The Feedback Loop

7.1 ▣ INTRODUCTION

Now that we are prepared with a good understanding of process dynamics, we can begin to address the technology for automatic process control. The goals of process control—safety, environmental protection, equipment protection, smooth operation, quality control, and profit—are achieved by maintaining selected plant variables as close as possible to their best conditions. The variability of variables about their best values can be reduced by adjusting selected input variables using feedback control principles. As explained in Chapter 1, feedback makes use of an output of a system in deciding the way to influence an input to the system, and the technology presented in this part of the book explains how to employ feedback. This chapter builds on the chapters in Part I of the book, which were more qualitative and descriptive, by establishing the key quantitative aspects of a control system.

It is important to emphasize that we are dealing with the control *system*, which involves the process and instrumentation as well as the control calculations. Thus, this chapter begins with a section on the feedback loop in which all elements are discussed. Then, reasons for control are reviewed, and because engineers should always be prepared to define measures of the effectiveness of their efforts, quantitative measures of control performance are defined for key disturbances; these measures are used throughout the remainder of the book. Because the process usually has several input and output variables, initial criteria are given for selecting the variables for a control loop. Finally, several general approaches to feedback

control, ranging from manual to automated methods, are discussed, along with guidelines for when to employ each approach.

7.2 □ PROCESS AND INSTRUMENT ELEMENTS OF THE FEEDBACK LOOP

All elements of the feedback loop can affect control performance. In this section, the process and instrument elements of a typical loop, excluding the control calculation, are introduced, and some quantitative information on their dynamics is given. This analysis provides a means for determining which elements of the loop introduce significant dynamics and when the dynamics of some fast elements can usually be considered negligible.

A typical feedback control loop is shown in Figure 7.1. This discussion will address each element of the loop, beginning with the signal that is sent to the process equipment. This signal could be determined using feedback principles by a person or automatically by a computing device. Some key features of each element in the control loop are summarized in Table 7.1.

The feedback signal in Figure 7.1 has a range usually expressed as 0 to 100%, whether determined by a controller or set manually by a person. When the signal is transmitted electronically, it usually is converted to a range of 4 to 20 milliamperes (mA) and can be transmitted long distances, certainly over one mile. When the signal is transmitted peumatically, it has a range of 3 to 15 psig and can only be transmitted over a shorter distance, usually limited to about 400 meters unless special signal reinforcement is provided. Pneumatic transmission would normally be used only when the controller is performing its calculations pneumatically, which is not common with modern equipment. Naturally, the electronic signal
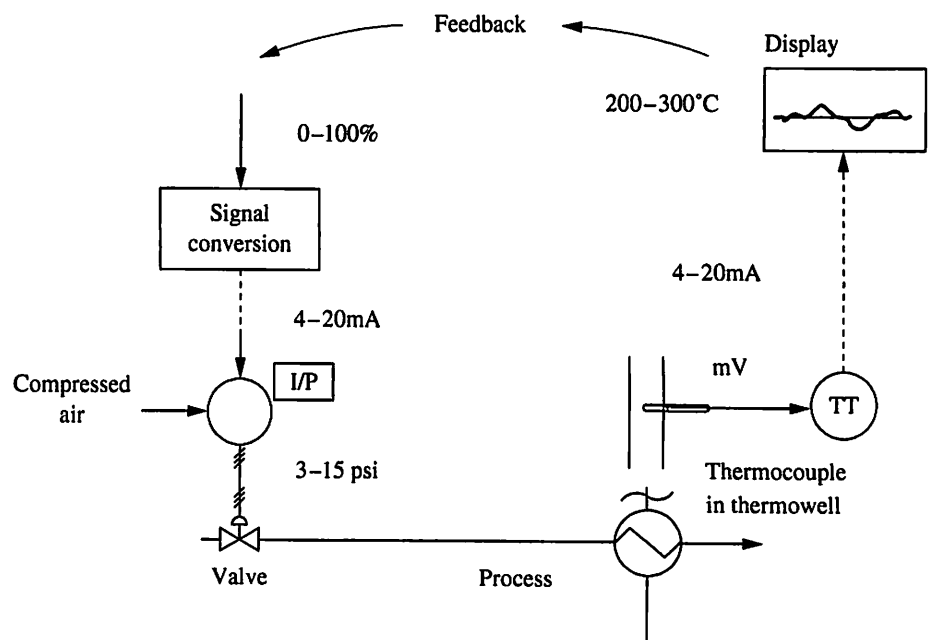

FIGURE 7.1

Process and instrument elements in a typical control loop.

TABLE 7.1

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Key features of control loop elements, excluding the process

Loop element*	Function	Typical range	Typical dynamic response, $t_{63\%}$ **
Controller output	Initiate signal at a remote location intended for the final element	Operator/controller use 0–100%	
Transmission	Carry signal from controller to final element and from the sensor to the controller	*Pneumatic:* 3–15 psig *Electronic:* 4–20 milliamp (mA)	*Pneumatic:* 1–5 s *Electronic:* Instantaneous
Signal conversion	Change transmission signal to one compatible with final element	*Electronic to pneumatic:* 4–20 mA to 3–15 psig *Sensor to electronic:* mV to 4–20 mA	0.5–1.0 s
Final control element	Implement desired change in process	*Valve:* 0–100% open	1–4 s
Sensor	Measure controlled variable	Scale selected to give good accuracy, e.g., 200–300°C	Typically from a few seconds to several minutes

*The terms *input* and *output* are with respect to a controller.
**Time for output to reach 63% after step input.

transmission is essentially instantaneous; the pneumatic signal requires several seconds for transmission. Note that the standard signal ranges (e.g., 4 to 20 mA) are very important so that equipment manufactured by different suppliers can be interchanged.

At the process unit, the output signal is used to adjust the final control element: the equipment that is manipulated by the control system. The final control element in the example, as in over 90 percent of process control applications, is a valve. The valve percent opening could be set by an electrical motor, but this is not usually done because of the danger of explosion with the high-amperage power supply a motor would require. The alternative power supply typically used is compressed air. The signal is converted from electrical to pneumatic; 3 to 15 psig is the standard range of the pneumatic signal. The conversion is relatively accurate and rapid, as indicated by the entry for this element in Table 7.1. The pneumatic signal is transmitted a short distance to the control valve, which is specially designed to adjust its percent opening based on the pneumatic signal. Control valves respond relatively quickly, with typical time constants ranging from 1 to 4 sec.

The general principles of a control valve are demonstrated in Figure 7.2. The process fluid flows through the opening in the valve, with the amount open (or resistance to flow) determined by the valve stem position. The valve stem
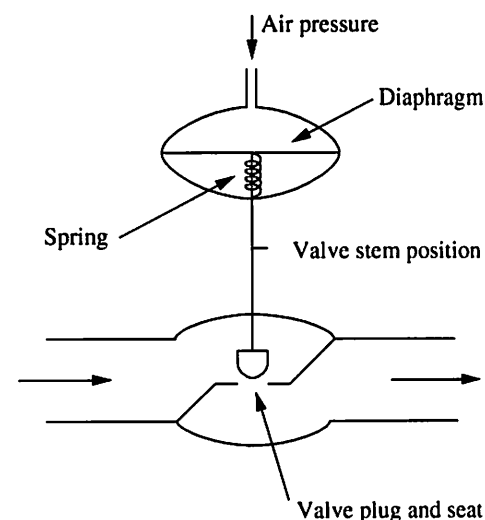

FIGURE 7.2

Schematic of control valve.

is connected to the diaphragm, which is a flexible metal sheet that can bend in response to forces. The two forces acting on the diaphragm are the spring and the variable pressure from the control signal. For a zero control signal (3 psig), the diaphragm in Figure 7.2 would be deformed upward because of the greater force from the spring, and the valve stem would be raised, resulting in the greatest opening for flow. For a maximum signal (15 psig), the diaphragm in Figure 7.2 would be deformed downward by the greater force from the air pressure, and the stem would be lowered, resulting in the minimum opening for flow. Other arrangements are possible, and selection criteria are presented in Chapter 12.

After the final control element has been adjusted, the process responds to the change. The process dynamics vary greatly for the wide range of equipment in the process industries, with typical dead times and time constants ranging from a few seconds (or faster) to hours. When the process is by far the slowest element in the control loop, the dynamics of the other elements are negligible. This situation is common, but important exceptions occur, as demonstrated in Example 7.1.

The sensor responds to the change in plant conditions, preferably indicating the value of a single process variable, unaffected by all other variables. Usually, the sensor is not in direct contact with the potentially corrosive process materials; therefore, the protective equipment or sample system must be included in the dynamic response. For example, a thin thermocouple wire responds quickly to a change in temperature, but the metal sleeve around the thermocouple, the thermowell, can have a time constant of 5 to 20 sec. Most sensor systems for flow, pressure, and level have time constants of a few seconds. Analyzers that perform complex physicochemical analyses can have much slower responses, on the order of 5 to 30 minutes or longer; they may be *discrete*, meaning that a new analyzer result becomes available periodically, with no new information between results. Physical principles and performance of sensors are diverse, and the reader is encouraged to refer to information in the additional resources from Chapter 1 on sensors for further details.

The sensor signal is transmitted to the controller, which we are considering to be located in a remote control room. The transmission could be pneumatic (3 to 15 psig) or electrical (4 to 20 mA). The controller receives the signal and performs its control calculation. The controller can be an *analog* system; for example, an electronic analog controller consists of an electrical circuit that obeys the same equations as the desired control calculations (Hougen, 1972). For the next few chapters, we assume that the controller is a continuous electronic controller that performs its calculations instantaneously, and we will see in Chapter 11 that essentially the same results can be obtained by a very fast digital computer, as is used in most modern control equipment.

EXAMPLE 7.1.

The dynamic responses of two process and instrumentation systems similar to Figure 7.1, *without the controller*, are evaluated in this exercise. The system involves electronic transmission, a pneumatic valve, a first-order-with-dead-time process, and a thermocouple in a thermowell. The dynamics of the individual elements are given in Table 7.2 with the time in seconds for two different systems, A and B. The dynamics of the entire loop are to be determined. The question could be stated, "How does a unit step change in the manual output affect the displayed variable,

TABLE 7.2

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Dynamic models for elements in Example 7.1

Element	Units*	Case A	Case B
Manual station	mA/% output	0.16	0.16
Transmission		1.0	1.0
Signal conversion	psi/mA	$0.75/(0.5s + 1)$	$0.75/(0.5s + 1)$
Final element	%open/psi	$8.33/(1.5s + 1)$	$8.33/(1.5s + 1)$
Process	°C/psi	$1.84e^{-1.0s}/(3s + 1)$	$1.84e^{-100s}/(300s + 1)$
Sensor	mV/°C	$0.11/(10s + 1)$	$0.11/(10s + 1)$
Signal conversion	mA/mV	$1.48/(0.51s + 1)$	$1.48/(0.51s + 1)$
Transmission		1.0	1.0
Display	°C/mA	$6.25/(1.0s + 1)$	$6.25/(1.0s + 1)$

*Time is in seconds.

which is also the variable available for control, in the control house?" Note that the two systems are identical except for the process transfer functions.

The physical system in this problem and shown in Figure 7.1 is recognized as a series of noninteracting systems. Therefore, equation (5.40) can be applied to determine the transfer function of the overall noninteracting series system. The result for Case B is

$$\frac{Y(s)}{X(s)} = \prod_{i=0}^{n-1} G_{n-i}(s)$$

$$\frac{Y(s)}{X(s)} = \frac{(0.16)(1.0)(0.75)(8.33)(1.84)(0.11)(1.48)(1.0)(6.25)e^{-100s}}{(0.5s + 1)(1.5s + 1)(300s + 1)(10s + 1)(0.51s + 1)(s + 1)}$$

Before the simulation results are presented for this example, it is worthwhile performing an approximate analysis, using the simple approximation introduced in Chapter 5 for series processes. The overall gains and approximate 63 percent times for both systems that relate the manual signal to the display are shown in the following table:

	Case A	Case B	
Process gain $K_p = \prod K_i$	1.84	1.84	°C/(% controller output)
Time to 63% $\approx \Sigma(\tau_i + \theta_i)$	≈ 17.5	≈ 413.5	seconds

The two cases have been simulated, and the results are plotted in Figure 7.3a and b. The results of the approximate analysis compare favorably with the simulations. Note that for system A, which involves a fast process, the sensor and final element contribute significant dynamics, resulting in a substantial difference between the true process temperature and the displayed value of the temperature, which would be used for feedback control. In system B the process dynamics are much slower,
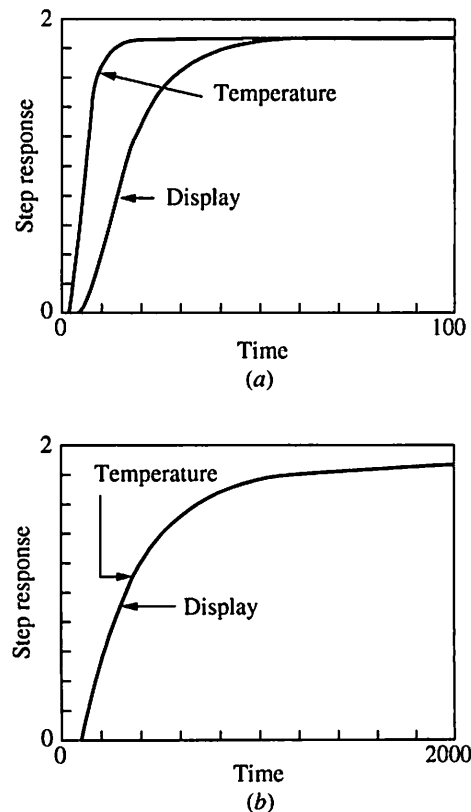

FIGURE 7.3

Transient response for Example 7.1 with a 1% step input change at time = 0. (*a*) Case A; (*b*) Case B.

and the dynamic effects of all other elements in the loop are negligible. This is a direct consequence of the time-domain solution to the model of this process for a step $(1/s)$ input, which has the form

$$Y'(t) = C_1 + C_2 e^{-t/\tau_2} + C_3 e^{-t/\tau_3} + \cdots$$

Clearly, a slow "mode" due to one especially long time constant will dominate the dynamic response, with the faster elements essentially at quasi-steady state. One would expect that a dynamic analysis that considered the process alone for control design would not be adequate for Case A but would be adequate for Case B.

It is worth recalling that the empirical methods for determining the "process" dynamics presented in Chapter 6 involve changes to the manipulated signal and monitoring the response of the sensor signal as reported to the control system. Thus, the resulting model includes *all elements in the loop,* including instrumentation and transmission. Since the experiments usually employ the same instrumentation used subsequently for implementing the control system, the dynamic model identified is between the controller output and input—in other words, the system "seen" by the controller. This seems like the appropriate model for use in design control systems, and that intuition will be supported by later analysis.

7.3 ▣ SELECTING CONTROLLED AND MANIPULATED VARIABLES

Feedback control provides a connection between the controlled and manipulated variables. Perhaps the most important decision in designing a feedback control system involves the selection of variables for measurement and manipulation. Some initial criteria are introduced in this section and applied to the continuous-flow chemical reactor in Figure 7.4. As more details of feedback control are presented, further criteria will be presented throughout Part III for a single-loop controller.

We begin by considering the controlled variable, which is selected so that the feedback control system can achieve an important control objective. The seven categories of control objectives were introduced in Chapter 2 and are repeated below.
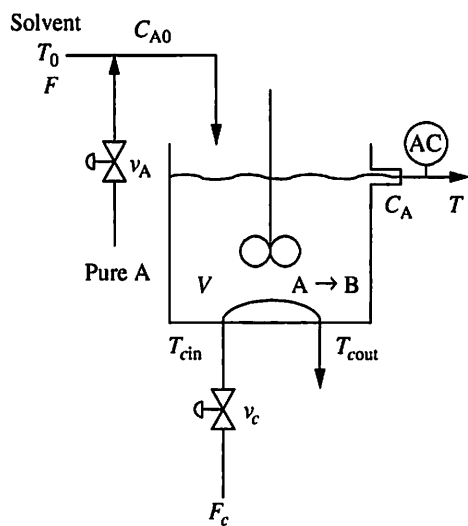

FIGURE 7.4

Continuous-flow chemical reactor example for selecting control loop variables.

Control objective	Process variable	Sensor
1. Safety		
2. Environmental protection		
3. Equipment protection		
4. Smooth plant operation and production rate		
5. Product quality ⟶	Concentration of ⟶ reactant A in the effluent	Analyzer in reactor effluent measuring the mole % A
6. Profit optimization		
7. Monitoring and diagnosis		

From none to several controlled variables may be associated with each control objective. Here, we consider the product quality objective and decide that the most important process variable associated with product quality is the concentration of reactant A in the reactor effluent. The process variable must be measured in real time to make it available to the computer, and the natural selection for the sensor would be an analyzer in the effluent stream. In practice, an onstream analyzer might not exist or might be too costly; for the next few chapters we will assume that a sensor is available to measure the key process variable and defer discussions of using substitute (inferential) variables, which are more easily measured, until later chapters.

The second key decision is the selection of the manipulated variable, because we must adjust some process variable to affect the process. First, we identify all input variables that influence the measured variable. The input variables are summarized below for the reactor in Figure 7.4.

Input variables that affect the measured variable	Selected adjustable flow	Manipulated valve
Disturbances:		
Feed temperature		
Solvent flow rate		
Feed composition, before mix		
Coolant inlet temperature		
Adjustable:		
Flow of pure A $\longrightarrow$	Flow of pure A $\longrightarrow$	v_A
Flow of coolant		

Six important input variables are identified and separated into two categories: those that cannot be adjusted (disturbances) and those that can be adjusted. In general, the disturbance variables change due to changes in other plant units and in the environment outside the plant, and the control system should compensate for these disturbances. Disturbances cannot be used as manipulated variables.

Only adjustable variables can be candidates for selection as a manipulated variable. To be an adjustable flow, a valve must influence the flow. (In general, manipulated variables include adjustable motor speeds and heater power, and so forth, but for the current discussion, we restrict the discussion to valves.) Criteria for selecting an adjustable variable include

1. Causal relationship between the valve and controlled variable (required)
2. Automated valve to influence the selected flow (required)
3. Fast speed of response (desired)
4. Ability to compensate for large disturbances (desired)
5. Ability to adjust the manipulated variable rapidly and with little upset to the remainder of the plant (desired)

As a method for ensuring that the manipulated variable has a causal relationship on the controlled variable, the dynamic model between the valve and controlled

variable must have a nonzero value, i.e., $\Delta C_A / \Delta FA = K_p \neq 0$. An important aspect of chemical plant design involves providing streams which accommodate the five criteria above; examples are cooling water, steam, and fuel gas, which are distributed and made available throughout a plant.

Two potential adjustable flows exist in this example, and based on the information available, either is acceptable. For the present, we will arbitrarily select the valve affecting the flow of pure component A, v_A. After we have analyzed the effects of feedback dynamics more thoroughly, we will reconsider this selection in Example 13.12.

> In conclusion, the feedback system for product quality control connects the effluent composition analyzer to the valve in the pure A line.

The next section discusses desirable features of dynamic behavior for a control system and how these features can be characterized quantitatively. The calculations performed by the controller to determine the valve opening are presented in the next chapter.

7.4 □ CONTROL PERFORMANCE MEASURES FOR COMMON INPUT CHANGES

The purpose of the feedback control loop is to maintain a small deviation between the controlled variable and the set point by adjusting the manipulated variable. In this section, the two general types of external input changes are presented, and quantitative control performance measures are presented for each.

Set Point Input Changes

The first type of input change involves changes to the *set point:* the desired value for the operating variable, such as product composition. In many plants the set points remain constant for a long time. In other plants the values may be changed periodically; for example, in a batch operation the temperature may need to be changed during the batch.

Control performance depends on the goals of the process operation. Let us here discuss some general control performance measures for a change in the controller set point on the three-tank mixing process in Figure 7.5. In this process, two streams, A and B, are mixed in three series tanks, and the output concentration of component A is controlled by manipulating the flow of stream A. Here, we consider step changes to the set point; these changes represent the situation in which the plant operator occasionally changes the value and allows considerable time for the control system to respond. A typical dynamic response is given in Figure 7.6. This is somewhat idealized, because there is no measurement noise or effect of disturbances, but these effects will be considered later. Several facets of the dynamic response are considered in evaluating the control performance.
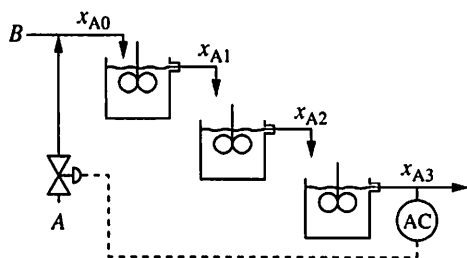

FIGURE 7.5

Example feedback control system, three-tank mixing process.

OFFSET. Offset is a difference between final, steady-state values of the set point and of the controlled variable. In most cases, a zero steady-state offset is highly
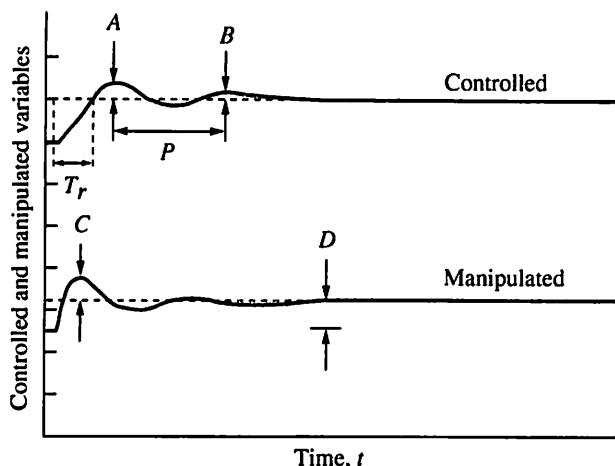
FIGURE 7.6

Typical transient response of a feedback control
system to a step set point change.

desired, because the control system should achieve the desired value, at least after a very long time.

RISE TIME. This (T_r) is the time from the step change in the set point until the controlled variable *first* reaches the new set point. A short rise time is usually desired.

INTEGRAL ERROR MEASURES. These indicate the cumulative deviation of the controlled variable from its set point during the transient response. Several such measures are used:

Integral of the absolute value of the error (IAE):

$$IAE = \int_0^\infty |SP(t) - CV(t)| \, dt \qquad (7.1)$$

Integral of square of the error (ISE):

$$ISE = \int_0^\infty [SP(t) - CV(t)]^2 dt \qquad (7.2)$$

Integral of product of time and the absolute value of error (ITAE):

$$ITAE = \int_0^\infty t \, |SP(t) - CV(t)| \, dt \qquad (7.3)$$

Integral of the error (IE):

$$IE = \int_0^\infty [SP(t) - CV(t)] dt \qquad (7.4)$$

The IAE is an easy value to analyze visually, because it is the sum of areas above and below the set point. It is an appropriate measure of control performance when the effect on control performance is linear with the deviation magnitude. The ISE is appropriate when large deviations cause greater performance degradation than small deviations. The ITAE penalizes deviations that endure for a long time. Note

that IE is not normally used, because positive and negative errors cancel in the integral, resulting in the possibility for large positive and negative errors to give a small IE. A small integral error measure (e.g., IAE) is desired.

DECAY RATIO (B/A). The decay ratio is the ratio of neighboring peaks in an underdamped controlled-variable response. Usually, periodic behavior with large amplitudes is avoided in process variables; therefore, a small decay ratio is usually desired, and an overdamped response is sometimes desired.

THE PERIOD OF OSCILLATION (P). Period of oscillation depends on the process dynamics and is an important characteristic of the closed-loop response. It is not specified as a control performance goal.

SETTLING TIME. Settling time is the time the system takes to attain a "nearly constant" value, usually ±5 percent of its final value. This measure is related to the rise time and decay ratio. A short settling time is usually favored.

MANIPULATED-VARIABLE OVERSHOOT (C/D). This quantity is of concern because the manipulated variable is also a process variable that influences performance. There are often reasons to prevent large variations in the manipulated variable. Some large manipulations can cause long-term degradation in equipment performance; an example is the fuel flow to a furnace or boiler, where frequent, large manipulations can cause undue thermal stresses. In other cases manipulations can disturb an integrated process, as when the manipulated stream is supplied by another process. On the other hand, some manipulated variables can be adjusted without concern, such as cooling water flow. We will use the overshoot of the manipulated variable as an indication of how aggressively it has been adjusted. The overshoot is the maximum amount that the manipulated variable exceeds its final steady-state value and is usually expressed as a percent of the change in manipulated variable from its initial to its final value. Some overshoot is acceptable in many cases; little or no overshoot may be the best policy in some cases.

Disturbance Input Changes

The second type of change to the closed-loop system involves variations in uncontrolled inputs to the process. These variables, usually termed *disturbances,* would cause a large, sustained deviation of the controlled variable from its set point if corrective action were not taken. The way the input disturbance variables vary with time has a great effect on the performance of the control system. Therefore, we must be able to characterize the disturbances by means that (1) represent realistic plant situations and (2) can be used in control design methods. Let us discuss three idealized disturbances and see how they affect the example mixing process in Figure 7.5. Several facets of the dynamic responses are considered in evaluating the control performance for each disturbance.

STEP DISTURBANCE. Often, an important disturbance occurs infrequently and in a sudden manner. The causes of such disturbances are usually changes to
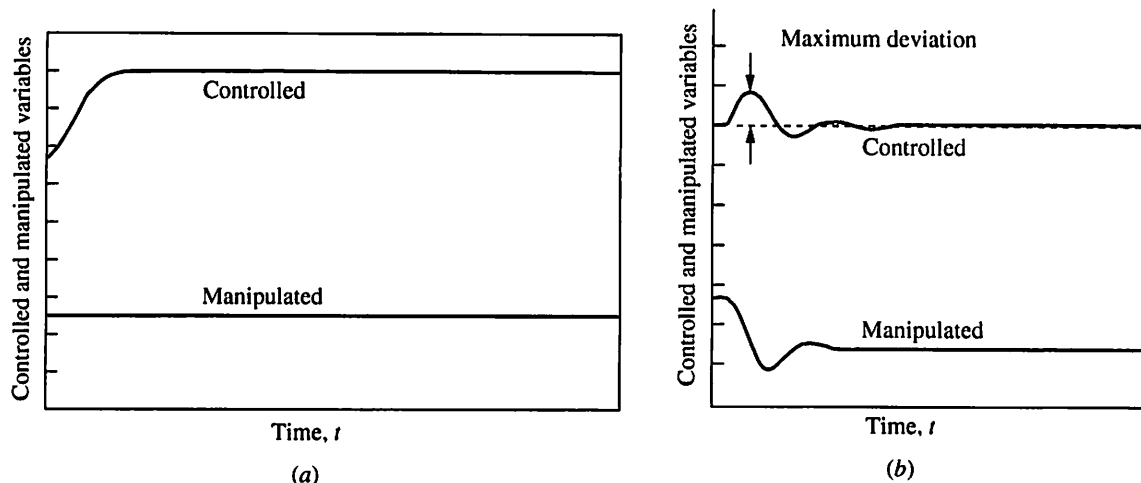
FIGURE 7.7

Transient response of the example process in Figure 7.5 in response to a step disturbance (*a*) without feedback control; (*b*) with feedback control.

other parts of the plant that influence the process being considered. An example of a step upset in Figure 7.5 would be the inlet concentration of stream *B*. The responses of the outlet concentration, without and with control, to this disturbance are given in Figure 7.7*a* and *b*. We will often consider dynamic responses similar to those in Figure 7.7 when evaluating ways to achieve good control that minimizes the effects of step disturbances. The explanations for the measures are the same as for set point changes except for rise time, which is not applicable, and for the following measure, which has meaning only for disturbance responses and is shown in Figure 7.7*b*:

MAXIMUM DEVIATION. The *maximum deviation* of the controlled variable from the set point is an important measure of the process degradation experienced due to the disturbance; for example, the deviation in pressure must remain below a specified value. Usually, a small value is desirable so that the process variable remains close to its set point.

STOCHASTIC INPUTS. As we recognize from our experiences in laboratories and plants, a process typically experiences a continual stream of small and large disturbances, so that the process is never at an exact steady state. A process that is subjected to such seemingly random upsets is termed a *stochastic system*. The response of the example process to stochastic upsets in all flows and concentrations is given in Figure 7.8*a* and *b* without and with control.

The major control performance measure is the variance, σ_{CV}^2, or standard deviation, σ_{CV}, of the controlled variable, which is defined as follows for a sample of *n* data points:

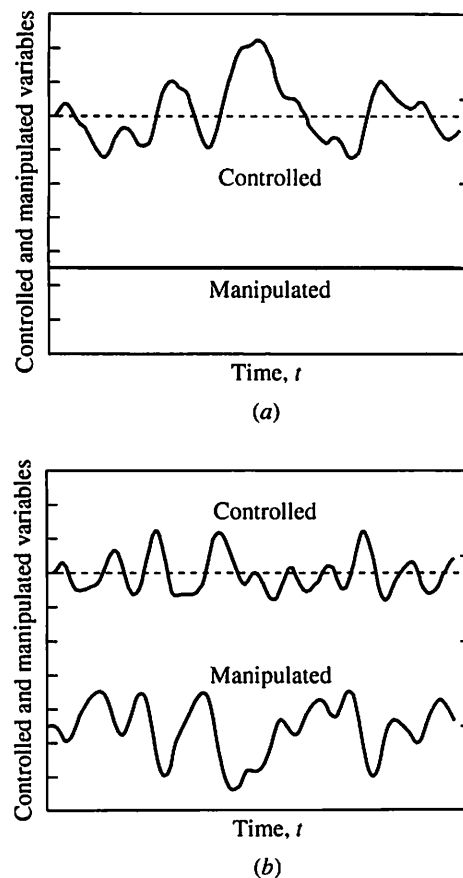$$\sigma_{CV} = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(\overline{CV} - CV_i)^2} \qquad (7.5)$$

FIGURE 7.8

Transient response of the example process (*a*) without and (*b*) with feedback control to a stochastic disturbance.

$$\text{With the mean} = \overline{CV} = \frac{1}{n}\sum_{i=1}^{n}CV_i \tag{7.6}$$

This variable is closely related to the ISE performance measure for step disturbances. The relationship depends on the approximations that (1) the mean can be replaced with the set point, which is normally valid for closed-loop data, and (2) the number of points is large.

$$\frac{1}{n-1}\sum_{i=1}^{n}(\overline{CV} - CV_i)^2 \approx \frac{1}{T}\int_0^T (SP - CV)^2 dt \tag{7.7}$$

Since the goal is usually to maintain controlled variables close to their set points, a small value of the variance is desired. In addition, the variance of the manipulated variable is often of interest, because too large a variance could cause long-term damage to equipment (fuel to a furnace) or cause upsets in plant sections providing the manipulated stream (steam-generating boilers). We will not be analyzing stochastic systems in our design methods, but we will occasionally confirm that our designs perform well with example stochastic disturbances by simulation case studies. As you may expect, the mathematical analysis of these statistical disturbances is challenging and requires methods beyond the scope of this book. However, many practical and useful methods are available and should be considered by the advanced student (MacGregor, 1988; Cryor, 1986).

SINE INPUTS. An important aspect of stochastic systems in plants is that the disturbances can be thought of as the sum of many sine waves with different amplitudes and frequencies. In many cases the disturbance is composed predominantly of one or a few sine waves. Therefore, the behavior of the control system in response to sine inputs is of great practical importance, because through this analysis we learn how the frequency of the disturbances influences the control performance. The responses of the example system to a sine disturbance in the inlet concentration of stream B with and without control are given in Figure 7.9a and b. Control performance is measured by the amplitude of the output sine, which is often expressed as the ratio of the output to input sine amplitudes. Again, a small output amplitude is desired. We shall use the response to sine disturbances often in analyzing control systems, using the frequency response calculation methods introduced in Chapter 4.

In summary, we will be considering two sources of external input change: set point changes and disturbances in input variables. Usually, we will consider the time functions of these disturbances as step and sine changes, because they are relatively easy to analyze and yield useful insights. The measures of control performance for each disturbance-function combination were discussed in this section.

It is important to emphasize two aspects of control performance. First, ideally good performance with respect to all measures is usually not possible. For example, it seems unreasonable to expect to achieve very fast response of the controlled variable through very slow adjustments in the manipulated variable. Therefore, control design almost always involves compromise. This raises the second aspect: that control performance must be defined with respect to the process operating objectives of a specific process or plant. It is not possible to define one set of universally applicable control performance goals for all chemical reactors or all
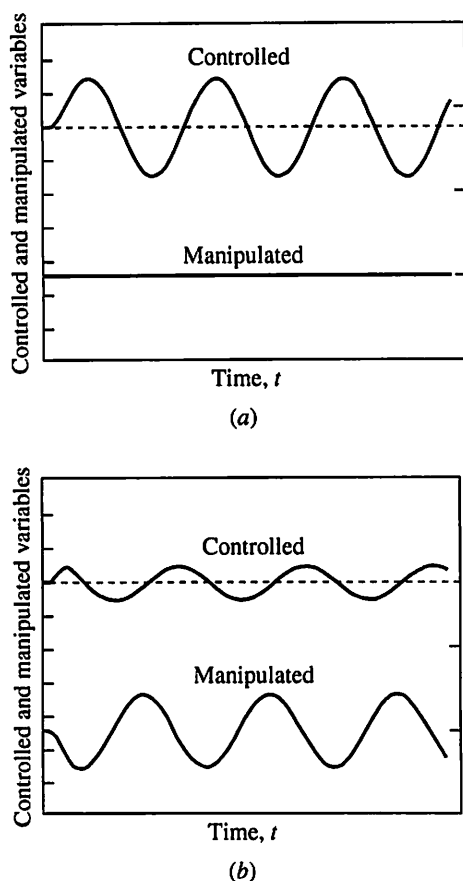

FIGURE 7.9

Transient response of the example system (a) without and (b) with control to a sine disturbance.

distillation towers. Guidance on setting goals will be provided throughout the book via many examples, with emphasis on the most common goals.

> Feedback reduces the variability of the controlled variable at the expense of increased variability of the manipulated variable.

Finally, the responses to all changes have demonstrated by example an important point that will be proved in later chapters. The application of feedback control does not eliminate variability in the process plant; in fact, the "total variability" of the controlled and manipulated variables may not be changed. This conclusion follows from the observation that a manipulated variable must be adjusted to reduce the variability in the output controlled variable. If these variables are selected properly, the performance of the plant, as measured by safety, product quality, and so forth, improves. The availability of manipulated variables depends on a skillful process design that provides numerous utility systems, such as cooling water, steam, and fuel, which can be adjusted rapidly with little impact on the performance of the plant.

EXAMPLE 7.2.

One of the example processes analyzed several times in Part III is the three-tank mixing process in Figure 7.5. This process is selected for its simplicity, which enables us to determine many characteristics of the feedback system, although it is complex enough to exhibit realistic behavior. The process design and model are introduced here; the linearized model is derived; and the selection of variables is discussed.
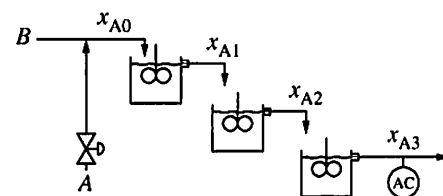
Goal. The outlet concentration is to be maintained close to its set point. Derive the nonlinear and linearized models and select controlled and manipulated variables.

Assumptions.
1. All tanks are well mixed.
2. Dynamics of the valve and sensor are negligible.
3. No transportation delays (dead times) exist.
4. A linear relationship exists between the valve opening and the flow of component A.
5. Densities of components are equal.

Data.

V = volume of each tank = 35 m^3

F_B = flow rate of stream B = 6.9 m^3 min

x_{Ai} = concentration of A in all tanks and outlet flow = 3% A (base case)

F_A = flow rate of stream A = 0.14 m^3/min (base case)

$(x_A)_B$ = concentration of stream B = 1% A (base case)

$(x_A)_A$ = concentration of stream A = 100% A

v = valve position = 50% open (base case)

Thus, the product flow rate is essentially the flow of stream B; that is, $F_B \gg F_A$.

Formulation. Since the variable to be controlled is the concentration leaving the last tank, component material balances on the mixing point and each mixing tank are given below.

$$x_{A0} = \frac{F_B(x_A)_B + F_A(x_A)_A}{F_B + F_A} \tag{7.8}$$

$$V_i\frac{dx_{Ai}}{dt} = (F_A + F_B)(x_{Ai-1} - x_{Ai}) \quad \text{for } i = 1, 3 \tag{7.9}$$

Note that the differential equations are nonlinear, because the products of flow and concentrations appear. (If you need a refresher, see Section 3.4 for the definition of linearity.) We will linearize these equations and determine how the process gains and time constants depend on the equipment and operating variables. The linearized models are now summarized, with the subscript s representing the initial steady state and the prime representing deviation variables.

$$F'_A = K_v v' \quad K_v = 0.0028\,\frac{\text{m}^3/\text{min}}{\%\text{ open}} \tag{7.10}$$

$$x'_{A0} = \left[\frac{F_{Bs}\{(x_{AA})_s - (x_{AB})_s\}}{(F_{Bs} + F_{As})^2}\right]F'_A \tag{7.11}$$

$$\frac{dx'_A}{dt} + \frac{F_{As} + F_{Bs}}{V}x'_{Ai} = \frac{F_{As} + F_{Bs}}{V}x'_{Ai-1} \quad \text{for } i = 1, 3 \tag{7.12}$$

The total flow is assumed to be approximately constant. By taking the Laplace transforms of these equations and performing standard algebraic manipulations, the feedback process relating the valve (v) to concentration (x_{A3}) transfer function can be derived:

Feedback: $$\frac{x_{A3}(s)}{v(s)} = G_p(s) = \frac{K_p}{(\tau s + 1)^3} \tag{7.13}$$

with $$K_p = K_v\left[\frac{F_{Bs}(x_{AA} - x_{AB})_s}{(F_{As} + F_{Bs})^2}\right] = 0.039\,\frac{\%\text{A}}{\%\text{ opening}} \tag{7.14}$$

$$\tau = \frac{V}{F_{Bs} + F_{As}} = 5.0 \text{ min} \tag{7.15}$$

It can be seen that the gain and all time constants are functions of the volumes and total flow. These expressions give an indication, which will be used in later chapters, of how the dynamic response changes as a result of changes in operating conditions.

The closed-loop block diagram also includes the disturbance transfer function $G_d(s)$: the effect of the disturbance if there were no control. This can be derived by assuming that the flows are all constant and that the important input variable that changes is $(x_A)_B$. The resulting model is

$$x'_{A0} = \left[\frac{F_B}{F_A + F_B}\right]x'_{AB} \approx x'_{AB} \tag{7.16}$$

This equation can be combined with equation (7.12) to give the disturbance transfer function,

Disturbance: $$\frac{x_{A3}(s)}{x_{AB}(s)} = G_d(s) = \frac{K_d}{(\tau s + 1)^3} \approx \frac{1.0}{(\tau s + 1)^3} \tag{7.17}$$

Notice that two models have been developed for the same physical system, and they both relate an external input variable to the dependent output variable. The

model $G_p(s)$ relates the *manipulated* valve to the concentration in the third tank. This provides the dynamic response for the feedback control system; as we shall see, favorable performance requires a large gain magnitude and fast dynamics. The model $G_d(s)$ relates the inlet concentration *disturbance* to the concentration in the third tank. This provides the disturbance response without control; favorable performance requires a small gain magnitude and slow dynamics. The reader should recognize and understand the difference between the two models.

The selection of the controlled variable is summarized in the following analysis.

Control objective	Process variable	Sensor
1. Safety		
2. Environmental protection		
3. Equipment protection		
4. Smooth plant operation and production rate		
5. Product quality ⟶	Concentration of reactant ⟶ A in the third tank	Analyzer in reactor effluent measuring the mole % A
6. Profit optimization		
7. Monitoring and diagnosis		

The reader will notice that the concentration of A in the upstream tanks has a direct influence on the third tank and might wonder if measuring concentration in these tanks might be useful. Feedback does not require other measurements, but additional measurements can improve the dynamic behavior, as explained in Chapters 14 (cascade) and 15 (feedforward).

The selection of the manipulated variable is straightforward, because only one valve exists. However, the analysis is presented here to complete the example for the reader.

Input variables that affect the measured variable	Selected adjustable flow	Manipulated valve
Disturbances: Solvent flow rate Feed composition, $(x_A)_B$ Composition of "pure A" stream		
Adjustable: Flow of pure A ⟶	Flow of pure A ⟶	v_A

The selection criteria presented in Section 7.3 are reviewed in the following steps.

1. Causal relationship (required). Yes, because $\Delta X_{A3}/\Delta v_A = K_p = 0.039 \neq 0$.
2. Valve to influence the selected flow (required). Yes, because a valve exists in the pure A pipe.
3. Fast speed of response (desired). We cannot evaluate this with the methods presented to this point in the book, but we will be evaluating this factor in Chapters 9 to 13.
4. Ability to compensate for large disturbances (desired). Yes, the reader can confirm that the exit concentration of 3 percent can be achieved for solvent flow rates of 0–13.8 m^3/min. If the solvent flow is larger, the valve will be 100 percent open and the effluent concentration will decrease below 3 percent.
5. Ability to adjust the manipulated variable rapidly and with little upset to the remainder for the plant (desired). Further information is required to evaluate this factor. We will assume that the pure A is taken from a large storage tank, so that changes in the flow of A do not disturb other parts of the plant.

> Because the three-tank mixing process is used in many examples in the remainder of the book, readers are *strongly encouraged* to fully understand the modelling and variable selection in Example 7.2.
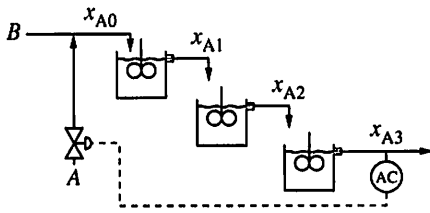
EXAMPLE 7.3.

Assume that the feedback control has been implemented on the mixing tanks problem with the goal of maintaining the outlet concentration near 3.0 percent. As an example of the control performance measures, the previous example is controlled using feedback principles. The disturbance was a step change in the feed concentration, x_{AB}, of magnitude +1.0 at time = 20. A feedback control algorithm explained in the next chapter was applied to this process with two different sets of adjustable parameters in Cases A and B, and the resulting control performance is shown in Figure 7.10a and b and summarized as follows.

Measure	Case A	Case B
Offset from SP	None	None
IAE	7.9	30.5
ISE	2.1	12.8
IE	−6.9	−30.5
CV maximum deviation	0.42	0.66
Decay ratio	<.1	(Overdamped)
Period (min)	37	(Overdamped)
MV maximum overshoot	6.9/25 = 28%	0% (expressed as % of steady-state change)

The controlled variable in Case A returns to its desired value relatively quickly, as indicated by the performance measures based on the error. This response requires a more "aggressive" (i.e., faster) adjustment of the manipulated variable.
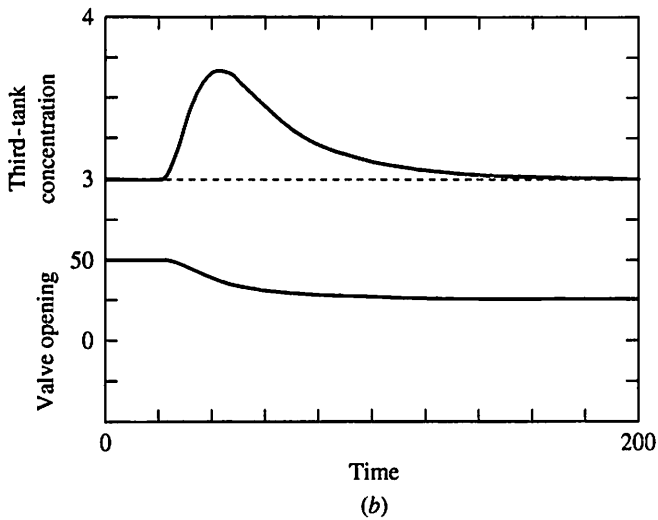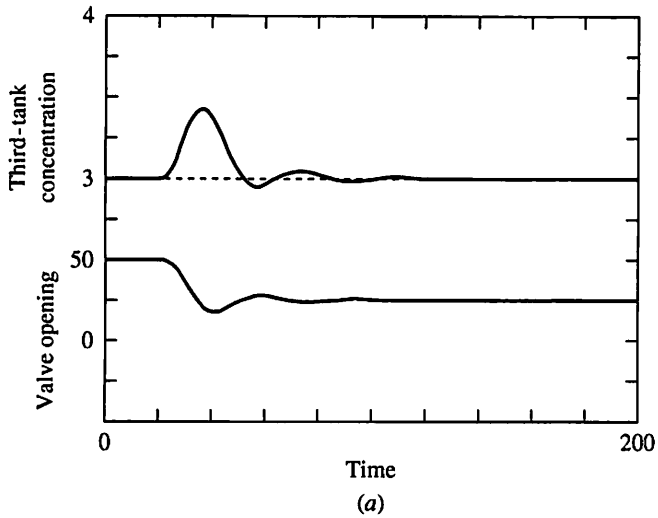
FIGURE 7.10

Feedback responses for Example 7.3. (*a*) **Case A;**
(*b*) **Case B.**

The general trend in feedback control is to require fast adjustments in the manipulated variable to achieve rapid return to the desired value of the controlled variable. One might be tempted to generally conclude that Case A provides better control performance, but there are instances in which Case B would be preferred. The final evaluation requires a more complete statement of control objectives.

Two important conclusions can be made based on Example 7.3.

1. The desired control performance must be matched to the process requirements.
2. Both the controlled and manipulated variables must be monitored in order to evaluate the performance of a control system.

7.5 ◾ APPROACHES TO PROCESS CONTROL

There could be many approaches to the control of industrial processes. In this section, five approaches are discussed so that the more common procedures are placed in perspective.

No Control

Naturally, the easiest approach is to do nothing other than to hold all input variables close to their design values. As we have seen, disturbances could result in large, sustained deviations in important process variables. This approach could have serious effects on safety, product quality, and profit and is not generally acceptable for important variables. However, a degrees-of-freedom analysis usually demonstrates that only a limited number of variables can be controlled simultaneously, because of the small number of available manipulated variables. Therefore, the engineer must select the most important variables to be controlled.

Manual Operation

When corrective action is taken periodically by operating personnel, the approach is usually termed *manual* (or *open-loop*) operation. In manual operation, the measured values of process variables are displayed to the operator, who has the ability to manipulate the final control element (valve) by making an adjustment in the control room to a signal that is transmitted to a valve, or, in a physically small plant, by adjusting the valve position by hand.

This approach is not always bad or "low-technology," so we should understand when and why to use it. A typical strategy used for manual operation can be related to the basic principles of statistical process control and can best be described with reference to the data shown in Figure 7.11. Along with the measured process variable, its desired value and upper and lower action values are plotted. The person observes the data and takes action only "when needed." Usually, the decision on when to take corrective action depends on the deviation from the desired value. If the process variable remains within an acceptable range of values defined by action limits, the person makes no adjustment, and if the process variable exceeds the action limits, the person takes corrective action. A slight alteration to this strategy could consider the consecutive time spent above (or below) the desired value but within the action limits. If the time continuously above is too long, a small corrective action can be taken to move the mean of the process variable nearer to the desired value.

This manual approach to process control depends on the person; therefore, the correct application of the approach is tied to the strengths and weaknesses of the human versus the computer. General criteria are presented in Table 7.3. They indicate that the manual approach is favored when the collection of key information is not automated and has a large amount of noise and when slow adjustments with "fuzzy," qualitative decisions are required. The automated approach is favored when rapid, frequent corrections using straightforward criteria are required. Also, the manual approach is favored when there is a substantial cost for the control effort; for example, if the process operation must be stopped or otherwise disrupted to effect the corrective action. In most control opportunities in the process industries, the corrective action, such as changing a valve opening or a motor speed, can be effected continuously and smoothly without disrupting the process.
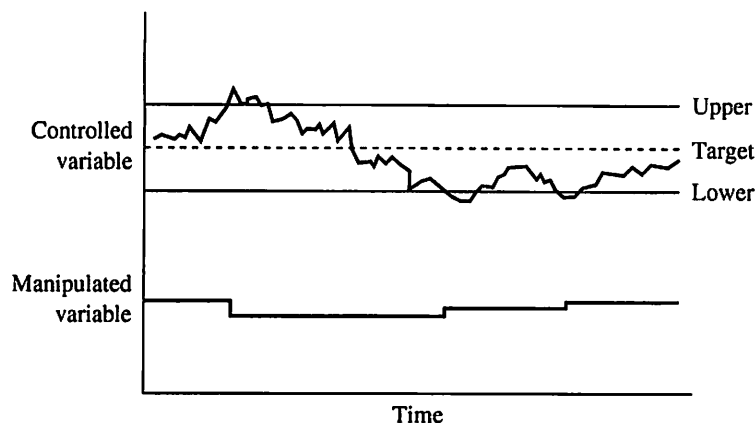
FIGURE 7.11

Transient response of a process under manual control to stochastic disturbances.

TABLE 7.3

Features of manual and automatic control

Control approach	Advantages	Disadvantages
Manual operation	Reduces frequency of control corrections, which is important when control actions are costly or disruptive to plant operation	Performance of controlled variables is usually far from the best possible
	Possible when control action requires information not available to the computer	Applicable only to slow processes
	Draws attention to causes of deviations, which can then be eliminated by changes in equipment or plant operation Keeps personnel's attention on plant operation	Personnel have difficulty maintaining concentration on many variables
Automated control	Good control perfomance for fast processes	Compensates for disturbances but does not prevent future occurrences
	Can be applied uniformly to many variables in a plant Generally low cost	Does not deal well with qualitative decisions May not promote people's understanding of process operation

Manual operation should be seen as complementary to the automatic approaches emphasized in this book. Statistical methods for monitoring, diagnosing, and continually improving process operation find wide application in the process

industries (MacGregor, 1988; Oakland, 1986), and they are discussed further in Chapter 26.

On-Off Control

The simplest form of automated control involves logic for the control calculations. In this approach, trigger values are established, and the control manipulation changes state when the trigger value is reached. Usually the state change is between on and off, but it could be high or low values of the manipulated variable. This approach is demonstrated in Figure 7.12 and was modelled for the common example of on-off control in room temperature control via heating in Example 3.4. While appealing because of its simplicity, on/off control results in continuous cycling, and performance is generally unacceptable for the stringent requirements of many processes. It is used in simple strategies such as maintaining the temperature of storage tanks within rather wide limits.

Continuous Automated Control

The emphasis of this book is on process control that involves the continuous sensing of process variables and adjustment of manipulated variables based on control calculations. This approach offers the best control performance for most process situations and can be easily automated using computing equipment. The types of control performance achieved by continuous control are shown in Figure 7.10a and b. The control calculation used to achieve this performance is the topic of the subsequent chapters in Part III. Since the control actions are performed continuously, the manipulated variable is adjusted essentially continuously. As long as the adjustments are not too extreme, constant adjustments pose no problems to valves and their associated process equipment that have been designed for this application.

Emergency Controls

Continuous control performs well in maintaining the process near its set point. However, continuous control does not ensure that the controlled variable remains
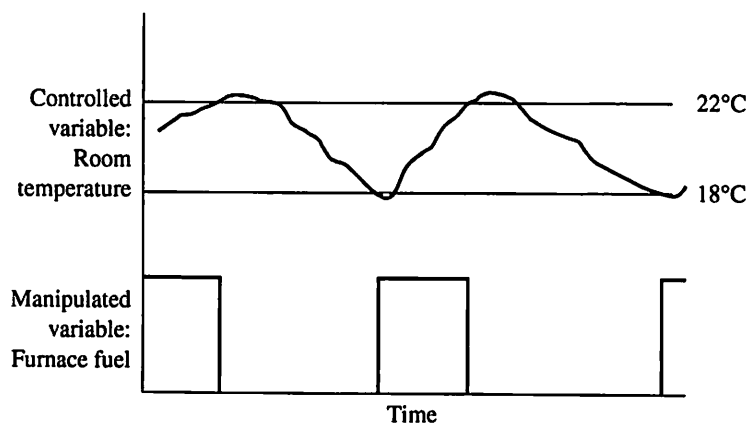

FIGURE 7.12

Example of a process under on/off control.

within acceptable limits. A large upset can result in large deviations from the set point, leading to process conditions that are hazardous to personnel and can cause damage to expensive equipment. For example, a vessel may experience too high a pressure and rupture, or a chemical reactor may have too high a temperature and explode. To prevent safety violations, an additional level of control is applied in industrial and laboratory systems. Typically, the emergency controls measure a key variable(s) and take extreme action before a violation occurs; this action could include stopping all or critical flow rates or dramatically increasing cooling duty.

As an example of an emergency response, when the pressure in a vessel with flows in and out reaches an upper limit, the flow of material into the vessel is stopped, and a large outflow valve is opened. The control calculations for emergency control are usually not complex, but the detailed design of features such as sensor and valve locations is crucial to safe plant design and operation. The topic of emergency control is addressed in Chapter 24. You may assume that emergency controls are not required for the process examples in this part of the book unless otherwise stated.

In industrial plants all five control approaches are used concurrently. Plant personnel continuously monitor plant performance, make periodic changes to achieve control of some variables that are not automated, and intervene when equipment or controls do not function well. Their attention is directed to potential problems by audio and visual alarms, which are initiated when a process measurement exceeds a high or low limiting value. Continuous controls are applied to regulate the values of important variables that can be measured in real time. The use of continuous controls enables one person to supervise the operation of a large plant section with many variables. The emergency controls are always in reserve, ready to take the extreme but necessary actions required when a plant approaches conditions that endanger people, environment, or equipment.

7.6 □ CONCLUSIONS

A review of the elements of a control loop and of typical dynamic responses of each element, with an example of transient calculation, shows that all elements in the loop contribute to the behavior of the controlled variable. Depending on the dynamic response of the process, the contributions of the instrument elements can be negligible or significant. Material in future chapters will clarify and quantify the relationship between dynamics and performance of the feedback system.

The principles and methods for selecting variables and measuring control performance discussed here for a single-loop system can be extended to processes with several controlled and manipulated variables, as will be shown in later chapters.

A key observation is that feedback control does not reduce variability in a plant, but it moves the variability from the controlled variables to the manipulated variables. The engineer's challenge is to provide adequate manipulated variables that satisfy degrees of freedom and that can be adjusted without significantly affecting plant performance.

The techniques used for continuous automated rather than manual control are emphasized because:

1. As demonstrated by its wide application, it is essential for achieving good operation for most plants.
2. It provides a sound basis for evaluating the effects of process design on the dynamic performance. A thorough understanding of feedback control performance provides the basis for designing more easily controlled processes by avoiding unfavorable dynamic responses.
3. It introduces fundamental topics in dynamics, feedback control, and stability that every engineer should master. The study of automatic control theory principles as applied to process systems provides a link for communication with other disciplines.

In this chapter the feedback controller has been left relatively loosely defined. This has allowed a general discussion of principles without undue regard for a specific approach. However, to build systems that function properly, the engineer will require greater attention to detail. Thus, the most widely used feedback control algorithm will be introduced in the next chapter.

REFERENCES

Cryor, J., *Time Series Analysis*, Duxbury Press, Boston, MA, 1986.

Hougen, J., *Measurements and Control—Applications for Practicing Engineers*, Cahners Books, Boston, MA, 1972.

MacGregor, J. M., "On-Line Statistical Process Control," *Chem. Engr. Prog. 84, 10*, 21–31 (1988).

Oakland, J., *Statistical Process Control*, Wiley, New York, 1986.

ADDITIONAL RESOURCES

Additional information on the dynamic responses of instrumentation can be found in

While, C., "Instrument Models for Process Simulation," *Trans. Inst. MC, 1, 4*, 187–194 (1979).

Additional references on the dynamic responses of pneumatic equipment can be found in

Harriott, P., *Process Control*, McGraw-Hill, New York, 1964, Chapter 10.

Instrumentation in the control loop performs many functions tailored to the specific process application. Therefore, it is difficult to discuss sensor systems in general terms. The reader is encouraged to refer to the instrumentation references provided at the end of Chapter 1.

The description of elements in the loop is currently accurate, but the situation is changing rapidly with the introduction of digital communication between the controller and the field instrumentation along with digital computation at the field equipment. For an introduction, see

Lindner, K., "Fieldbus—A Milestone in Field Instrumentation Technology," *Meas. and Cont., 23*, 272–277 (1990).

For a discussion of the interaction between the plant personnel and the automation equipment, see

Rijnsdorp, J., *Integrated Process Control and Automation,* Elsevier, Amsterdam, 1991.

> Many important decisions can be made based on the understanding of feedback control, without consideration of the control calculation. These questions give some practice in thinking about the essential aspects of feedback.

QUESTIONS

7.1. Consider the CSTR in Figure Q7.1. No product is present in the feed stream, a single chemical reaction occurs in the reactor, and the heat of reaction is zero. Determine whether each of the following single-loop control designs is possible. [Hint: Does a causal process relationship exist?] Consider each question separately.

(a) Control the product concentration in the reactor by adjusting the valve in the pure A pipe.

(b) Control the product concentration in the reactor by adjusting the valve in the coolant flow pipe.

(c) Control the product concentration in the reactor by adjusting the valve in the solvent pipe.

(d) Control the temperature in the reactor by adjusting the valve in the pure A pipe.

(e) Control the temperature in the reactor by adjusting the valve in the coolant flow pipe.

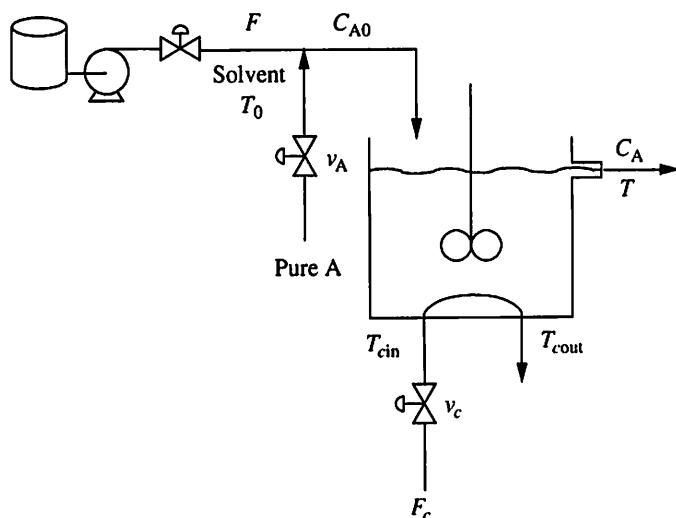(f) Control the temperature in the reactor by adjusting the valve in the solvent pipe.

FIGURE Q7.1

CSTR process

7.2. Elements in a control loop in Figure 1.7d are given in Table Q7.2 with their individual dynamics. The output signal is 0 to 100%, and the displayed controlled variable is 0 to 20 weight %. Determine the response of the indicator (or controller input) to a step change in the output signal from the manual station (or controller output).

(a) The time unit in the models is not specified. Using engineering judgment, what units would expect to be correct: seconds, minutes, or hours?

(b) First estimate the response, $t_{63\%}$, using an approximate method.

(c) Give an estimate for how much the sensor, transmission, and valve dynamics affect the overall response.

(d) Determine the response by solving the entire system numerically.

TABLE Q7.2

Dynamic models

Element	Units	Case A	Case B
Manual station	psi/% output	0.083	0.083
Transmission		$1.0/(1.3s + 1)$	1.0
Signal conversion	psi/mA	$0.75/(0.5s + 1)$	$0.75/(0.5s + 1)$
Final element	%open/psi	$8.33/(1.5s + 1)$	$8.33/(1.5s + 1)$
Process	m³/psi	$0.50e^{-0.5s}/(30s + 1)$	$0.50e^{-20s}/(30s + 1)$
Sensor		$1.0/(1s + 1)$	$1.0/(10s + 1)$
Signal conversion	mA/mV	—	—
Transmission		1.0	1.0
Display	wt%/mA	$1.25/(1.0s + 1)$	$1.25/(1.0s + 1)$

7.3. For the series reactors in Figure Q7.3, the outlet concentration is controlled at 0.414 mole/m³ by adjusting the inlet concentration. At the initial base case operation, the valve is 50% open, giving $C_{A0} = 0.925$ mole/m³. One first-order reaction $A \rightarrow B$ occurs; the data are $V = 1.05$ m³, $F = 0.085$ m³/min, and $k = 0.040$ min⁻¹. The process transfer function is derived in Example 4.2 as $C_{A2}(s)/C_{A0}(s) = 0.447/(8.25s + 1)^2$; the additional model relates the valve to inlet concentration, which for a linear valve and small flow of A ($F \gg F_A$) gives $C_{A0}(s)/v(s) = 0.925/50 = 0.0185$ (mole/m³)/% open; you may assume for this question that the sensor dynamics are negligible. Answer the following questions about the operating window of the process:

(a) Can the desired value of $C_{A2} = 0.414$ mole/m³ be achieved if the solvent flow changes from its base value of 0.085 m³/min to 0.12 m³/min?

(b) Can the desired value of $C_{A2} = 0.414$ mole/m³ be achieved if the concentration of A in the solvent changes from its base value of 0.0 to 1.0 mole/m³?

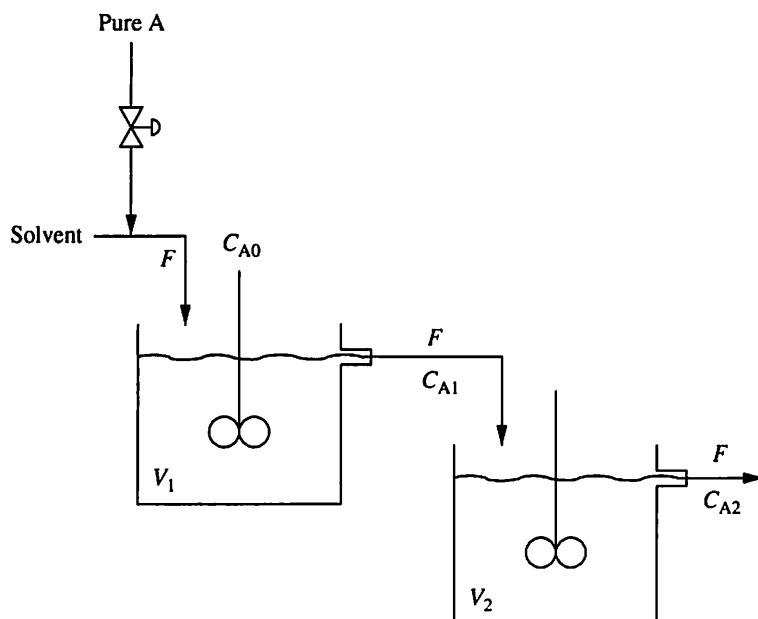(c) Can the outlet concentration of A be increased to 0.828 mole/m³?

FIGURE Q7.3

7.4. (a) Discuss the three types of disturbances described in this chapter and give a process example of how each could be generated by an upstream process.

(b) An alternative disturbance is a pulse function. Describe a pulse function, give control performance measures for a pulse disturbance, and give a process example of how it could be generated by an upstream process.

7.5. Dynamic responses for several different control systems in response to a change in the set point are given in Figure Q7.5. Discuss the control performance of each with respect to the measures explained in Section 7.4. (Note that the control performance cannot be evaluated exactly without a better definition of control objectives. Further exercises will be given in later chapters, when the objectives can be more precisely defined.)

7.6. A process with controls is shown in Figure Q7.6. The objective is to achieve a desired composition of B in the reactor effluent. The process consists of a feed tank of reactant A, which is maintained within a range of temperatures and is fed into the reactor, where the following reactions take place.

$$A \rightarrow B$$
$$A \rightarrow C$$
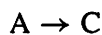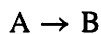
If the reactor level is too high, the pump motor should be shut off to prevent spilling the reactor contents. Identify at least one variable that is controlled by each of the five approaches to control presented in this chapter. Discuss why the approach is (or is not) a good choice.

7.7. Note that the electrical and pneumatic transmission ranges have a *nonzero* value for the lowest value of the range. Why is this a good selection for the range; that is, what is the advantage of this range selection?
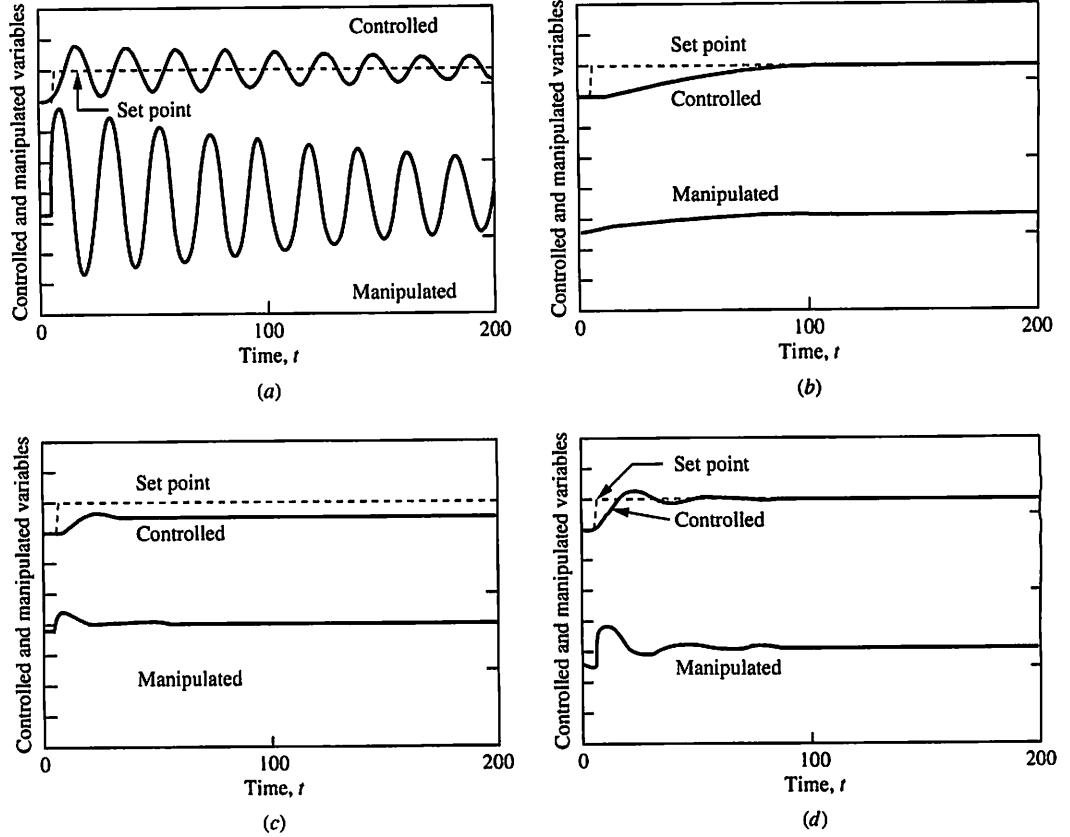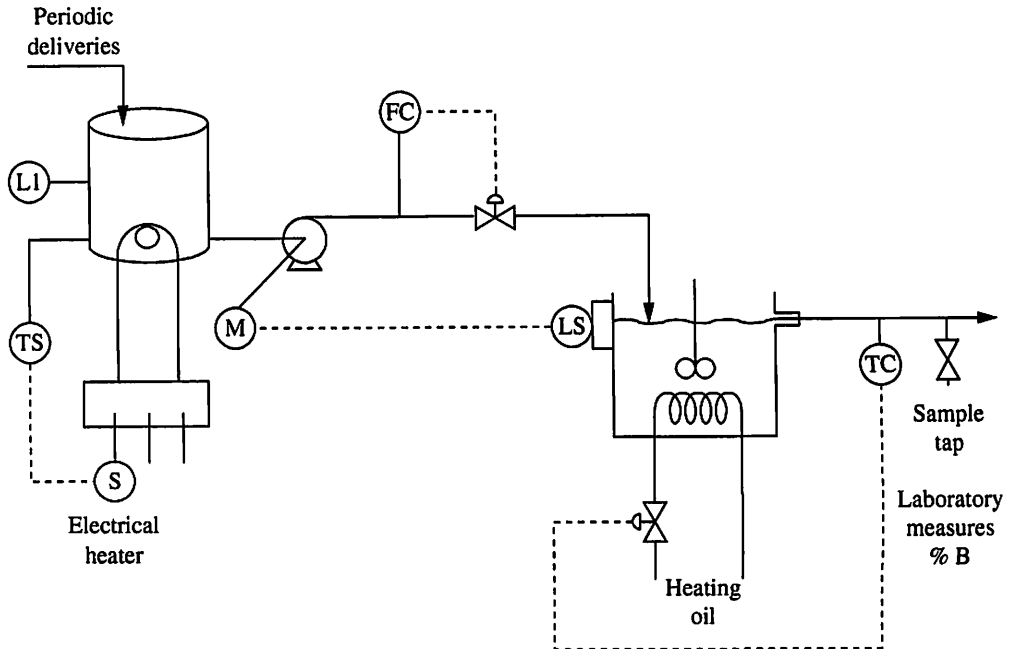
FIGURE Q7.5

FIGURE Q7.6

Schematic drawing of process and control design.

7.8. Confirm that the gains in the instrument models used in Example 7.1 are reasonable. The sensor is an iron-constantan thermocouple.

7.9. The proposal was made to select the control pairing for one single-loop controller for the nonisothermal CSTR in Section 3.6 and Figure 3.17. Evaluate each using the criteria in Section 7.3.
   (*a*) Control the reactor temperature by adjusting the coolant flow rate.
   (*b*) Control the reactant concentration in the reactor by adjusting the coolant flow rate.
   (*c*) Control the coolant outlet temperature by adjusting the coolant flow rate.

7.10. The proposal was made to make one of the control pairings for the binary distillation tower in Example 5.4. Evaluate each using the criteria in Section 7.3.
   (*a*) Control the distillate composition by adjusting the reboiler heating flow.
   (*b*) Control the distillate composition by adjusting the distillate flow.
   (*c*) Control both the distillate and bottoms compositions simultaneously by adjusting the reboiler heating flow.

7.11. Answer the following questions, which address the range of a control system.
   (*a*) The process in Example I.1 (in Appendix I) is to control the process temperature after the mix by adjusting the flow ratio. Over what range of inlet temperatures T_0 can the outlet temperature T_3 be maintained at 90°C?
   (*b*) The nonisothermal CSTR in Section C.2 (in Appendix C) is to be operated at 420 K and 0.20 kmole/m^3. Can this condition be achieved for the range of inlet concentration (C_{A0}) of 1.0 to 2.0 mole/m^3 and coolant flow rate (F_c) of 0 to 16 m^3/min? If not, which range(s) has to be expanded and by how much?
   (*c*) For the CSTR in Example 3.3, can the outlet concentration of reactant be controlled at 0.85 mole/m^3 by adjusting the inlet concentration? By adjusting the temperature of one reactor?

7.12. Answer the following questions on selecting control variables. Are there any limitations to the operating conditions for your answers?
   (*a*) In Example I.2 (in Appendix I), can the outlet concentration be controlled by adjusting the solvent flow rate?
   (*b*) How many valves influence the liquid level in the flash drum in Figure 1.8? Which of these valves would you recommend for use in feedback control?
   (*c*) In Figure 2.6, through adjustments of the air flow rate, can (i) the efficiency *and* (ii) the excess oxygen in the flue gas be controlled?

7.13. Evaluate the control design in Figure Q7.6.
   (*a*) Prepare a table for the selection of measured controlled variables based on the seven control objectives using the format presented in Section

7.3. Do you find measured control variables in Figure Q7.6 to be correctly selected?

(b) Prepare a table for the selection of a control valve (final element) to be connected to each controlled variable using the format presented in Section 7.3. Do you find the connections in Figure Q7.6 to be correctly selected?

7.14. For the process shown in Figure 1.8,

(a) Prepare a table for the selection of measured controlled variables based on the seven control objectives using the format presented in Section 7.3.

(b) Prepare a table for the selection of a control valve (final element) to be connected to each controlled variable using the format presented in Section 7.3.

(Note: This is a challenging exercise, but it will help you to understand the manner that many single-loop controllers can be used to control a complex process. Do the best you can at this point; multiple-loop systems are addressed in detail later in the book.)

7.15. Sketch the operating window for the three-tank mixing process. The variables on the axes, which define the operating window, are (1) the outlet concentration (defining the range of achievable desired product) and (2) the concentration of A in the feed B, $(x_A)_B$ (defining the range of disturbances that can be compensated by adjusting the valve). Discuss the shape of the window; is it rectangular?