Unit 1: Sequences and Series Review

Name ____________________

Date ____________________

Potentially useful formulas: $\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$ $\qquad$ $\sum_{k=1}^{n} k^3 = \frac{n^2(n+1)^2}{4}$

1. Short answer

a. If for some arithmetic sequence $A_3 = 7$ and $A_4 = 4$, find A_{11}.

b. Again, if for some arithmetic sequence $A_3 = 7$ and $A_4 = 4$, evaluate $\sum_{k=0}^{25} A_k$

c. Write the following sum in sigma notation: $\frac{2}{7} - \frac{4}{8} + \frac{6}{9} - \frac{8}{10} + \quad \ldots \quad + \frac{22}{17}$

d. Find the term at position 0 of a geometric sequence whose 2^{nd} term is 3 and whose 6^{th} term is 75.

e. Find a rule that defines the sequence below. **The rule must be an equation, but it may be in any form** – recursive, explicit or "mixed".

$\{-8, \quad -5, \quad 1, \quad 10, \quad 22, \quad \ldots\}$

f. Recall that the partial sums of some sequence *t* are given by $S_n = \sum_{k=1}^{n} t_k$. If the first few partial sums of some sequence are $S_1 = 5$, $S_2 = 20$, $S_3 = 30$, $S_4 = 36$, what is the value of t_3? If you can't find this value, explain why not.

2. Evaluate the following series:

a. $\sum_{k=3}^{17}\left(2(k+1)^2+3\left(\frac{4}{3}\right)^k\right)=$

b. $\left[\sum_{j=1}^{15}(j^5-7j)\right]-\left[\sum_{m=1}^{15}(m^5-9)\right]=$

c. $\sum_{k=1}^{3}\left(\sum_{m=k+1}^{k+4}(m+k)\right)$

3. **The Cantor Set.** The Cantor middle-thirds set is obtained by starting with the closed interval $[0,1]$ and deleting the open middle third interval $\left(\frac{1}{3},\frac{2}{3}\right)$. This produces two closed intervals, so we can then remove the middle third of each of these, and so on. Let C_0 be the original interval $[0,1]$, C_1 the set obtained after one iteration (consisting of two closed intervals), and so on. The first few iterations are shown below:

   a. How many segments does C_n consist of?

   b. What is the length of each segment in C_n?

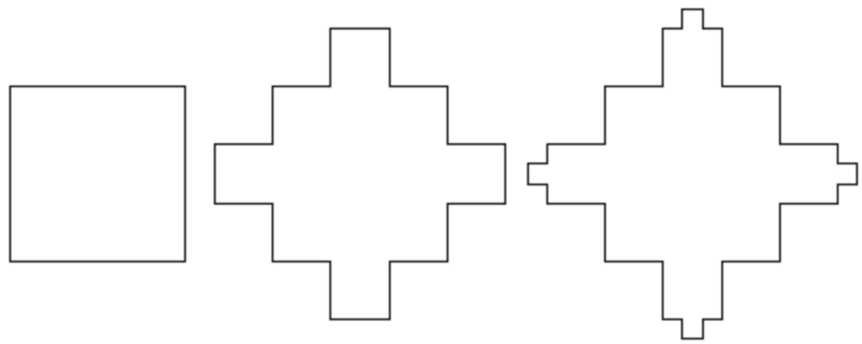
   c. What is the total length of C_n?

   d. Find the total length of the intervals <u>removed</u> to obtain C_n.

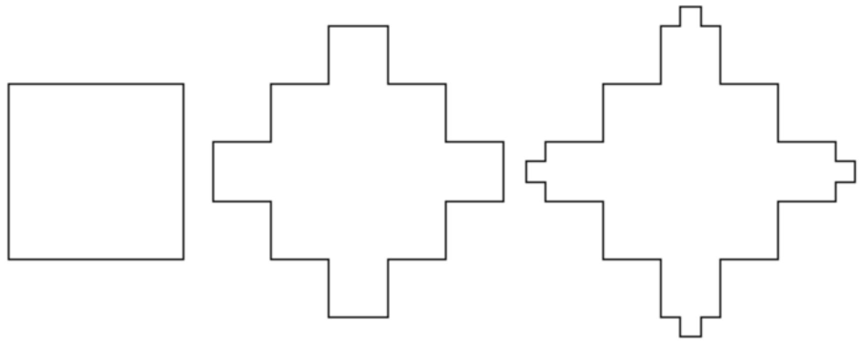
   e. If we iterate this process forever (in other words, let $n \to \infty$), will there be anything left? Discuss.

4. **More snowflakes**

The diagrams below show the first 3 stages of another "snowflake" that was inspired by the Koch snowflake. To make the stage $n + 1$ snowflake, you take the stage n snowflake and add a new square to the left-most, right-most, top-most and bottom-most sides. The new squares are centered on the old sides and have a side length that is 1/3 the length of the side they are added to. Assume the side length of the first snowflake ($n = 1$) is one unit.



a. If A_n denotes the area of the stage *n* snowflake, analyze $\lim_{n\to\infty} A_n$.
Be as precise as possible. If it diverges, how do you know? If it converges, what does it converge to? If you can't find a specific value, can you put upper and/or lower bounds on the value?



b. If P_n denotes the perimeter of the stage n snowflake, analyze $\lim_{n \to \infty} P_n$.
Be as precise as possible. If it diverges, how do you know? If it converges, what does it converge to? If you can't find a specific value, can you put upper and/or lower bounds on the value?

5. Evaluating Series

a. Evaluate the series $36 + 64 + 100 + 144 + \quad ... \quad + 10{,}000$

b. Evaluate $\sum_{j=2}^{\infty} \sum_{k=1}^{\infty} \left(\frac{1}{2}\right)^{j} \left(\frac{1}{3}\right)^{k}$. If you rewrite this as a different, equivalent sigma expression, you must justify your reformulation.