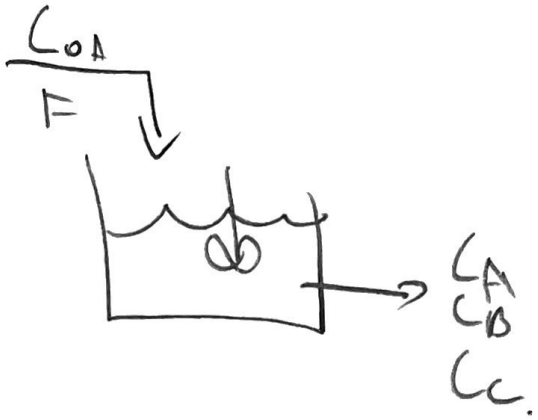
CSTR with reaction $A \rightarrow B \rightleftarrows C$

Feed contains A only




Disturbance in $C_{0A}$ at $t \Rightarrow$

Conservation Equations

$$V\frac{dC_A}{dt} = F(C_{0A} - C_A) - Vk_AC_A$$

$$V\frac{dC_B}{dt} = -FC_B + Vk_AC_A - Vk_BC_B + Vk_CC_C$$

$$V\frac{dC_C}{dt} = -FC_C + Vk_BC_B - Vk_CC_C$$

Use Deviation Variables

$$V\frac{dC_A'}{dt} = F(\Delta C_{0A} - C_A') - Vk_AC_A'$$

$$V\frac{dC_B'}{dt} = -FC_B' + Vk_AC_A' - Vk_BC_B' + Vk_CC_C'$$

$$V\frac{dC_C'}{dt} = -FC_C' + Vk_BC_B' - Vk_CC_C'$$

## Rearrange and Solve for $C_A'(s)$:

$$\frac{dC_A'}{dt} + \underbrace{\frac{F+Vk_A}{V}}_{1/\tau_A} C_A' = \underbrace{\frac{F}{V}}_{K_{pA}/\tau_A} \Delta C_{oA}$$

Laplace Transform:

$$sC_A' + 1/\tau_A C_A' = \frac{K_{pA}}{\tau_A} \Delta C_{oA}(s)$$

$$\boxed{C_A' = \frac{K_{pA}}{(\tau_A s+1)} \Delta C_{oA}(s)}$$

1st order expression

## Rearrange and Solve for $C_c'$:

$$\frac{dC_c'}{dt} + \underbrace{\frac{F+Vk_c}{V}}_{1/\tau_c} C_c' = \frac{k_B}{V} C_B'$$

Laplace Transform:

$$sC_c' + 1/\tau_c C_c' = K_B C_B'$$

$$C_c' = \frac{K_B \tau_c}{(\tau_c s+1)} C_B'$$

Need to solve for $C_B'$ to find $C_c'$

Re-arrange and solve for $C_B'$:

$$\frac{dC_B'}{dt} + \underbrace{\frac{F+Vk_B}{V}}_{1/\tau_B} C_B' = k_A C_A' + k_C C_C'$$

Laplace Transform:

$$sC_B' + 1/\tau_B C_B' = k_A C_A' + k_C C_C'$$

Plug in expressions for $C_A'$ and $C_C'$:

$$C_B'(\tau_B s+1) = \frac{k_A \tau_B k_{pA}}{(\tau_A s+1)} \Delta C_{OA}(s) + \frac{k_C \tau_B k_B \tau_C}{(\tau_C s+1)} C_B'$$

$$C_B'\left(\tau_B s+1 - \frac{k_B k_C \tau_B \tau_C}{(\tau_C s+1)}\right) = \frac{k_A \tau_B k_{pA}}{\tau_A s+1} \Delta C_{OA}(s)$$

$$C_B'\left(\frac{(\tau_B s+1)(\tau_C s+1) - k_B k_C \tau_B \tau_C}{(\tau_C s+1)}\right) = \frac{k_A \tau_B k_{pA}}{\tau_A s+1} \Delta C_{OA}(s)$$

$$\boxed{C_B' = -\frac{k_A \tau_B k_{pA}(\tau_C s+1)}{(\tau_A s+1)(\tau_B s+1)(\tau_C s+1) - (\tau_A s+1) k_B k_C \tau_B \tau_C} \Delta C_{OA}(s)}$$

This is a third order expression

$$C_C' = \frac{k_A k_B \tau_B \tau_C K_{PA}}{(\tau_A s+1)(\tau_B s+1)(\tau_C s+1) - (\tau_A s+1) k_B k_C \tau_B \tau_C} \Delta C_{OA}(s)$$

this is a 3rd. order expression.

Use Syms variables and ilaplace function in Matlab to plot solution

```
Syms τA τB τC kA kB kC kPA s      % Define Symbolic vars.
Vars = s                          % define transform variable.
ilaplace( (kPA/(τA s+1)) (DcA/s) )   % Find inverse Laplace Transform
                                     for ΔCOA(s) = DcA/s
                                     Step function in ΔCOA
```