Finance 5350, Fall 2018
Project 1 Description

This example is taken from Chapter 4 of the book *Implementing Derivatives Models* by Clewlow and Strickland. In this project you will price a European arithmetic Asian call option. This option pays the difference (if positive) between the arithmetic average of the asset price A_T and the strike price K at the maturity date T. The arithmetic average is taken on a set of observations (called fixings) of the asset price S_{t_i} (which we assume follows geometric Brownian motion) at dates $t_i; i = 1 \ldots N$.

$$A_T = \frac{1}{N}\sum_{i=1}^{N} S_{t_i}$$

There is no analytical solution for the price of an arithmetic Asian option; however, there is a simple analytical formula for the price of a geometric Asian option. A geometric Asian call option pays the difference (again, only if positive) between the geometric average of the asset price G_T and the strike price K at the maturity date T. The geometric average is defined as

$$G_T = \left(\prod_{i=1}^{N} S_{t_i}\right)^{1/N}$$

Since the geometric average is essentially the product of lognormally distributed variables it is also lognormally distributed. Therefore the price of the geometric Asian call option is given by a modified Black–Scholes formula:

$$C_{GA} = exp(-rT)\left(exp(a + \frac{1}{2}b)N(x) - KN(x - \sqrt{b})\right)$$

where

$$
\begin{aligned}
a &= \ln(G_t) + \frac{N-m}{N}\left(\ln(S) + \upsilon(t_{m+1}-t) + \frac{1}{2}\upsilon(T-t_{m+1})\right) \\
b &= \frac{(N-m)^2}{N^2}\sigma^2(t_{m+1}-t) + \frac{\sigma^2(T-t_{m+1})}{6N^2}(N-m)(2(N-m)-1) \\
\upsilon &= r - \delta - \frac{1}{2}\sigma^2 \\
x &= \frac{a - \ln(K) + b}{\sqrt{b}}
\end{aligned}
$$

where G_t is the current geometric average and m is the last known fixing. The geometric Asian option makes a good static hedge style control variate for the arithmetic Asian option. To implement the Monte Carlo method we simulate the difference between the arithmetic and geometric Asian options or a hedged portfolio which is long one arithmetic Asian and short one geometric Asian option. This is much faster than using the delta of the geometric Asian option to generate a delta hedge control variate because we do not have to compute the delta at every time step and it is equivalent to a continuous delta hedge.

For this project we will price a 1–year maturity, European Asian call option with a strike price of \$100, a current asset price at \$100 and a volatility of 20%. The continuously compounded interest rate is assumed to be 6% per annum, the asset pays a continuous dividend yield of 3% per annum, and their are 10 equally spaced fixing dates. The simulation has 10 time steps and 10,000 simulations; $K = \$100$, $T = 1$ year, $S = \$100$, $\sigma = 0.2$, $r = 0.06$, $\delta = 0.03$, $N = 10$, $M = 10000$.

The deliverable for this project, in addition to the project source code, is a table that presents the results of the simulation. In the table present the price, standard error, and relative computation time for the following:

- Simple Monte Carlo
- Control Variate Monte Carlo (using the Geometric Asian formula as control variate)