Integrating Factor

APPENDIX

B

A single energy or material balance on a well-mixed system results in a first-order ordinary differential equation. Since this equation is often linearized in dynamic analysis, a linear first-order differential equation results. The differential equation is useful because it provides analytical relationships between the process equipment and operating parameters and key dynamic parameters such as time constants and gains. Often, an analytical solution is desired for the (open-loop) system output in response to one or more relatively simple input forcing functions. The integrating factor can be used to evaluate the analytical solution.

The general linearized model will be of the form

$$a(t)\frac{dY}{dt} + b(t)Y = c(t) \tag{B.1}$$

The functions $a(t)$, $b(t)$, and $c(t)$ are known functions of time, t. When the function $a(t) \neq 0$ during the time considered in the solution, equation (B.1) can be rearranged to give

$$\frac{dY}{dt} + f(t)Y = g(t) \tag{B.2}$$

with $g(t)$ the forcing function. This ordinary differential equation is linear and first-order but not separable. However, it can be modified to be separable, and directly solvable, by multiplying by a term called the *integrating factor*, IF.