residence: US
 Affiliation: Department of Economics, University of Minnesota
 Address: 1008 Heller Hall, Minneapolis, MN 55455
 Tel: 612-625-5808
 E-mail: hurwicz@umn.edu
 Education: Warsaw University, Poland, **L.L.M.** 1938



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Nobel Prize in Economic Sciences 2007

Eric S. Maskin

Born: December 12, 1950
 Place of birth: New York, NY, USA
 Residence: US
 Affiliation: Institute for Advanced Study, Princeton University

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 School of Social Science
 Institute for Advanced Study
 Einstein Drive
 Princeton, NJ 08540
 Tel: (609) 734-8309
 Fax: (609) 951-4457
 E-mail: maskin@ias.edu

Education:
 Harvard University, A.B. Mathematics 1972
 Harvard University, A.M. Applied Mathematics 1974
 Harvard University, **Ph.D. Applied Mathematics 1976**
 Cambridge University, M.A. Honorary 1972



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Nobel Prize in Economic Sciences 2007

Roger B. Myerson

Born: March 29, 1951
 Place of birth: Boston, Massachusetts, USA
 Residence: US
 Affiliation: University of Chicago, IL, USA

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 Department of Economics
 University of Chicago
 1126 East 59th Street
 Chicago, IL 60637, USA
 Tel: (773) 834-9071
 Fax: (773) 702-8490
 E-mail: myerson@uchicago.edu

Education:
 Harvard University, A.B and S.M. Applied Mathematics 1973
 Harvard University, **Ph.D. Applied Mathematics 1976**





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Nobel Prize in Economic Sciences 2012

Lloyd S. Shapley
 (1923-)
 UCLA, USA

PhD (math) 1953, Princeton
 A.B. 1948, Harvard

Alvin E. Roth
 (1951-)
 Harvard, USA

PhD 1974, Stanford (OR)
 MS 1973, Stanford (OR)
 BS 1971, Columbia (OR)

"for the theory of **stable allocations** and the practice of **market design**."

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Nobel Prize in Economic Sciences 2012


Lloyd S. Shapley
 Dept. of Economics / Dept. of **Mathematics**, UCLA

Born: Cambridge, Massachusetts, June 2, 1923

Educated:
 Harvard University, A.B., 1948
 Princeton University, Ph.D. (**math.**), 1953

Dissertation:
Additive and Nonadditive Set Functions (A. W. Tucker), 1953

Employment:
U.S. Army, 1943-1945
 Rand Corporation, **research mathematician**, 1948-49, 1954-81
 Princeton University, Fine Instructor, 1952-54
 University of California, Los Angeles, 1981-



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Nobel Prize in Economic Sciences 2012

Alvin E. Roth
 Dept. of Economics / HBS, Harvard

DATE OF BIRTH: 18 December 1951

EDUCATION:
 Stanford University, Ph.D., 1974 (**Operations Research**)
 Stanford University, M.S., 1973 (**Operations Research**)
 Columbia University, B.S., 1971 (**Operations Research**)

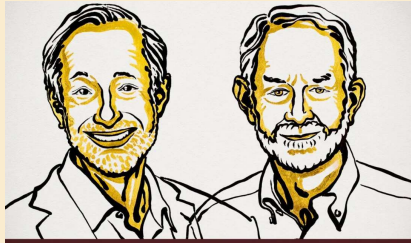
PRINCIPAL EMPLOYMENT:
 8/98-present: George Gund Professor, Harvard.
 8/82 – 8/98: Professor, University of Pittsburgh.
 8/79 – 8/82: Professor, University of Illinois.
 8/74 – 7/79: Assi/Asso Professor, University of Illinois.



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Nobel Prize in Economic Sciences 2020



Paul R. Milgrom Robert B. Wilson

"for improvements to auction theory and inventions of new auction formats"

Both are Professors in Stanford University

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GT: Interactive Decision Theory

- Decision theory: single decision makers
 - You are self-interested and “selfish”
- Game theory: multiple decision makers
 - So is everyone else

“If it’s true that we are here to help others, then what exactly are the others here for?”
 - George Carlin (George Bernard Shaw?)

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Why Study Game Theory?

- “No man is an island”
- Study of rational behavior in *interactive* or *interdependent* situations
- Bad news:
 - Knowing game theory does not guarantee winning
- Good news:
 - Framework for thinking** about strategic interaction

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Why Study Game Theory?

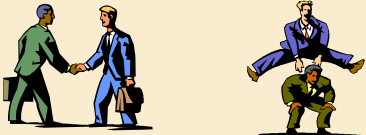
- Because we can formulate effective strategy...
- Because we can predict the outcome of strategic situations...
- Because we can **select or design** the best game for us to be playing...

The Golden Rule: COMMANDMENT
Never assume that your opponents’ behavior is fixed.
Predict their reaction to your behavior.

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Game models



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Primary Concepts

- Recognizing the game
 - Rules of the game:** players, strategies, payoffs
 - Simultaneous games** (例如: 石头剪刀布)
 - Anticipating rival’s moves
 - Sequential games** (例如: 围棋)
 - Looking forward – reasoning back
 - Mixed strategies** (vs. pure strategies)
 - Sensibility of being unpredictable
 - Repeated games** (vs. one-shot games)
 - Cooperation and agreeing to agree (or disagree)

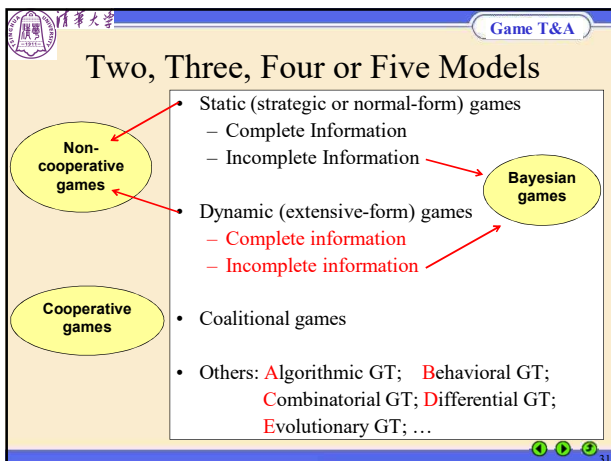
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Other important issues

- Winning the game
 - Commitment** (承诺 / 担当)
 - Credibility, threats, and promises, reputation
 - Information**
 - Strategic use of information
 - Bargaining** (讨价还价 / 谈判)
 - Gaining the upper hand in negotiation
 - Mechanism design** (e.g. Auctions (拍卖))
 - Design and Participate

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Summary - The Matrix of Game Theory

	Non-cooperative	Cooperative
Static	Non-cooperative (Static) Game Theory	Cooperative Game Theory
Dynamic	Non-cooperative Dynamic Games	Cooperative Dynamic Games

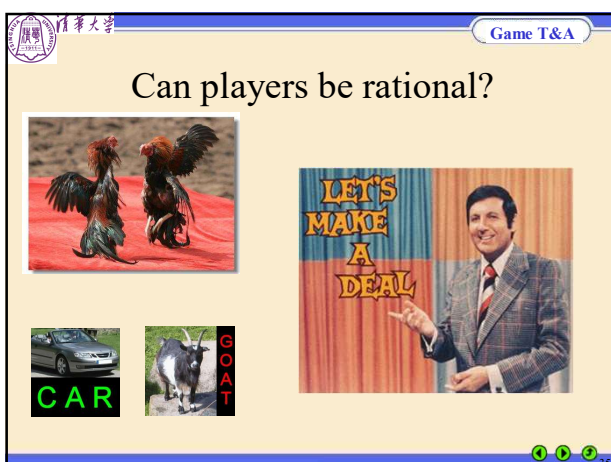
Summary - The Matrix of Non-cooperation Game

	Complete (Full) Information	Incomplete Information
Simultaneous Move (Static)	Nash Equilibrium (Nash, 1950/51)	Bayesian (Nash) Equilibrium (Harsanyi, 1967/68)
Sequential Move (Dynamic)	Subgame Perfect (Nash) Equil. (Selten, 1965)	Subgame Perfect Bayesian Equil. (Selten, 1975; etc.)

Disclaim

- A game describes the **strategies** that the players **can** (or actually **should**) take.
("normative / prescriptive / coherence theories")
- A game does not specify the **actions** that the players **do** take.
("positive / descriptive / correspondence theories")

---- Situation changed in: **Behavioral Game Theory**



Challenging: Behavioral GT

- Characteristics of Human behavior
 - "Bounded Rationality" (H. A. Simon)
 - Altruism (Kin ~, Group ~, Reciprocal ~); Inequity aversion
 - Prospect Theory (D. Kahneman): Reference dependent; loss aversion; ...
 - Mental Accounting (Richard H. Thaler)
 -
- Behavioral (Experimental) GT
- Behavioral (Experimental) Econ.; Neuroeconomics
- Behavioral Marketing; Behavioral Financing
- Behavioral Operations Management (BOM)
- Behavioral Operations Research (BOR)?

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Summary

- What is Game Theory about?
 - Interactive decision theory (models)
- Why should one study Game Theory?
 - Extensive applications (can be found almost everywhere)
- Basic concepts
 - Players, strategies, payoffs
 - Sequence of (decision) events
 - Information structure
 - Cooperative vs. non-cooperative
 - ...

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基本要求

- 出勤表现
 - ✓ 交流 / 答疑: 课堂、网络学堂、email、Office hour
- 平时作业 (纸版、电子版均可)
- 课程报告 (电子版; 口头报告初步安排在15周)
 - ✓ 可以是文献阅读报告, 也可以是自主选题小论文
 - ✓ **研究生必须参加(每人报告15-20分钟), 本科生自愿**
- 期末考试 (第16周最后一次课随堂考; 没有期中考试)
- 最后总评: 考试成绩约占50%
 - ✓ 平均分一般不低于85, 也不超过90 → 转为等级制
- 根据兴趣, 可有不同的关注重点 (课上难以面面俱到)
 - ✓ 理论: 数学理论证明与推广, 算法设计与分析, ...
 - ✓ 应用: 实际问题建模与分析, 模型解释与检验, ...

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基本参考书

- J. González-Díaz, I. García-Jurado and M. G. Fiestras-Janeiro, **An introductory course on mathematical game theory**, AMS, 2010. (GSM-115)
- M. J. Osborne and A. Rubinstein, **A Course in Game Theory**, MIT Press, 1994. (中译本: 中国社会科学出版社, 2000) (OR-1994)
- R. Gibbons, **Game Theory for Applied Economists** (also called **A Primer in Game Theory**), Princeton University Press, 1992. (中译本: 中国社会科学出版社, 1999). --- 简明清晰、例子丰富 (Gibbons-1992)
- 肖条军, **博弈论及其应用**, 上海三联书店, 2004. (Xiao-2004)
- 张维迎, **博弈论与信息经济学**, 上海人民出版社, 2004. (Zhang-2004)




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Other useful books

- M. J. Osborne, **An Introduction to Game Theory**, Oxford Univ. Press, 2004. (影印版: 上海财经大学出版社, 2005) --- 例子丰富
- D. Fudenberg and J. Tirole, **Game Theory**, MIT Press, 1991. (中译本: 中国人民大学出版社, 2002) --- 技术细节
- E. Rasmusen, **Games and Information: An Introduction to Game Theory, 4th Edition**, Wiley-Blackwell, 2006. (第3版中译本: 北京大学出版社, 2003) --- 信息经济学
- C. F. Camerer, **Behavioral Game Theory**, Princeton University Press, 2003. --- 行为博弈论
-
- 五花八门的各类专著、教材和科普书籍
 - 可能关注理论, 也可能完全不讲数学理论



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作业

内容 (不交书面作业)

自我阅读参考书的相关内容;
了解博弈论的基本概念、基本模型。

GAME NEVER OVER

Acknowledgments:
本课程课件主要参考了台湾大学吕学一教授的讲课材料, 同时也参考、引用了大量其他学者的书籍和讲课材料, 特此致谢!

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