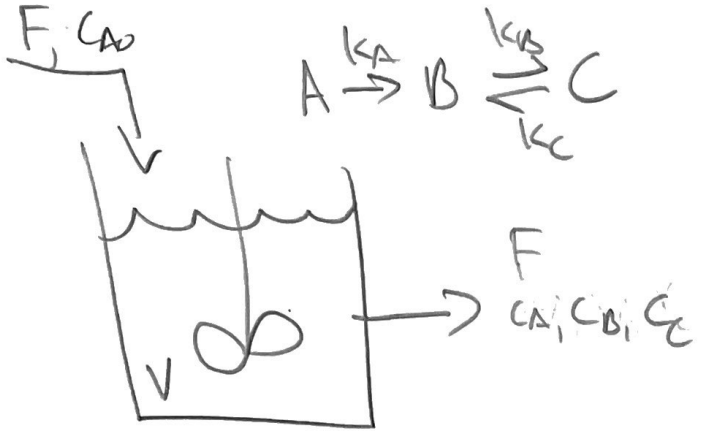
CHBE 470 - Homework #2

Problem 1



Balance for A:

$$V\frac{dC_A}{dt} = F(C_{A0} - C_A) - Vk_A C_A$$

Balance for B:

$$V\frac{dC_B}{dt} = -FC_B + Vk_A C_A - Vk_B C_B + Vk_C C_C$$

Balance for C:

$$V\frac{dC_C}{dt} = -FC_C + Vk_B C_B - Vk_C C_C$$

The three equations above are the solution to the first part

The solution for C_A will be as for the examples discussed in class

$$C_A' = K_{PA}\left(1 - e^{-t/\tau_A}\right)\Delta C_{A0}, \quad \tau_A = \frac{V}{F + VK_A} \quad K_{PA} = \frac{F}{F + VK_A}$$

This can then be used to solve for C_B and C_C.

$$\frac{dC_B}{dt} + \underbrace{\frac{F + K_B}{V}}_{1/\tau_B} C_B = k_A C_A + k_C C_C$$

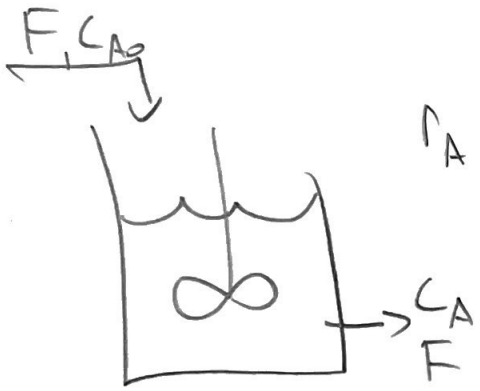
$$\frac{dC_B'}{dt} + 1/\tau_B C_B' = k_A k_{PA}\left(1 - e^{-t/\tau_A}\right)\Delta C_{A0} + k_C C_C'$$

Similarly:

$$\frac{dC_C'}{dt} + \frac{F + VK_C}{V} C_C = k_B C_B'$$

Note that these cannot be solved using the integrating factor method

3.15



$$r_A = -\frac{k_1 C_A}{1+k_2 C_A}$$

a) $$V\frac{dC_A}{dt} = F(C_{A0} - C_A) - \underbrace{\frac{V k_1 C_A}{1+k_2 C_A}}_{\text{nonlinear term}}$$

b) $$\frac{C_A}{1+k_2 C_A} \simeq \frac{C_{As}}{1+k_2 C_{As}} + \left(\frac{1}{1+k_2 C_{As}} - \frac{k_2 C_{As}}{(1+k_2 C_{As})^2}\right)(C_A - C_{As})$$

$$= \frac{C_{As}}{1+k_2 C_{As}} + \left(\frac{1+k_2 C_{As} - k_2 C_{As}}{(1+k_2 C_{As})^2}\right)(C_A - C_{As})$$

$$= \frac{C_{As}}{1+k_2 C_{As}} + \frac{1}{(1+k_2 C_{As})^2} C_A'$$

c) Plug linearized expression into balance

$$V\frac{dC_A}{dt} = F C_{A0} - F C_A - V k_1 \left(\frac{C_{As}}{1+k_2 C_{As}} + \frac{1}{(1+k_2 C_{As})^2} C_A'\right)$$

Write in terms of deviation variables

$$V\frac{dC_A'}{dt} = F\Delta C_{A0} - F C_A' - \frac{V k_1}{(1+k_2 C_{As})^2} C_A'$$

Rearrange and solve:

$$\frac{dC_A'}{dt} + \underbrace{\frac{F + \frac{Vk_1}{(1+k_2C_{A0})^2}}{V}}_{1/\tau} C_A' = \underbrace{\frac{F}{V}}_{k_p/\tau} \Delta C_{A0}$$

$$C_A' = k_p \Delta C_{A0}\left(1 - e^{-t/\tau}\right)$$

$$\tau = \frac{V}{F + \frac{Vk_1}{(1+k_2C_{A0})^2}} \qquad k_p = \frac{F}{F + \frac{Vk_1}{(1+k_2C_{A0})^2}}$$