Approximate Dynamic Models

APPENDIX

D

D.1 ■ METHOD OF MOMENTS

Real processes have complex dynamic responses and require models with many parameters to be characterized accurately. However, the engineer often seeks a simple model with few parameters to describe the main aspects of the dynamic behavior. Examples throughout this book demonstrate that the first-order-with-dead-time model is adequate for the process control analysis of many, but not all, processes. In this section a method is developed for determining a few parameters that can be used to fit a model to the expected dynamic behavior; this is the *method of moments*. The application of the method of moments described in this appendix was demonstrated by Paynter and Takahashi (1956) and Gibilaro and Lee (1969).

The basic approach is to evaluate several *moments* of the output behavior and use these to characterize the dynamic behavior. Thus, the first step is to define a moment.

The nth moment of a variable $Y(t)$ is

$$M_n = \frac{\int_0^\infty t^n Y(t)\, dt}{\int_0^\infty Y(t)\, dt} \tag{D.1}$$

Further moments are usually defined with respect to the first moment, which is the *mean*; thus, the moments of the variable $Y(t)$ about its mean are