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Comparative study of isotropic and anisotropic sky models to estimate solar radiation incident on tilted surface: A case study for Bhopal, India

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ABSTRACT

The purpose of this study is to compare the different empirical models used for estimation of solar radiation on tilted surface. For this, three isotropic and same number of anisotropic sky models were employed by using average monthly mean value of solar radiation on daily basis at Bhopal, local climatic condition, located in central region of India. The tilt angle was fixed at 23.26° N (latitude of Bhopal). The models results were compared with ground measured data from one sample statistical test. It was found that Hays and Davis model (HD) estimated the highest amount of incident solar radiation in the whole year whereas Badescu model (BA) established the lowest among all isotropic as well as anisotropic models. Finally, Badescu model (BA) was preferred for estimation of solar radiation incident on tilted surface with smallest statistical errors among all models and closed agreement with measured data.

1. Introduction

Solar radiation data are the best source of information for estimating average incident radiation necessary for proper design and the assessment of solar energy conversion systems (Sabziparvar, 2008). There are several forms of solar radiation data, which could be used for a variety of purposes in the design and development of solar energy systems (Jakhrani et al., 2012). Daily data is often available and hourly radiation can be estimated from available daily data. The availability of more comprehensive solar radiation data is invaluable for the design and evaluation of solar-based conversion systems. Particularly, the basic solar radiation data for the surfaces of interests are not readily available in most developing countries (Li et al., 2008; El-Sebaii et al., 2010). Because of not being able to afford the measuring equipments and techniques involved. Therefore, it is necessary to develop methods to estimate the solar radiation on the basis of the more readily available meteorological data (El-Sebaii et al., 2010).

Several models have been developed to estimate the amount of global solar radiation on horizontal surfaces using various climatic parameters, such as sunshine duration, cloud cover, humidity, maximum and minimum ambient temperatures, and wind speed (El-Sebaii et al., 2010).

Wu et al. (2007) used the metrological data from 1994 to 2005 of Nanchang station (China) to predict daily global solar radiation from sunshine hours, air temperature, total precipitation and dew point. Wu et al. (2007) and Bulut and Buyukalaca (2007) recently proposed a simple model for estimation of monthly average of daily global solar radiation on horizontal surface for 68 provinces of Turkey with a high accuracy (Bulut and Buyukalaca, 2007). Janjai et al. (2009) proposed a model for calculating the monthly average hourly global radiation in the tropics with high aerosol load using satellite data. This model was employed to generate hourly solar radiation maps in Thailand (Janjai et al., 2009).

It is rather important to determine the beam and diffuse components of total radiation incident on a horizontal surface. Once these components are determined, they can be transposed over tilted surfaces, and hence, the short as well as the long term performances of tilted flat plate collectors, photovoltaic modules and other solar devices can be estimated. Many authors have presented empirical correlations to estimate the monthly average daily diffuse radiation on a horizontal surface. El-Sebaii and Trabea (2003) proposed correlations for estimating horizontal diffuse radiation in Egypt by correlating (H_d/H_g) and (H_d/H_o) with K_T and $(S/S_{\max})$ (El-Sebaii and Trabea, 2003). Solanki and Sangani (2008)

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Nomenclature

$\bar{H}_o$: Monthly Average daily extraterrestrial solar radiation (kWh/m^2-day)
I_{SC}: Solar constant 1.367 kW/m^2
N: Day of the year
$\bar{H}_g$: Monthly average daily global solar radiation (kWh/m^2-day)
a, b: Angstrom constants (for Bhopal $a = 0.26$, $b = 0.05$)
s: Monthly average daily hours of bright sunshine (hours)
$S_{\max}$: Monthly average of the maximum possible daily hours (day length) of bright sunshine
$\bar{H}_d$: Monthly average daily defused radiation (kWh/m^2-day)
K_T: Monthly average clearness index
$\bar{H}_T$: Total incident solar radiation on tilted surface (kWh/m^2-day)
$\bar{H}_{T,b}$: Beam radiation on tilted surface (kWh/m^2-day)
$\bar{H}_{T,d}$: Defused radiation on tilted surface (kWh/m^2-day)
$\bar{H}_{T,r}$: Ground reflected radiation on tilted surface (kWh/m^2-day)
$\bar{H}_b$: Monthly average daily beam radiation on horizontal surface (kWh/m^2-day)
$\bar{R}_b$: View factor for beam radiation
$\bar{R}_r$: View factor for ground reflected radiation
$\bar{H}_{d,\text{iso}}$: Isotropic diffused radiation
$\bar{H}_{d,\text{cs}}$: Circumsolar component of diffused radiation
$\bar{H}_{d,\text{hz}}$: Horizon brightening component of diffused solar radiation
$\bar{H}_{\text{gm}}$: Metrological ground measured global solar radiation at horizontal surfaces (kWh/m^2-day)
$\bar{H}_{\text{gmt}}$: Metrological ground measured tilted global solar radiation (kWh/m^2-day)
MAPE: Mean Absolute Percentage Error (%)
MBE: Mean Bias Error (kWh/m^2-day)
RMSE: Root Mean Square Error (kWh/m^2-day)
t-set: t-statistics error
HD: Hay and Davies Model
BA: Badescu Model
LJ: Liu and Jordan Model
KO: Koronakis Model
RE: Reindl et al. Model
HDKR: Hay and Davies, Klucher Model
IMD: Indian Metrological Department
A: Anisotropy index
F: Modulating factor
$F_{c\text{-}s}$: View factor for circum solar diffused radiation
$F_{c\text{-hz}}$: View factor for horizon brightening solar diffused radiation

Greek symbols

γ: Azimuth angle (degree)
β: Tilted angle (degree)
ω: Hour angle (degree)
ω_s: Sunset hour angle for mean day of month (degree)
Φ: Latitude angle (degree)
θ: Angle of incidence (degree)
θz: Zenith angle (degree)
ε: Elevation angle (degree)

proposed a new method which may be used for estimating H_b on the basis of calculation of the elevation angle constant (ε) for a given location and time (Solanki and Sangani, 2008). Ozan and Tuncay (2009) proposed artificial neural-network using satellite data were also used to estimate monthly mean daily average of horizontal direct and diffuse radiation in different cities of Turkey (Ozan and Tuncay, 2009).

Furthermore, meteorological stations usually measure solar global and diffuse radiation intensities on horizontal surfaces. Measured solar radiation data on tilted surfaces are rarely available. Consequently, the solar radiation incident on a tilted surface must be determined by converting the solar radiation intensities measured on a horizontal surface to that incident on the tilted surface of interest in order to design the system size and estimate its long term performance.

It is generally known that in the northern hemisphere, the optimum collector orientation in south facing ($\gamma = 0$) and the optimum tilt depend upon the latitude and the day of the year. In winter month, The optimum tilt is greater (usually latitude + 15) whilst in summer months the optimum tilt is less (usually latitude − 15). There are many papers in the literature which make different recommendations for the optimum tilt based only on the latitude Sudhakar et al. (2013). In practice the collector plate is usually oriented south facing and latitudinal fixed tilt angle which is set to maximize the average energy collected over a year (Ahmad and Tiwari, 2009).

Total radiation incident on a tilted surface consists of three components: beam radiation, diffuse radiation and ground reflected radiation. The beam radiation on a tilted surface can be computed by the relatively simple geometrical relationship between the horizontal and tilted surfaces. The ground reflected radiation can be estimated with good accuracy with the aid of an isotropic model using a simple algorithm. This is not the case regarding the diffuse component, since diffuse radiation has no define or (singular) angle of incidence on a horizontal surface. There exist a relatively large number of models that attempt to correlate the diffuse radiation on a tilted surface to that measured on a horizontal surface. Generally, these models may be classified as isotropic and anisotropic sky models.

The isotropic models assume that the intensity of diffuse sky radiation is uniform over the sky dome. Hence, the diffuse radiation incident on a tilted surface depends on a fraction of the sky dome seen by it. The anisotropic models on the other hand, presume that the anisotropy of the diffuse sky radiation in the circumsolar region (sky near the solar disk) plus the isotropically distributed diffuse component from the rest of the sky dome (horizon brightening fraction) (Noorian et al., 2008). In general, the diffused fraction of radiation on inclined surfaces is composed of isotropic, circum solar and horizon brightening factors.

The main objectives of this paper are:

1. To estimate the monthly average daily global, diffused and beam solar radiation on horizontal surface in Bhopal using the different empirical relations.
2. To calculate the total solar radiation incident on tilted surface at tilt angle 23.26°N (latitude of Bhopal) using 6 selected empirical models.
3. Compare each model with measured data using statistical tests which includes namely MAPE, MBE, RMSE, and t-stat.

Measured solar radiation data comprising of monthly average daily global solar radiation for Bhopal have been collected average of last several years from Indian Meteorological Department (IMD) for horizontal and latitudinal tilted surface (Karakoti et al., 2012).

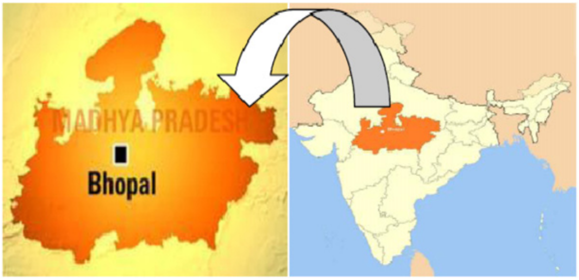

Fig. 1. Study location Bhopal map.
Source: www.mapsofindia.com.

2. Material and methods

2.1. Study location

The geographical location of the Bhopal City lies within North Latitude 23°17′, East Longitude 77°36′ and Altitude 501 m at sea level. The location of Bhopal falls in the north western portion of Madhya Pradesh. It seen in the Map of India, Bhopal occupies the central most region of the country as shown in Fig. 1.

The climate of Bhopal is subtropical, with hot and humid summer and a cool but dry winter. The average temperature during the day is around 30 °C, whereas in the month of May, it rises to 40 °C. Humidity always remains high during this time and hence the atmosphere remains sweaty. Monsoons usually start from June and last till September end. The total rainfall of the city does not exceed 1200 mm, accompanied by frequent thunderstorms and occasional floods. In brief about the Bhopal district:

- On average, the temperatures are always high.
- Most rainfall is seen in June, July, August and September.
- It has dry periods in January, February, March, April and December.
- The warmest month is May.
- The coolest month is December.
- The wettest month is August.

2.2. Solar radiation on horizontal surface

The monthly average daily extraterrestrial solar radiation ($\bar{H}_o$) on the horizontal surface is determined by the following empirical relationship:

$$\bar{H}_o = \frac{24}{\pi} I_{sc}\left(1 + 0.033\cos\frac{360N}{365}\right) \times \left(\frac{\pi\omega_s}{180}\sin\varnothing\sin\delta + \cos\varnothing\cos\delta\sin\omega_s\right)\ \text{kWh/m}^2\text{-day} \quad (1)$$

where I_{sc} is the solar constant 1.367 kW/m^2, N is the day of the year, ω_s is the sunshine hour angle for the mean day of the month (degrees), Φ is the latitude angle (degrees) and δ is the declination angle (degrees).

The declination angle (δ) can be mathematically presented by the Cooper's (1969) equation (Cooper, 1969):

$$\delta = 23.34\sin\frac{360}{365}(284 + N) \quad (2)$$

where N is the day of the year starting from January as shown in Table 1.

The sunshine hour angle (ω_s) for a location is a function of solar declination angle and the latitude (Solanki, 2011) is given by:

$$\omega_s = \cos^{-1}(-\tan\delta\tan\varnothing). \quad (3)$$

Table 1
Days on which extraterrestrial radiation is equal to monthly mean value Klein (1977).

Month	Day	Day of the year (N)
Jan.	17	17
Feb.	16	47
Mar.	16	75
Apr.	15	105
May	15	135
Jun.	11	162
Jul.	17	198
Aug.	16	228
Sept.	15	258
Oct.	15	288
Nov.	14	318
Dec.	10	344

The monthly average daily global solar radiation on a horizontal surface $\bar{H}_g$ is given by Angstrom (1924):

$$\frac{\bar{H}_g}{\bar{H}_o} = a + b\left(\frac{S}{S_{\max}}\right) \quad (4)$$

where S is the monthly average daily hours of bright sunshine. For Bhopal local climatic condition value of S is 9.42 h. $S_{\max}$ is monthly average of the maximum possible daily hours of bright sun shine, which is given by the equation:

$$s_{\max} = \left(\frac{2}{15}\right)\omega_s \quad (5)$$

a, b are constants known as angstrom constants and they are empirical and obtained by the curve fitting data:

$$\left.\begin{aligned} a &= 0.409 + 0.5016\sin(\omega_s - 60) \\ b &= 0.6609 + 0.4767\sin(\omega_s - 60) \end{aligned}\right\}. \quad (6)$$

The value of constants a and b are given by Modi and Sukhatme (1979) for many Indian cities (Bhopal $a = 0.26$, $b = 0.5$) (Modi and Sukhatme, 1979).

In Indian context (Garg and Garg, 1985) have examined radiation data for 11 Indian cities and proposed the equation for estimation of diffused radiation that is:

$$\frac{\bar{H}_d}{\bar{H}_g} = 0.8677 - 0.7365\left(\frac{S}{S_{\max}}\right). \quad (7)$$

Monthly average beam radiation on horizontal surface is usually estimated by subtracting diffused solar radiation from global solar radiation on horizontal surfaces which is

$$\bar{H}_b = \bar{H}_g - \bar{H}_d. \quad (8)$$

2.3. Solar radiation on the tilted surface

The incident solar radiation on a tilted surface is the sum of the set of radiation streams including direct or beam radiation, radiation reflected from the various surfaces seen by the tilted surfaces and the three components of diffused radiation from the sky. The total incident solar radiation on the tilted surface ($\bar{H}_T$) can be written as in the following forms:

$$\bar{H}_T = \bar{H}_{T,b} + \bar{H}_{T,r} + \bar{H}_{T,d} \quad (9)$$

where $\bar{H}_T$ is the monthly total daily incident solar radiation, $\bar{H}_{T,b}$ is the beam radiation, $\bar{H}_{T,r}$ is the ground reflected radiation and $\bar{H}_{T,d}$ is the diffused radiation on tilted surface.

Beam radiation ($\bar{H}_{T,b}$):

Beam radiation on tilted surface is given by:

$$\bar{H}_{T,b} = \bar{H}_b\bar{R}_b \quad (10)$$

where $\bar{H}_b$ is the monthly average daily beam radiation on horizontal surface and it is usually estimated by subtracting the diffused radiation from global radiation on horizontal surfaces ($\bar{H}_b = \bar{H}_g - \bar{H}_d$), $\bar{R}_b$ is the ratio of mean daily beam radiation on the tilted surface to that on horizontal.

Basically $\bar{R}_b$ is a function of transmittance of atmosphere, which is equal to $\left(\frac{\bar{H}_{T,b}}{\bar{H}_b}\right)$ and be determined by the following expression for the surface that are sloped towards the equator in the northern hemisphere or 180° in the southern hemisphere (most favorable azimuth angle $\gamma = 0$, for collector of PV module) (Kreith and Goswami, 2011) therefore the value of $\bar{R}_b$ is computed by:

$$\bar{R}_b = \frac{\cos\theta}{\cos\theta z} = \frac{\sin\delta\sin(\emptyset-\beta)+\cos\delta\cos(\emptyset-\beta)}{\sin\delta\sin\emptyset+\cos\delta\cos\emptyset\cos\omega} \tag{11}$$

where δ is the declination angle, $\emptyset$ is the latitude angle, β is the inclination of tilted surface and ω is the hour angle. (All are in degrees.)

Reflected radiation ($\bar{H}_{T,r}$):

Reflected radiation is the part of total solar radiation that is reflected by the surface of the earth and by any other surface intercepting object such as trees, terrain or buildings on to a surface exposed to the sky is termed as ground reflected radiation (Jakhrani et al., 2013).

A tilted surface at slope from the horizontal has a view factor ($\bar{R}_r$) to the ground and $\bar{R}_r = \frac{(1-\cos\beta)}{2}$. Assuming that the reflection of the beam and diffuse radiation falling on the ground is isotropic and that the surroundings have a diffuse reflectance of ρ for the total solar radiation. Subsequently, the reflected radiation ($\bar{H}_{T,r}$) from the surroundings on the surface will be:

$$\bar{H}_{T,r} = \bar{H}_g\rho\frac{(1-\cos\beta)}{2} \tag{12}$$

where β is the slope of tilted surface, ρ is the constant which depends on the type of ground surrounding tilted surfaces and is called the ground reflectance. The value of the ground reflectance most commonly used $\rho = 0.2$ for hot and humid tropical location, $\rho = 0.5$ for dry tropical locations and $\rho = 0.9$ for snow covered ground (Muneer, 2004).

Diffused radiation ($\bar{H}_{T,d}$):

Diffused radiation ($\bar{H}_{T,d}$) is that fraction of total solar radiation which is received from the sun when its direction has been changed by atmospheric scattering (Kondratev, 1969). The direction of diffused radiation is highly variable and difficult to determine. It is function of condition of cloudiness and atmospheric clearness which are extremely unpredictable. The diffused radiation fraction is also the combination of three components namely isotropic, circumsolar and horizon brightening. The isotropic diffuse radiation component is received evenly from the entire sky dome. The circumsolar diffuse part is received from onward dispersion of solar radiation and concentrated in the section of the sky around the sun (Widen, 2009). The horizon brightening component is concentrated near the horizon and it is most obvious in the clear skies (Robinson and Stone, 2004). In general the diffuse fraction of radiation on inclined surface is composed of isotropic, circumsolar and horizon brightening factors as given by

$$\bar{H}_{T,d} = \bar{H}_{d,\mathrm{iso}}F_{c-s} + \bar{H}_{d,\mathrm{cs}}\bar{R}_b + \bar{H}_{d,\mathrm{hz}}F_{c-\mathrm{hz}}. \tag{13}$$

If the diffuse radiation is considered to be only isotropic, then:

$$F_{c-s} = \frac{(1+\cos\beta)}{2}. \tag{14}$$

Thus Eq. (9) for calculating $\bar{H}_T$ can be rewritten as:

$$\bar{H}_T = \bar{H}_b\bar{R}_b + \bar{H}_g\rho\frac{(1-\cos\beta)}{2} + \bar{H}_{d,\mathrm{iso}}\left(\frac{1+\cos\beta}{2}\right) + \bar{H}_{d,\mathrm{cs}}\bar{R}_b + \bar{H}_{d,hz}F_{c-\mathrm{hz}}. \tag{15}$$

2.4. Description of isotropic and anisotropic sky models for diffuse radiation

The models used to predict the diffuse radiation on a tilted surface are broadly classified as isotropic and anisotropic sky models. Several isotropic and anisotropic models are available in literature: Temps and Coulson (1977); Steven and Unsworth (1980); Perez et al. (1987); Skartveit and Olseth (1986); Perez et al. (1990) and Tian et al. (2001). For this study, total six empirical models were chosen, and their results were compared for selection of suitable and appropriate model for this area. Out of six, three isotropic models namely Liu and Jordan (1960), Koronakis (1986), and Badescu model (2002), and three anisotropic models namely Hay and Davies (1980), Reindl et al. (1990) and HDKR (2006) model were investigated.

A brief description of the isotropic and anisotropic sky models selected for comparison of estimated results is given below:

Liu and Jordan model (LJ):

In this model, the solar radiation on tilted surface is considered to be composed of three parts such as; beam, reflected from ground and diffuse fraction. It was assumed that the diffuse radiation is isotropic only; whereas, circumsolar and horizon brightening were taken as zero. Hence, $\bar{H}_{T,d} = \bar{H}_d\left(\frac{1+\cos\beta}{2}\right)$, and the overall formula for computing the total radiation on tilted surface is proposed as sum of beam, earth reflected and isotropic diffuse radiation. Thus, $\bar{H}_T$ is given as follows:

$$\bar{H}_T = \bar{H}_b\bar{R}_b + \bar{H}_g\rho\left(\frac{1-\cos\beta}{2}\right) + \bar{H}_d\left(\frac{1+\cos\beta}{2}\right). \tag{16}$$

Koronakis model (KO):

Koronakis modified the assumption of isotropic sky diffuse radiation and proposed that the slope $\beta = 90°$ provides 66.7% of diffuse solar radiation of the total sky dome, for example $F_{c-s} = \left(\frac{2+\cos\beta}{3}\right)$. Thus, following correlation was suggested to measure incident radiation on tilted surface.

$$H_T = \bar{H}_b\bar{R}_b + \bar{H}_g\rho\left(\frac{1-\cos\beta}{2}\right) + H_d\left(\frac{2+\cos\beta}{3}\right). \tag{17}$$

Badescu model (BA):

Badescu demonstrated model for the solar diffuse radiation on a tilted surface, and considered the view factor, $F_{c-s} = \left(\frac{3+\cos 2\beta}{4}\right)$. Therefore, the total radiation on a tilted surface was expressed as:

$$H_T = \bar{H}_b\bar{R}_b + \bar{H}_g\rho\left(\frac{1-\cos\beta}{2}\right) + \bar{H}_d\left(\frac{3+\cos 2\beta}{4}\right). \tag{18}$$

Hay and Davies model (HD):

Hay and Davies assumed that the diffuse radiation from the sky is composed of an isotropic and circumsolar component only, whereas, the horizon brightening part was not taken into account. It was assumed that the diffuse parts coming directly from the sun's direction is circumsolar and the diffuse component reaching through the rest of the sky dome isotropically. These components were weighted according to an anisotropy index (A). The anisotropy index was used to quantify a portion of diffuse radiation treated as circumsolar with remaining part of the diffuse radiation assumed to be isotropic. The reflected part is dealt with same as suggested by Liu and Jordan. The total radiation on a tilted surface is proposed as follows.

$$\bar{H}_T = \left(\bar{H}_b + \bar{H}_d A\right)\bar{R}_b + \bar{H}_g\rho\left(\frac{1-\cos\beta}{2}\right) + \bar{H}_d\left(\frac{1+\cos\beta}{2}\right)(1-A) + A\bar{R}_b \tag{19}$$

where A is anisotropy index, which is the function of transmittance of the atmosphere for beam radiation and defined as:

$$A = \frac{\bar{H}_{b,n}}{\bar{H}_{0,n}} = \frac{\bar{H}_b}{\bar{H}_0}. \tag{20}$$

Reindl et al. model (RE):

In this model, horizon brightening factor was added to isotropic diffuse and circumsolar radiation component. Beam and reflected fraction of solar radiation was taken as same, which were proposed by Liu and Jordan and other authors. A definition of anisotropy index (A) was introduced as proposed by Hay and Davies. The modulating factor $f = \sqrt{\frac{\bar{H}_b}{\bar{H}_g}}$ was also added to multiply the term of $\sin^3\left(\frac{\beta}{2}\right)$ for horizon brightening factor. They considered all three components of diffuse fraction, such as $\bar{H}_{T,d,\mathrm{iso}}$, $\bar{H}_{T,d,\mathrm{hz}}$ and $\bar{H}_{T,d,\mathrm{cs}}$ and their proposed model is given below:

$$\bar{H}_T = \left(\bar{H}_b + \bar{H}_d A\right)\bar{R}_b + \bar{H}_g\rho\left(\frac{1-\cos\beta}{2}\right) + \bar{H}_d\left\{(1-A)\right. \\ \left.\times\left(\frac{1+\cos\beta}{2}\right)\left[1+\sqrt{\frac{\bar{H}_b}{\bar{H}_g}}\sin^3\left(\frac{\beta}{2}\right)\right] + A\bar{R}_b\right\}. \tag{21}$$

Hay and Davies, Klucher and Reindl models (HDKR):

If the beam reflected and all terms of diffuse radiation such as isotropic, circumsolar and horizon brightening are added to the solar radiation equation, a new correlation develops called HDKR model. It is basically the combination of Hay and Davies, Klucher and Reindl models. The solar energy irradiation on tilted surface is then determined as:

$$\bar{H}_T = \left(\bar{H}_b + \bar{H}_d A\right)\bar{R}_b + \bar{H}_g\rho\left(\frac{1-\cos\beta}{2}\right) \\ + \bar{H}_d\left\{(1-A)\left(\frac{1+\cos\beta}{2}\right)\left[1+\sin^3\left(\frac{\beta}{2}\right)\right]\right\}. \tag{22}$$

2.5. Methods of models evaluation

In this study, estimated global solar radiation data and tilted global solar radiation data at Bhopal climatic conditions are compared with the data measured by Indian meteorological department. The comparison is done by four statistical tests:

- Mean Absolute Percentage Error (MAPE),
- Mean Bias Error (MBE),
- Root Mean Square Error (RMSE),
- t-statistic

These tests evaluate the accuracy of the correlations described above.

Mean Absolute Percentage Error (MAPE):

Mean absolute percentage error is an indicator of accuracy which usually expresses accuracy as a percentage of the data. It may be expressed as:

$$\mathrm{MAPE} = \frac{1}{n}\sum_{i=1}^{n}\left(\frac{H-H_P}{H}\right)\times 100 \tag{23}$$

where H is Real Value, H_P is Predicted Value, and n is the total number of observations. The ideal value of Mean absolute percentage error is zero.

Mean Bias Error (MBE):

The mean bias error provides information on the long-term performances of the correlations by allowing a comparison of the actual deviation between calculated and measured values term by term. In other words, it is an indicator for the average deviation of the predicted values from the measured data. Mean bias error is given by:

$$\mathrm{MBE} = \frac{1}{n}\sum_{i=1}^{n}(H_{Pi}-H_i) \tag{24}$$

where H_i is ith Real Value, H_{pi} is ith Predicted Value and n is the total number of observations.

The mean bias error provides information on the long term performance. A low MBE is desired. Ideally a zero value of MBE should be obtained. A positive value gives the average amount of over-estimation in the calculated value and vice versa. A drawback of this test is that over estimation of an individual observation will cancel under estimation in a separate observation.

Root Mean Square Error (RMSE):

Provides information on the short-term performance and is a measure the variation of the predicted values around the measured data. The Root Mean Square Error may be computed from the following equation:

$$\mathrm{RMSE} = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(H_{Pi}-H_i)^2} \tag{25}$$

where H_i is ith Real Value, H_{pi} is ith Predicted Value and n is the total number of observations.

The RMSE is always positive, a zero value is ideal. This test provides information on short-terms performance of the correlation by arranging a term by term comparison of the actual deviation between the calculated value and the measured value. The smaller the value, the better the model's performance.

***t*-statistic (*t*-stat)**:

MBE and RMSE separated do not represent a reliable assessment of the models performance and can lead to the false selection of the best model from a set of candidates (Stone, 1993).

$$t\text{-stat} = \sqrt{\frac{(n-1)\,\mathrm{MBE}^2}{\mathrm{RMSE}^2-\mathrm{MBE}^2}}. \tag{26}$$

It is obvious that each test by itself may not be an adequate indicator of a model's performance. It is possible to have a large RMSE value and at the same time a small MBE (a large scatter about the line of perfect estimation). On the other hand, it is also possible to have a relatively small RMSE and a relatively large MBE (a consistently small over- or under-estimation).

3. Result and discussion

3.1. Regression constant for Bhopal

Input parameters for estimation of solar radiation on horizontal and tilted surfaces are shown in Table 2. From this it is observed that declination angle (δ) varies according to the cooper's model (1969), $-23.04°$ (December solstice) and $+23.08°$ (June solstice). Twice in year the value of declination angle becomes zero on two equinoxes (in March and September).

Sunrise and sunset hour angle varies according to the latitude and both will be the same due to symmetry. For Bhopal location average sunshine hour angle (ω_s) is observed approximately 87° which is very good for estimation of solar radiation in this location. From Table 2, it is found that percentage sunshine duration ($s/s_{\max}$) is about 79% thought the year. Employing these parameters the regression constant a and b are obtained from the Angstrom Eq. (4) as $a = 0.27$ and $b = 0.50$ for Bhopal.

Table 2
Input parameters for estimation of monthly average daily global solar radiation at Bhopal, Madhya Pradesh, India.

Month	Δ degree	ω_s degree	S hours	S_{max} hours	$\frac{S}{S_{max}}$ %
Jan.	−20.81	80.59	10	10.24	0.88
Feb.	−12.95	84.60	11	11.28	0.83
Mar.	−2.41	88.96	11	11.86	0.79
Apr.	9.41	135.15	12	11.45	0.82
May	18.79	98.39	13	13.11	0.72
Jun.	23.08	100.55	10	13.40	0.71
Jul.	21.18	80.41	5	10.72	0.88
Aug.	13.45	84.10	5	11.21	0.84
Sept.	2.21	89.07	8	11.87	0.79
Oct.	−9.59	85.84	10	11.44	0.83
Nov.	−18.91	81.53	9	10.87	0.87
Dec.	−23.04	79.47	9	10.59	0.89

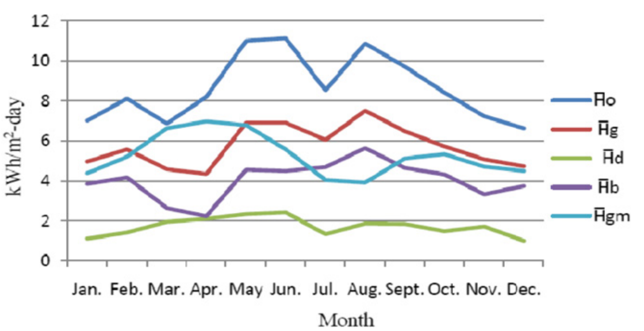

Fig. 2. Variation of monthly average daily solar radiation (H_o, H_g, H_d, H_b and H_{gm}) on horizontal surface at Bhopal.

3.2. Variation of solar radiation on horizontal surface

Input parameters like declination angle δ, sunshine hour angle ω_s and day length S_{max} are inserted in Eq. (4) and to estimate the extraterrestrial solar radiation in daily basis ($\bar{H}_o$) as shown in Fig. 2. H_o is observed to be maximum in May 11.00 kWh/m^2-day and minimum in December 6.62 kWh/m^2-day.

Global solar radiation ($\bar{H}_g$) on monthly average daily basis for horizontal surface is estimated with the help of Angstrom Eq. (4) and regression constant for Bhopal ($a = 0.27$ and $b = 0.50$). Estimated value of $\bar{H}_g$ are compared with the measured value H_{gm} and found that Hg is 5.72 kWh/m^2-day whereas measured value is 5.22 kWh/m^2-day as shown in Fig. 2.

The diffused solar radiation based on Liu and Jordan is commonly recommended for predicting daily diffused radiation at location across the world. However in Indian context Modi and Sukhatme (1979) and Garg and Garg (1985) proposed the modified equation. From the estimated result it is seen that $\bar{H}_d$ is 1.70 kWh/m^2-day which is 30% of total global radiation. That much availability of average monthly solar radiation is encouraging from utilization point of view as shown in Fig. 2.

Monthly average beam radiation on horizontal surface is usually estimated by subtracting diffused solar radiation from global solar radiation on horizontal surface. It is observed that average value of beam radiation is 4.01 kWh/m^2-day which is 70% of the total global solar radiation as shown in Fig. 2.

3.3. Sky condition of Bhopal

Clearness index is the parameter which indicates the transparency of the atmosphere and indicated by fraction of extraterrestrial radiation that reaches the earth surface as global solar radiation. It is measurement of the degree of clearness of the sky. Clearness Index K_T is defined as $K_T = \frac{\bar{H}_g}{\bar{H}_o}$. From the estimated value of $\bar{H}_o$ and $\bar{H}_g$ for Bhopal, K_T is calculated and it is very encouraging to note that the sky over Bhopal is very clear almost throughout the year ($K_T > 0.66$). The transmission through atmosphere K_T along with the diffused radiation and global radiation is shown in Fig. 3. The dip in the value of K_T is accordance with the high value of $\bar{H}_d/\bar{H}_g$ for the same month. The sky is fairly clear during winter months when the solar radiation is demand for utilization purpose for photovoltaic application. Empirical coefficients K_T, $\frac{\bar{H}_d}{\bar{H}_o}$ and $\frac{\bar{H}_d}{\bar{H}_g}$ for Bhopal are shown in Fig. 3.

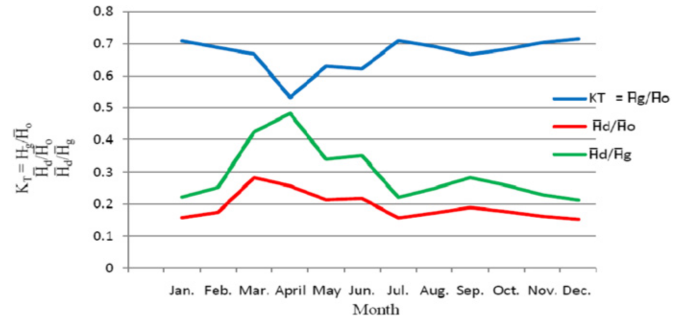

Fig. 3. Monthly variation of clearness index $K_T = H_g/H_o$, H_d/H_o and H_d/H_g at Bhopal.

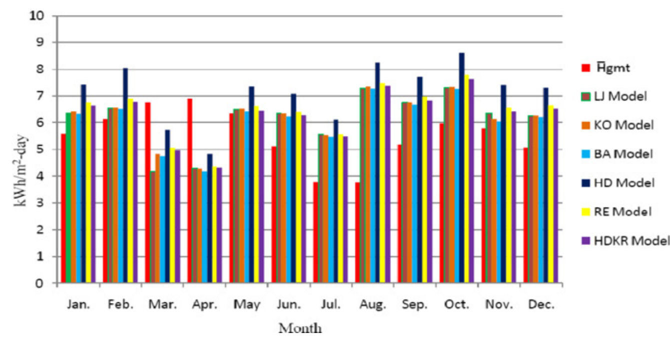

Fig. 4. Comparison of different models with measured monthly average daily solar radiation on tilted surface at Bhopal.

3.4. Variation of estimated solar radiation on tilted surface with using different models

The comparison of results revealed that Liu and Jordan model (LJ) and Koronakis Model (KO) demonstrated almost same results. However Reindl et al. Model (RE) and HDKR Model execute slightly more values than Liu and Jordan model and (LJ) and Koronakis Model (KO) as shown in Fig. 4. This was due to addition of the circumsolar components in diffused radiation fraction in both Reindl et al. Model (RE) and HDKR Model.

Hay and Davies Model (HD) displayed highest estimated values among all models. It may be considerations of all diffuse components individually in this model and incorporation of modulating factor, which was multiplied by the term used for horizon brightening. Badescu Model (BA) demonstrated lowest estimated results than all isotropic and anisotropic models.

It was found from analysis that all models predicted more incident solar energy irradiation on tilted surfaces ($\bar{H}_T$) than on horizontal surface radiation ($\bar{H}_g$) due to the optimization of slope. The slope was fixed at 23.26°N to make the incident angle close to the beam radiation in worst months of year as shown in Table 3.

3.5. Statistical analysis of models

The results of statistical analysis of the solar radiation model are shown in Figs. 5–8. The global component of solar radiation

Table 3
Estimated monthly average daily incident solar radiation (kWh/m^2-day) on tilted surface by different models and measured data at Bhopal.

Month	$\bar{H}_{gm}$ (kWh/m^2-day)	$\bar{H}_{gmt}$ (kWh/m^2-day)	Estimated incident solar radiation on tilted surfaces ($\bar{H}_T$)					
			(LJ) model	(KO) model	(BA) model	(HD) model	(RE) model	HDKR model
Jan.	4.38	5.59	6.36	6.41	6.33	7.43	6.75	6.63
Feb.	5.21	6.12	6.54	6.56	6.51	8.04	6.90	6.77
Mar.	6.62	6.75	4.18	4.82	4.74	5.73	5.06	4.95
April.	6.97	6.90	4.28	4.27	4.18	4.82	4.36	4.31
May	6.78	6.34	6.50	6.52	6.42	7.33	6.62	6.44
Jun.	5.57	5.12	6.35	6.35	6.24	7.08	6.40	6.28
Jul.	4.03	3.78	5.56	5.52	5.46	6.10	5.56	5.47
Aug.	3.91	3.77	7.28	7.34	7.26	8.24	7.49	7.36
Sept.	5.11	5.18	6.75	6.75	6.66	7.71	6.97	6.83
Oct.	5.33	5.97	7.30	7.32	7.25	8.62	7.78	7.63
Nov.	4.70	5.79	6.36	6.12	6.04	7.42	6.56	6.41
Dec.	4.49	5.07	6.26	6.27	6.22	7.29	6.64	6.52
Avg.	5.22	5.53	6.14	6.18	6.10	7.15	6.42	6.30

Fig. 5. Mean Absolute Percentage Error (MAPE) for different models.

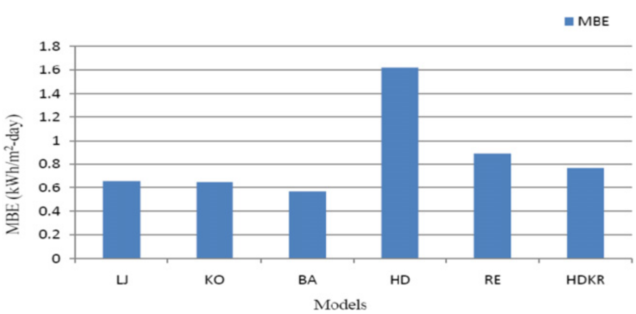

Fig. 6. Mean Bias Errors (MBE) for different models.

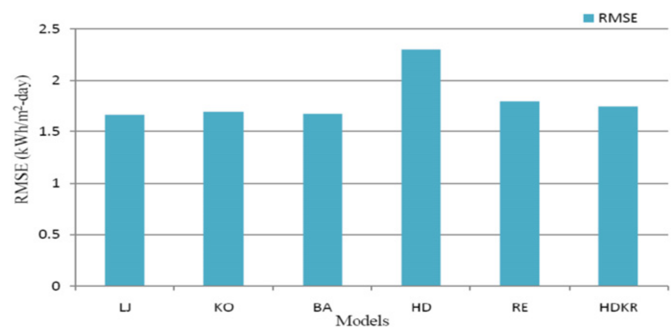

Fig. 7. Root Mean Square Error (RMSE) for different models.

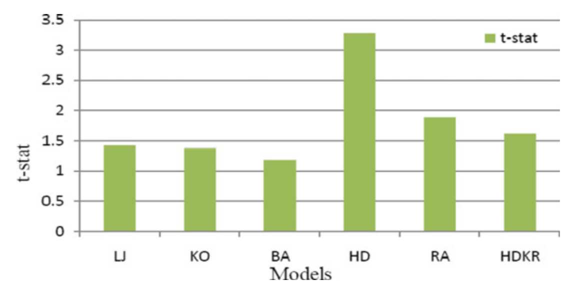

Fig. 8. t-stat for different models.

on the tilted surface was determined from estimated horizontal data using 6 models and compared with the measured tilted data. Ground reflectance was taken as 0.2.

It can be observed from Fig. 5. Mean Absolute Percentage Error (MAPE) is ranging from −10.30% to −29.29% for Badescu Model (BA) and Hay and Davies Model (HD) respectively whereas for other models: Liu and Jordan model (LJ) −11.93%, Koronakis Model (KO) −11.75%, Reindl et al. Model (RE) −16.09% and HDKR Model −13.92% lies in between Badescu Model (BA) and Hay and Davies Model (HD).

The Mean Bias Error (MBE) provides information on the long-term performances of the models by allowing a comparison of the actual deviation between estimated and measured values term by term. In other words, it is an indicator for the average deviation of the predicted values from the measured data. In Fig. 6 it can observed that less MBE was executed by Badescu Model (BA) 0.57 kWh/m^2-day and 2.30 kWh/m^2-day for Hay& Davies Model (HD). Liu and Jordan model (LJ) and Koronakis Model (KO) demonstrated similar behavior with 0.66 kWh/m^2-day and 0.65 kWh/m^2-day respectively. Reindl et al. Model (RE) and HDKR Model scored 0.89 kWh/m^2-day and 0.77 kWh/m^2-day MBE error.

Root Mean Square Error (RMSE) provides information on short-term performance of the models. The smaller the value, the better the model performance. However, a few large errors in the sum can produce significant increase in RMSE. The RMSE is always positive, a zero value is ideal.

As shown in Fig. 7, Hay and Davies Model (HD) scored highest value 2.30 kWh/m^2-day of RMSE whereas Liu and Jordan model (LJ) generating a low RMSE 1.66 kWh/m^2-day which is closed to Badescu Model (BA) 1.67 kWh/m^2-day. Other models Koronakis Model (KO), Reindl et al. Model (RE) and HDKR Model were executed 1.69, 1.79 and 1.74 kWh/m^2-day RMSE error.

It is obvious that MAPE, MBE and RMSE itself may not be an adequate indicator of the models performance since it can be possible to have a large RMSE value and at the same time small MBE (a large scatter about the line of perfect estimation). On the other hand, it is also possible to have a, relatively small RMSE and a relatively large MBE (a consistently small over-or under-estimation). However, all through these statistical indicators provide a reasonable procedure to compare models, they do not objectively indicate whether a model estimate is statistically significant, i.e. not significantly different from their measured counter parts. t-statistical indicator allows models to be compared and at the same time indicate whether or not a model's estimates are statistically significant at a particular confidence level (Stone, 1993). It was seen that the t-statistic used in addition to the RMSE and MBE gave more reliable and explanatory results (Togrul, 1993). Smaller the value of t-stat the better the model performance. From Fig. 8 for Hay and Davies Model (HD) t-stat seen to be 3.28 whereas for Badescu Model (BA) it is executed as 1.19. Liu and Jordan model (LJ) value obtained from the same figure is 1.43 whereas Koronakis Model (KO) value is 1.38. The estimated value obtained for Reindl et al. Model (RE) and HDKR is 1.88 and 1.62 respectively. It is observed that isotropic models having t-state values lower as anisotropic models.

4. Conclusions

The estimation of solar radiation in central part of India, Bhopal for horizontal and tilted surfaces was carried out by considering different input parameters, inclination angle, sun shine hour and day length. The following conclusions are drawn from the comparative study of 6 different models at a tilt angle of 23.26° (latitude of Bhopal).

1. Average of estimated exterritorial solar radiation ($\bar{H}_o$), global solar radiation ($\bar{H}_g$) and diffused solar radiation ($\bar{H}_d$) were found to be 8.63, 5.73 and 1.63 kWh/m^2-day on horizontal surface respectively.
2. Hay and Davies Model (HD) predicted the highest (7.15 kWh/m^2-day) and Badescu Model (BA) demonstrated the lowest values (6.10 kWh/m^2-day) of average solar energy irradiation on tilted surface among all isotropic as well as anisotropic sky models.
3. Most isotropic model predicted lower solar radiation availability in worst month and established higher results in the good weather condition from August to February.
4. Liu and Jordan model (LJ) and Koronakis Model (KO) displayed almost same results 6.19 kWh/m^2-day and 6.18 kWh/m^2-day.
5. The results of statistical analysis revealed that Badescu Model (BA) executed smaller mean absolute percentage error (MAPE) (−10.30%), mean bias error (MBE) (0.57 kWh/m^2-day), root mean square error (RMSE) (1.67 kWh/m^2-day) and t-stat (1.19) among all 6 models.
6. Badescu Model (BA) predicted radiation more close to the measured value and all the statistical errors found to be lowest. Therefore Badescu Model (BA) can be preferred for the estimation of solar radiation on the titled surface in Bhopal.
7. The research work "Estimation of solar radiation on tilted surface at local Bhopal climatic conditions with different isotropic and anisotropic sky models" is the first of its type in the associated field.
8. These 6 models can be implemented all over the country where ground measured data is rarely available. Also monthly average hourly solar radiation can be estimated from this estimated daily data and can be used for near future in Solar PV applications.

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