Partial Fractions and Frequency Response

APPENDIX H

Dynamic models involve differential equations that are best analyzed using Laplace transform methods. In Chapter 4, the partial fraction method was introduced as a way to invert Laplace transforms, and, more importantly, to establish a basis for determining key system properties like stability and frequency response directly from the transfer function. The methods were explained in Chapters 4 and 10 and applied throughout subsequent chapters. The *proofs* of the methods are provided in this appendix.

H.1 ■ PARTIAL FRACTIONS

The Laplace transform method for solving differential equations could be limited by the availability of entries in Table 4.1, and with so few entries, it would seem that most models could not be solved. However, many complex Laplace transforms can be expressed as a linear combination of a few simple transforms through the use of partial fraction expansion. Once the Laplace transform can be expressed as a sum of simpler elements, each can be inverted individually using the entries in Table 4.1, thus greatly increasing the number of differential equations, that can be solved. More importantly, the application of partial fractions provides generalizations about the forms of solutions to a wide range of differential equation models, and these generalizations enable us to establish important characteristics about a system's time-domain behavior *without* determining the complete transient solution.

The partial fraction expansion can be applied to a Laplace transform that can be expressed as a ratio of polynomials in s. This does not pose a severe limitation,