Normal_Distibution

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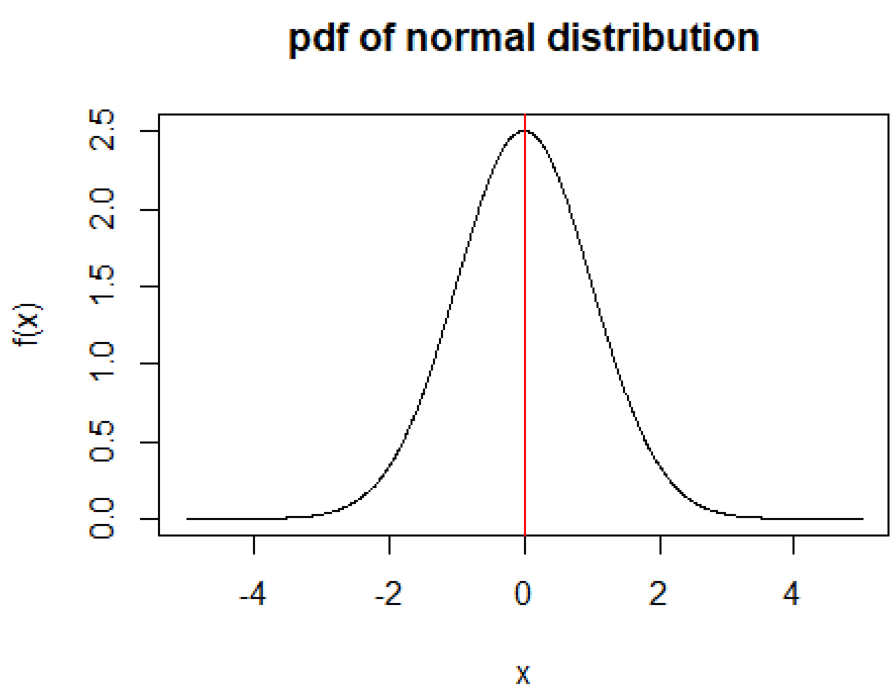
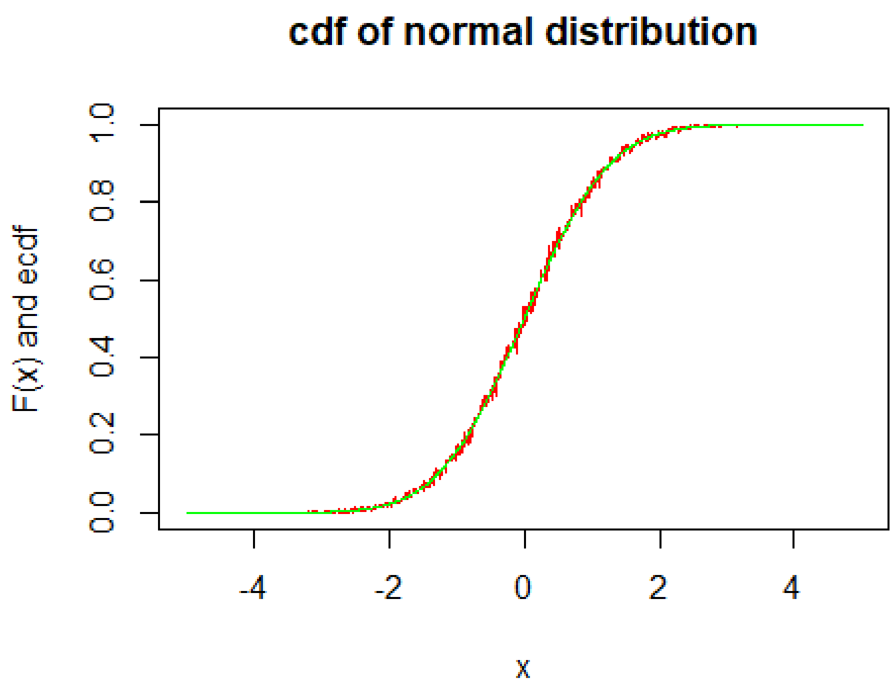
```
#### pdf of normal distribution
x=seq(-5,5,by=0.01)
sigma=1
mu=0   ###### mean
f=(1/sigma*sqrt(2*pi))*exp((-1/2)*((x-mu)/sigma)^2)
plot(x,f,type="l",xlab="x",ylab="f(x)",main="pdf of normal distribution")
abline(v=mu,col="red")
```




```
#### symmetric about mean

###### cdf of normal distribution using ecdf and the_cdf
x=seq(-5,5,by=0.01)
sigma=1
mu=0
n=1000
ecdf=c()
for(i in 1:length(x))
{
samp=rnorm(n)
```

```
ecdf[i]=length(which(samp<=x[i]))/n
}
the_cdf=pnorm(x)
plot(x,ecdf,xlab="x",ylab="F(x) and ecdf",type="l",main="cdf of normal
distribution",col="red")
lines(x,the_cdf,col="green")
```

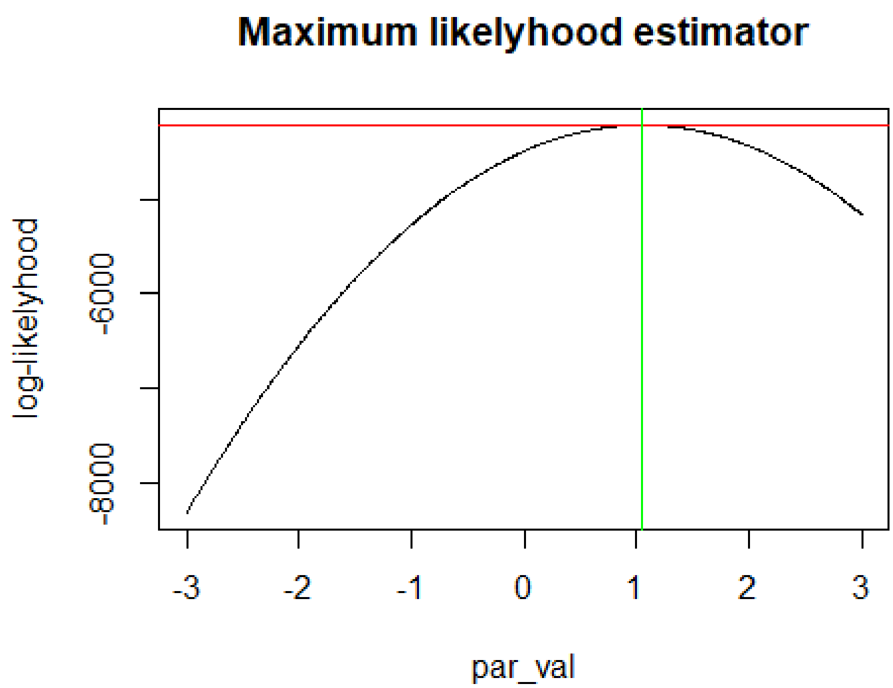



```
#legend(locator(1),legend=c("ecdf","the_cdf"),fill=c("red","green"))
```

Result : As sample size increases ecdf convergres to theorotical cdf

```
########## Obtain MLE for normal Disrtibution
######### Case:1 when mu is unknown and sigma square(variance) is known
mu=1
sigma=2
n=2000
samp=rnorm(n,mu,sigma)
tn=sum(samp)/n
print(tn)

## [1] 1.05167

par_val=seq(-3,3,by=0.01)
logl=c()
for(i in 1:length(par_val))
{
logl[i]=(n*log(1/(sigma*sqrt(2*pi))))-(1/2)*sum(((samp-par_val[i])/sigma)^2)
}
```

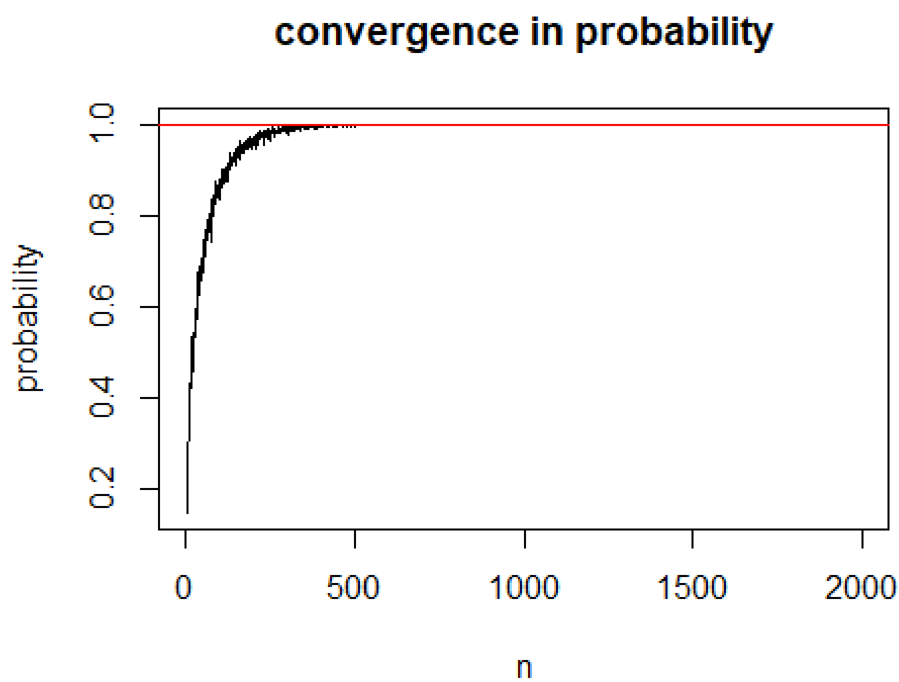
```r
m=max(logl)
print(m)

## [1] -4217.457

ind=which(logl==m)
mle=par_val[ind]
print(mle)

## [1] 1.05

plot(par_val,logl,type="l",xlab="par_val",ylab="log-likelyhood",main="Maximum
likelyhood estimator")
abline(h=m,col="red")
abline(v=mle,col="green")
```



Result : The value of parameter which maximise the loglikelyhood is 1

```r
############### consistency property
mu=1
n=2000
elp=0.3
sigma=2
it=500
prob=c()
for(i in 1:n)
{
est=c()
```

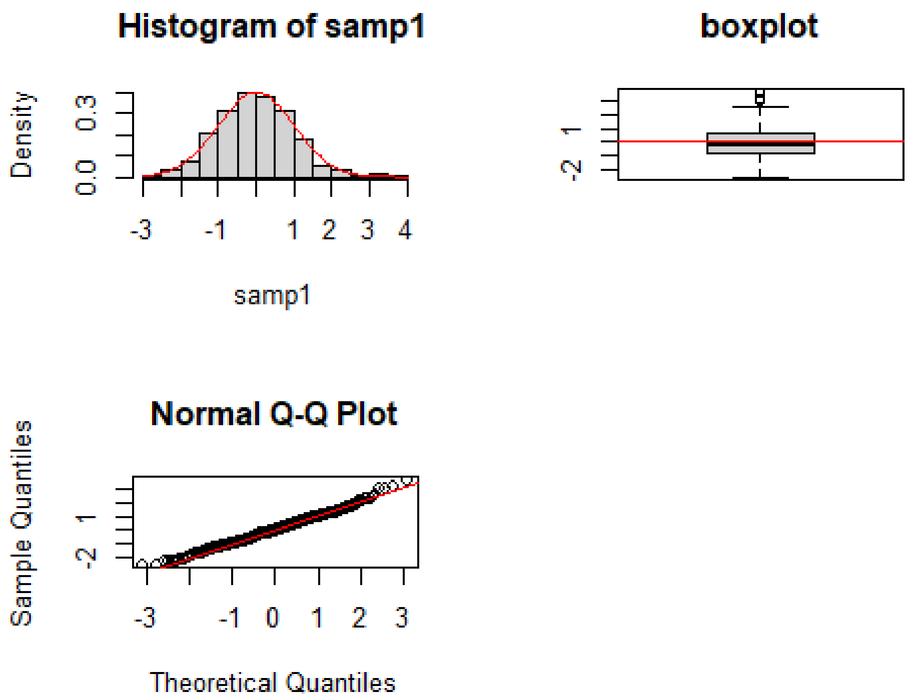
```r
for(j in 1:it)
{
samp=rnorm(i,mu,sigma)
est[j]=sum(samp)/i
}
vec=abs(est-mu)
prob[i]=length(which(vec<=elp))/it
#print(i)
}
plot(1:n,prob,xlab="n",type="l",ylab="probability",main="convergence in
probability")
abline(h=1,col="red")
```



Result : As sample size increases the the probability that absolute difference between the estimator and parameter value less than absilon is converges to one.

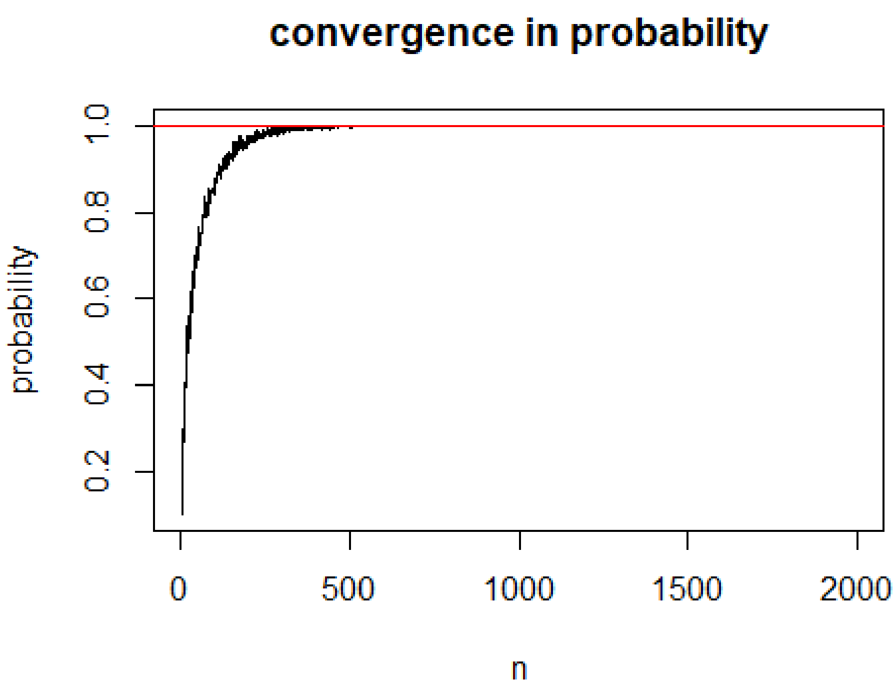
```r
################ Asymptotic normality
inf=1/(sigma)^2  #### information function
samp1=c()
for(i in 1:it)
{
samp=rnorm(n,mu,sigma)
tn=sum(samp)/n
samp1[i]=(sqrt(n*inf))*(tn-mu)
}
par(mfrow=c(2,2))
hist(samp1,prob=TRUE)
```

```
x=seq(-3,3,by=0.01)
curve(dnorm(x),col="red",add=TRUE)
################# Boxplot
boxplot(samp1,main="boxplot")
abline(h=0,col="red")
############## QQ plot
qqnorm(samp1)
qqline(samp1,col="red")
```

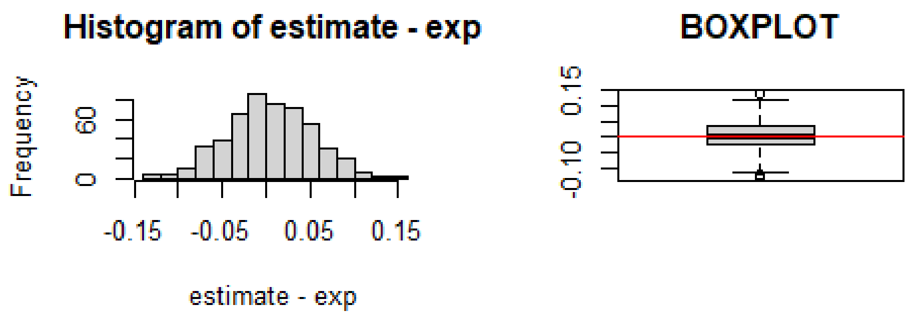


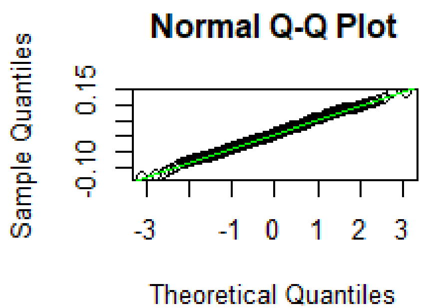

```
############# Moment based estimator property
par(mfrow=c(1,1))
mu=1
sigma=2
exp=mu    ### expected value
mu=1
n=2000
esp=0.3
it=500
prob=c()
for(i in 1:n)
{
est=c()
for(j in 1:it)
{
samp=rnorm(i,mu,sigma)
est[j]=mean(samp)
}
```

```
vec=abs(est-exp)
prob[i]=length(which(vec<=esp))/it
#print(i)
}
plot(1:n,prob,xlab="n",type="l",ylab="probability",main="convergence in
probability")
abline(h=1,col="red")
```




```
###### 2) Asymptotic Normality
n=2000
it=500
sigma=2
mu=1
exp=mu
estimate=c()
for(i in 1:it)
{
samp=rnorm(n,mu,sigma)
estimate[i]=mean(samp)
}
par(mfrow=c(2,2))
hist(estimate-exp)
boxplot(estimate-exp,main="BOXPLOT")
m=median(estimate-exp)
abline(h=m,col="red")
qqnorm(estimate-exp)
qqline(estimate-exp,col="green")
```

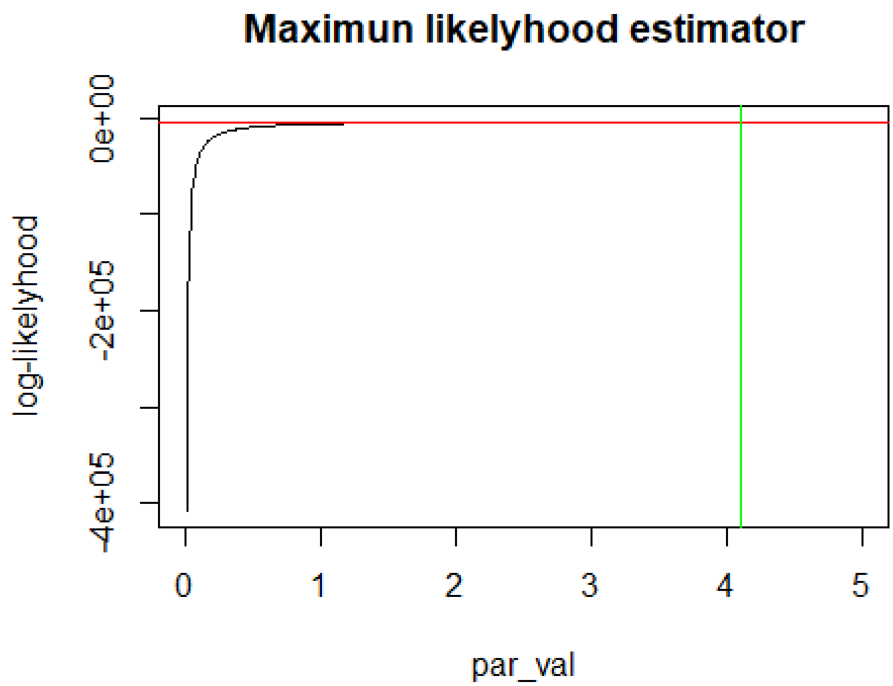





```
################################### case:2
############# when mu is known and variance is unknown
par(mfrow=c(1,1))
mu=1
sigma=2
n=2000
samp=rnorm(n,mu,sigma)
tn=sum((samp-mu)^2)/n
print(tn)

## [1] 4.107112

par_val=seq(0.01,5,by=0.01)  #### sigma square value
logl=c()
for(i in 1:length(par_val))
{
logl[i]=(n*log(1/((par_val[i])^0.5*sqrt(2*pi))))-((1/2)*sum(((samp-
mu)/(par_val[i])^0.5)^2))
}
m=max(logl)
print(m)

## [1] -4250.598

ind=which(logl==m)
mle=par_val[ind]
print(mle)
```

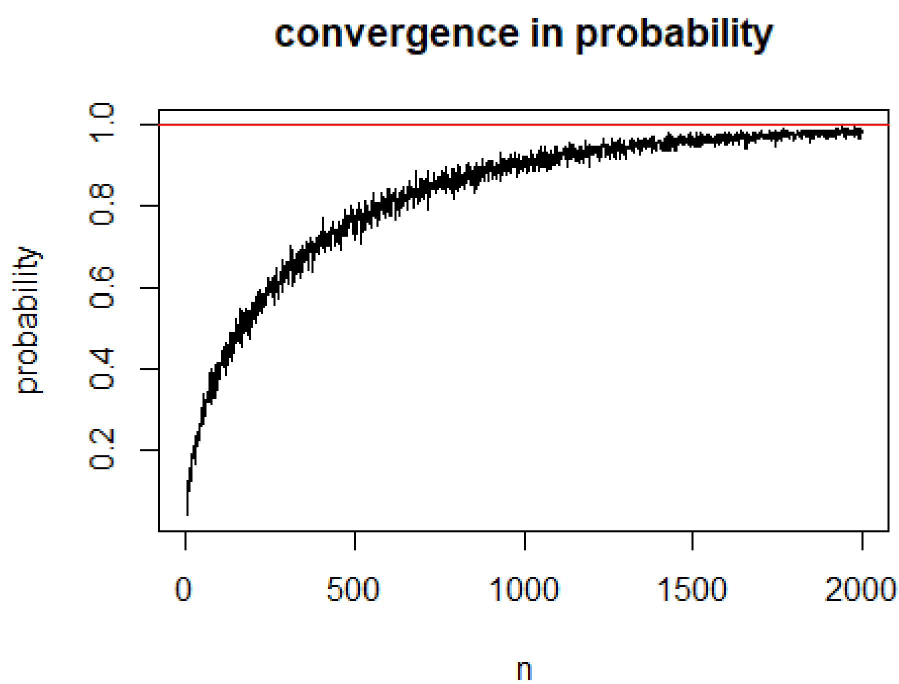
```
## [1] 4.11
```

```
plot(par_val,logl,type="l",xlab="par_val",ylab="log-likelyhood",main="Maximun likelyhood estimator")
abline(h=m,col="red")
abline(v=mle,col="green")
```




```
################## consistency property
n=2000
mu=1
it=500
sigma=2
p=(sigma)^2
prob=c()
for(i in 1:n)
{
est=c()
for(j in 1:it)
{
samp=rnorm(i,mu,sigma)
est[j]=sum((samp-mu)^2)/i
}
vec=abs(est-p)
prob[i]=length(which(vec<elp))/it
#print(i)
}
plot(1:n,prob,xlab="n",type="l",ylab="probability",main="convergence in
```

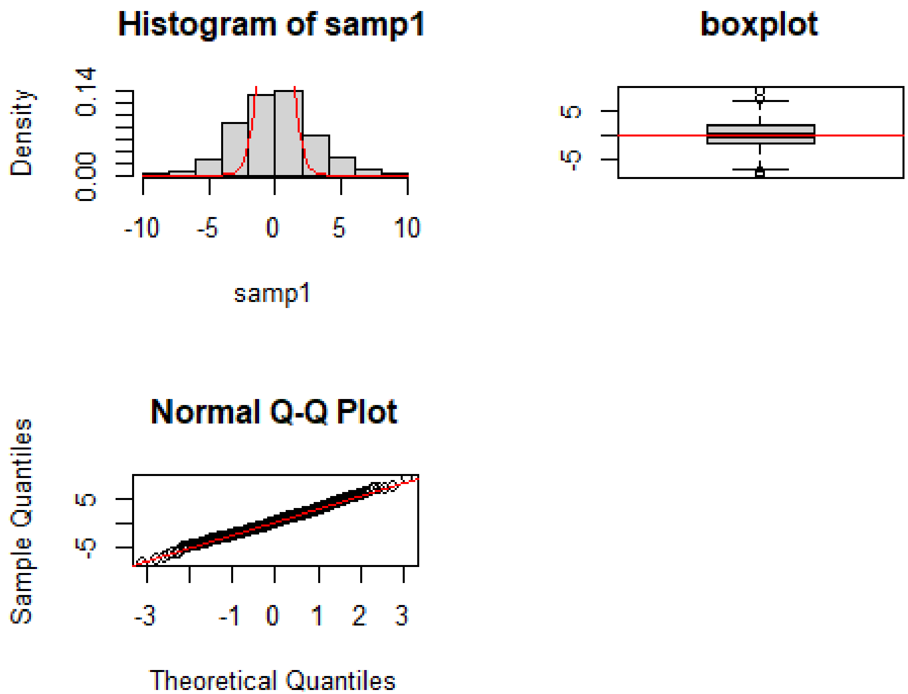
```
probability")
abline(h=1,col="red")
```




```
##############################
```

Result : As sample size increses the probability converges to one.

```
################ Asymptotic normality
inf=1/(sigma)^2  #### information function
p=(sigma)^2
samp1=c()
for(i in 1:it)
{
samp=rnorm(n,mu,sigma)
tn=sum((samp-mu)^2)/n
samp1[i]=(sqrt(n*inf))*(tn-p)
}
par(mfrow=c(2,2))
hist(samp1,prob=TRUE)
x=seq(-5,5,by=0.01)
curve(dnorm(x),col="red",add=TRUE)
################# Boxplot
boxplot(samp1,main="boxplot")
abline(h=0,col="red")
############## QQ plot
qqnorm(samp1)
qqline(samp1,col="red")
##############################################################
```


Histogram of samp1
Density
0.00
0.14
-10
-5
0
5
10
samp1
boxplot
5
-5
Normal Q-Q Plot
Sample Quantiles
5
-5
-3
-1
0
1
2
3
Theoretical Quantiles