1^{st} Order, Linear $\rightarrow$ $C_A' = K_p \Delta C_{A0}(1 - e^{-t/\tau})$

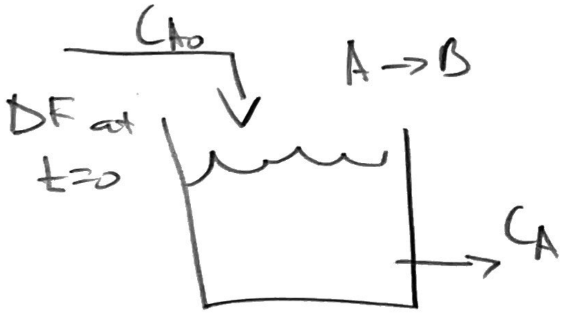
2^{nd} Order Linear $\rightarrow$ $C_A' = K_p' \Delta C_{A0}(1 - e^{-t/\tau}) - K_p' \Delta C_{A0} \, t/\tau \, e^{-t/\tau}$

$3^{rd}, 4^{th}, 5^{th}$ order $+ \, t^2 e^{-t/\tau}, \; t^3 e^{-t/\tau} \ldots$

Non linear $\rightarrow$ in many cases, can linearize to a 1^{st} order linear. (e.g. non linear rate).

- What if we have multiple disturbances?
- what if we cannot easily solve using integrating factor method?
- Is there a better way to solve and analyze dynamic systems? (yes!)

Reactor with disturbance in F



Balance:

$$V \frac{dC_A}{dt} = F C_{A0} - \underbrace{F C_A}_{\text{non-linear need to linearize}} - V K_1 C_A$$

FC_A is non-linear: Need to linearize.

$$f(F, C_A) \approx f(F_s, C_{As}) + \left.\frac{\partial f}{\partial F}\right|_{F_s, C_s} (F - F_s) + \left.\frac{\partial f}{\partial C_A}\right|_{F_s, C_s} (C_A - C_{As})$$

$$FC_A \approx F_s C_{As} + C_{As}(F - F_s) + F_s(C_A - C_{As})$$

Plug into balance:

$$V\frac{dC_A}{dt} = FC_{A0} - F_s C_{As} - C_{As}(F - F_s) - F_s(C_A - C_{As}) - Vk_1 C_A$$

Write in terms of deviation variables ΔF, C_A'

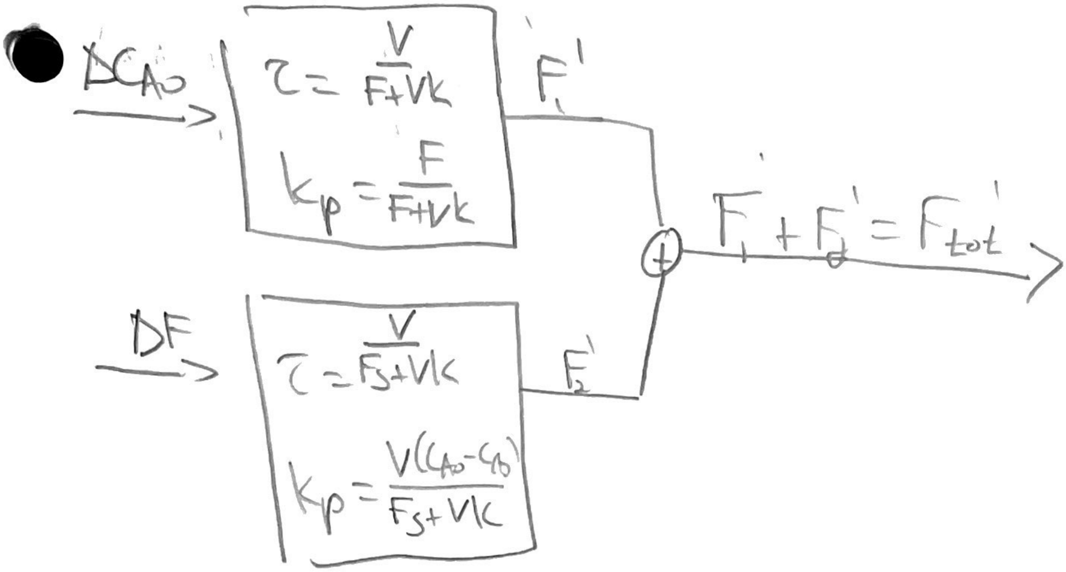
$$V\frac{dC_A'}{dt} = \Delta F C_{A0} - C_{As}\Delta F - F_s C_A' - Vk_1 C_A'$$

$$\frac{dC_A'}{dt} + \underbrace{\frac{F_s + Vk_1}{V}}_{1/\tau} C_A' = \frac{1}{V}\underbrace{(C_{A0} - C_{As})}_{K_p/\tau}\Delta F$$

$$C_A' = K_p \Delta F\left(1 - e^{-t/\tau}\right), \quad \tau = \frac{V}{F_s + Vk_1}$$

$$K_p = \frac{C_{A0} - C_{As}}{F_s + Vk_1}$$

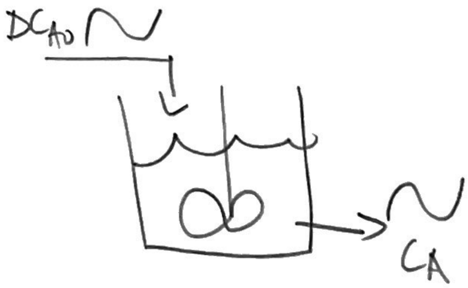
What if there are changes in both ΔC_{A0} and ΔF?



add together deviations due to different disturbances

Problem 3.5:

What if ΔC_{A0} is not a step function. $\Delta C_{A0} = A\sin(\omega t)$.



Mixing tank:

$$V\frac{dC_A}{dt} = F(C_{A0}-C_A)$$

$$\frac{dC_A'}{dt} + \frac{1}{\tau}C_A' = \frac{1}{\tau}\Delta C_{A0} = \frac{1}{\tau}A\sin(\omega t)$$

Solve as before.

$$\frac{d}{dt}\left(e^{t/\tau}C_A'\right) = \frac{1}{\tau}A\sin(\omega t)e^{t/\tau}$$

$$e^{t/\tau}C_A' = \frac{A}{\tau}\int e^{t/\tau}\sin(\omega t)\,dt + I$$

$$e^{t/\tau} C_A' = \frac{A}{\tau}\left[\frac{\frac{1}{\tau}\sin\omega t - \omega\cos\omega t}{1/\tau^2 + \omega^2}\right] e^{t/\tau} + I$$

$$C_A' = \frac{A}{\tau}\left[\frac{\tau \sin\omega t - \omega\tau^2\cos\omega t}{1+\omega^2\tau^2}\right] + I e^{-t/\tau}$$

Set $C_A' = 0$ at $t=0$ gives $I = \dfrac{A\omega\tau}{1+\omega^2\tau^2}$

$$C_A' = \frac{A}{1+\omega^2\tau^2}\left(\sin\omega t - \omega\tau\cos\omega t\right] + \frac{A\omega\tau}{1+\omega^2\tau^2} e^{-t/\tau}$$

$$AR = \frac{1}{\sqrt{1+\omega^2\tau^2}}$$

another example.

$$\frac{dC_A}{dt} + \left(\frac{1}{t}+k\right)C_A = \frac{1}{t} C_{A0}$$

not a constant

use integrating factor $\exp\left(\int\left(\frac{1}{t}+k\right)dt\right) = \exp(\ln t + kt)$

$$= t\exp(kt)$$

$$\frac{d}{dt}\left(C_A t\exp(kt)\right) = \frac{t\exp(kt)}{t} C_{A0}$$

Numerical Solutions

In some cases, it is not easy to linearize an equation, or perhaps you want a plot of the non-linear result. It is easy to calculate a numerical solution.

1) Derive the conservation equation

$$V\frac{dC_A}{dt} = F(C_{A0} - C_A) - VkC_A$$

2) Find the initial steady state values

Set $\frac{dC_A}{dt} = 0$, $C_{A0} = C_{A0,init}$

$$0 = F(C_{A0,init} - C_A) - VkC_A$$

$$C_A = \underbrace{\frac{F}{F+Vk}}_{K_p} C_{A0,init}$$

3) Define a time step, $t = 0.001\,s$

4) Calculate $\frac{dC_A}{dt}$ at $t=0$

$$\frac{dC_A}{dt} = \frac{F}{V}(C_{A0} - C_{A(t=0)}) - VkC_{A(t=0)}$$

5) Find ΔC_A

$$\Delta C_A = \left.\frac{dC_A}{dt}\right|_{t=0} \cdot \Delta t$$

6) This gives $C_A(t=0.001)$ Repeat Process