WinBUGS Tutorial Outline

Ed Merkle & Trisha Van Zandt

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- WinBUGS overview
- Description of Example Data
- Example 1: Confidence in Signal Detection
- Example 2: Mean RT Model
- Example 3: Estimating RT Distributions
- Conclusions/Things to Remember

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- **Aside from direct sampling, other methods generally work by sampling from distributions similar to the posterior, then accepting the sample with some probability.**

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  - **The generated values can cause the program to crash, though!**
- **Similarity to R/Splus**

Model Specification

- **Define data distributions, prior distributions**
  - **Distributions defined using ~ operator**
  - **Assignment defined using the <- operator**

Short SDT example:

```
model
{
# x is vector of observed responses (1/2)
# y is vector of correct response

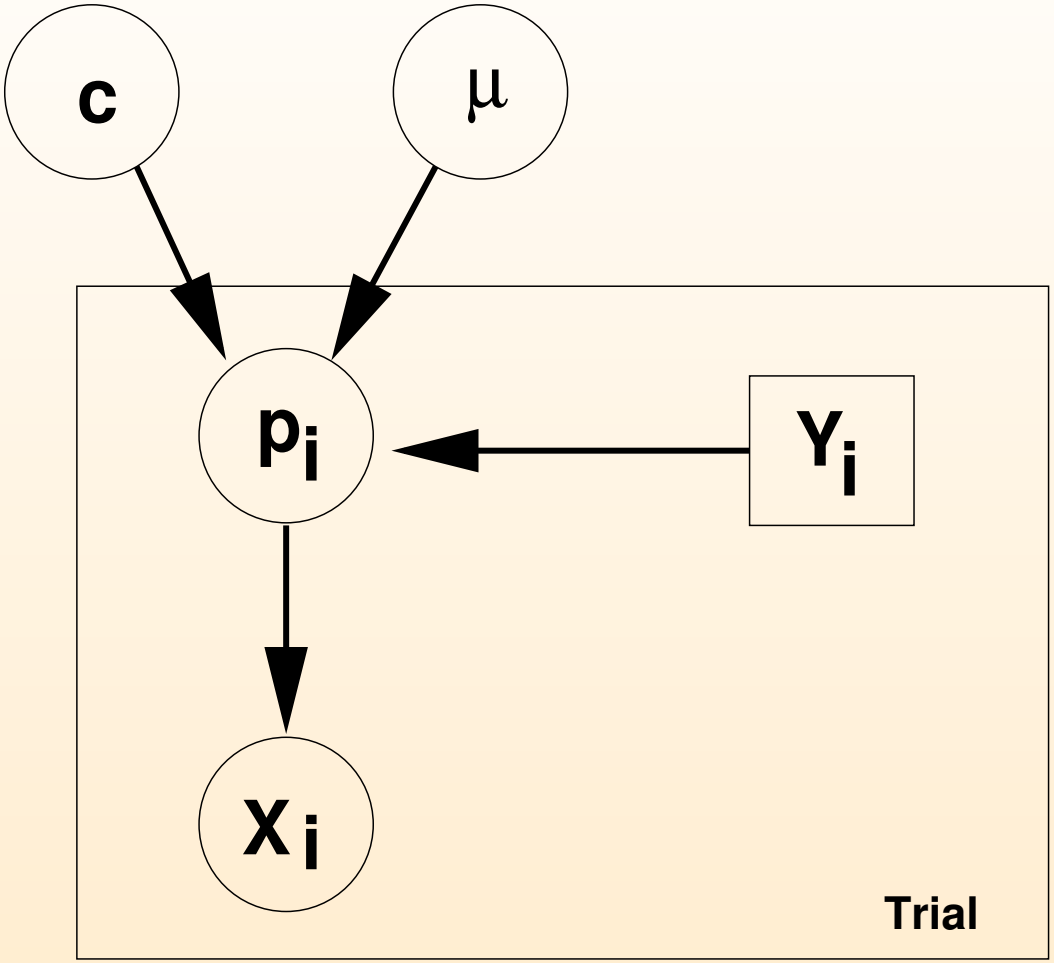
# Define x as categorical with probability of response
# determined by SDT model.
for (i in 1:N){
  x[i] ~ dcat(p[i,])

  # Define SDT probabilities, assuming variance of 0.5
  p[i,1] <- phi((c - mu*y[i])/0.5)
  p[i,2] <- 1 - p[i,1]
}

# Define prior distributions on model parameters.
mu ~ dnorm(0,0.0001)
c ~ dnorm(0,0.0001)
}

list(mu=0.5,c=0.25)
```

SDT Doodle



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- **Starting value formats are just like data formats.**

Initializing the Model

- **Submit model specification & data to WinBUGS using Model Specification Tool:**
  - **Highlight "model", click "check model".**
  - **Highlight "list", click "load data".**
  - **Type number of chains, click "compile".**
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- **Error messages appear in lower right-hand corner.**
- **When "model is initialized" message appears, you are ready to simulate.**

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- **Click "Samples..." off the inference menu to bring up the Sample Monitor Tool.**
  - **Type individual parameter names, click "set"**
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- **Click "Samples..." off the inference menu to bring up the Sample Monitor Tool.**
  - **Type individual parameter names, click "set"**
  - **If sampling method requires an adaptive phase, not all options will initially be available to you.**
- **Click "Update..." off the model menu.**
  - **Type desired number of iterations & refresh frequency, then click update button to run the simulation.**
  - **If program crashes, changing the starting values can solve the problem.**

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- **Initial sampled parameters are usually discarded to allow for burn-in/convergence to posterior distribution.**

Summarizing Results

- **Sample monitor tool provides many useful options:**
  - **"stats" yields summary statistics.**
  - **"history", "auto cor" yield time series, autocorrelation plots.**
  - **"bgr diag" yields Gelman-Rubin statistic plots.**

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- **Comparison Tool provides summary plots for parameter vectors, plots of observed vs. predicted data.**
- **DIC Tool provides DIC measure of model complexity.**

Example Data

- **Data from same experiment are used for all 3 examples.**

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- **Data from same experiment are used for all 3 examples.**
- **Participants presented with dots clusters, where number of dots arose from one of two normal distributions differing in mean number of dots.**
- **For each cluster, participants state confidence that dots arose from the high distribution.**
- **Collected data:**
  - **Confidence: From 5% to 95% in increments of 10%.**
  - **Correct Distribution: True distribution from which the dots arose.**
  - **Condition: Easy condition $\mu_H = 55$ and $\mu_L = 45$; Hard condition $\mu_H = 53$ and $\mu_L = 47$.**
  - **Implied Response: Implied choice based on the confidence judgments.**
  - **Number of Dots: Number of dots in a specific dots cluster.**
  - **RT: The participant's confidence response time in msec.**

Disclaimer

In the examples that follow, we estimate relatively simple models using relatively simple data sets. The models are intended to introduce WinBUGS to a general math psych audience; they are not intended to be optimal for the given data sets. Furthermore, we do not recommend blind application of these models to future data sets.

Example 1: Confidence in Signal Detection

- **Signal detection model of confidence:**
  - **Partitions placed on signal/noise distributions**
  - **Confidence determined by which two partitions a given stimulus falls between.**
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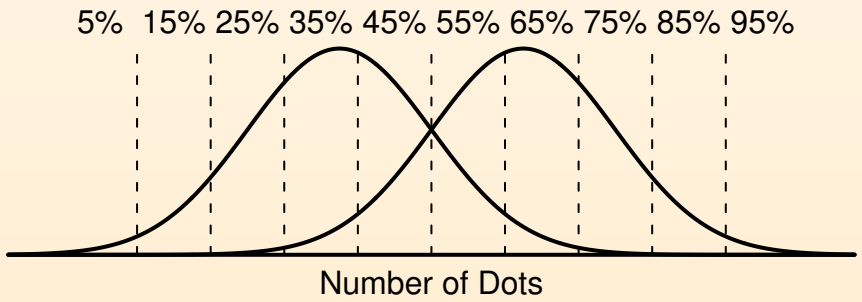

The Model

- **Mathematically:**
  - x **is vector of confidence judgments**
  - γ **is vector of partitions**
  - K **is distribution from which dots actually arose**
  - **Mean (**μ_k**) of each distribution and common standard deviation (**s**) are assumed known.**
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- $P(x_i = j \mid K = k) = \Phi((\gamma_j - \mu_k)/s) - \Phi((\gamma_{j-1} - \mu_k)/s)$**.**

Model Specification

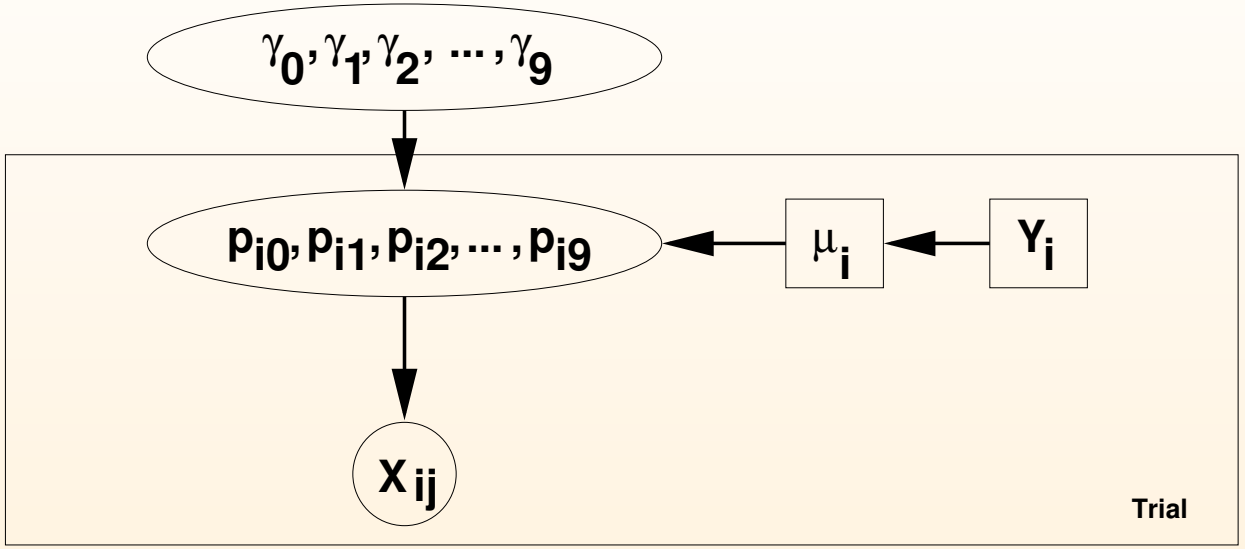
```
model
{
# y is vector of distribution from which dots arose
# x is vector of raw confidence data
mu[1] <- 45
mu[2] <- 55

for (i in 1:2){
  p[i,1] <- phi((gamm[1] - mu[i])/5)
  for (j in 2:9){
    p[i,j] <- phi((gamm[j]-mu[i])/5) - phi((gamm[j-1]-mu[i])/5)
  }
  p[i,10] <- 1 - phi((gamm[9] - mu[i])/5)
}

for (k in 1:N){
  x[k] ~ dcat(p[y[k]+1,])
}

gamm[1] ~ dnorm(50, 1.0E-06)I(,gamm[2])
for (j in 2:8){
  gamm[j] ~ dnorm(50, 1.0E-06)I(gamm[j-1],gamm[j+1])
}
gamm[9] ~ dnorm(50, 1.0E-06)I(gamm[8],)
}
```

Model Doodle



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- **Ordering of the parameters in γ is controlled by the prior distributions. Each parameter's prior distribution is essentially flat and is censored by the other threshold parameters surrounding it using the "I(,)" specification. For another example on the use of ordered thresolds, see the "Inhalers" example included with WinBUGS.**

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- **Sampling from this model is slow; it takes a couple minutes per 1000 iterations. It is thought that the censoring causes this delay.**

Results

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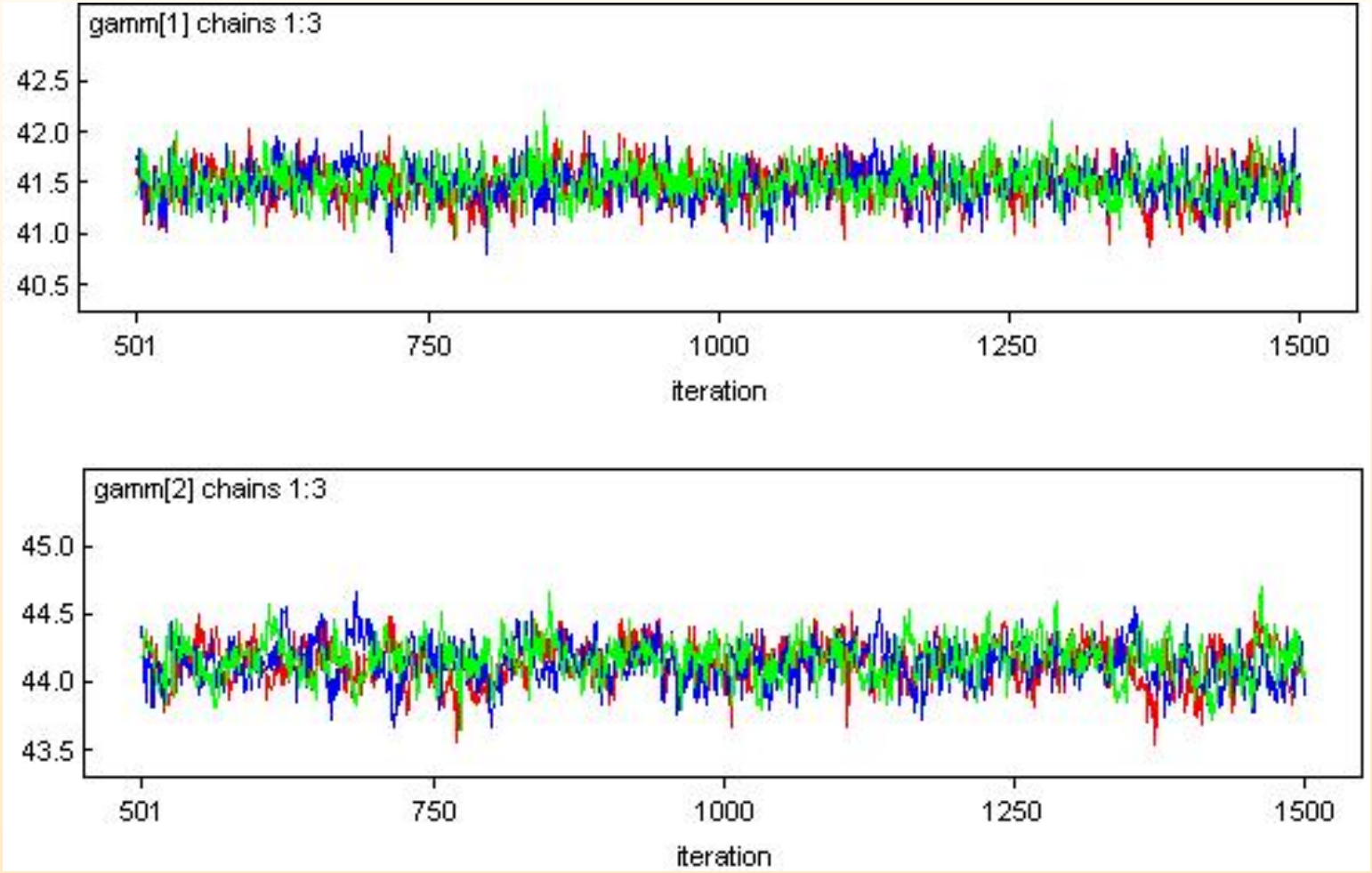

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- **Summary statistics for threshold parameters:**

Node statistics

node	mean	sd	MC error	2.5%	median	97.5%	start	sample
gamm[1]	41.49	0.1859	0.00568	41.11	41.49	41.84	501	3000
gamm[2]	44.14	0.1526	0.006595	43.84	44.15	44.43	501	3000
gamm[3]	46.3	0.1393	0.007265	46.03	46.31	46.58	501	3000
gamm[4]	47.71	0.1348	0.007639	47.45	47.71	47.98	501	3000
gamm[5]	49.02	0.1285	0.007446	48.77	49.01	49.27	501	3000
gamm[6]	50.45	0.124	0.006738	50.2	50.44	50.69	501	3000
gamm[7]	52.19	0.127	0.006412	51.95	52.19	52.44	501	3000
gamm[8]	54.38	0.1345	0.005889	54.11	54.38	54.65	501	3000
gamm[9]	57.09	0.1566	0.005353	56.77	57.09	57.39	501	3000

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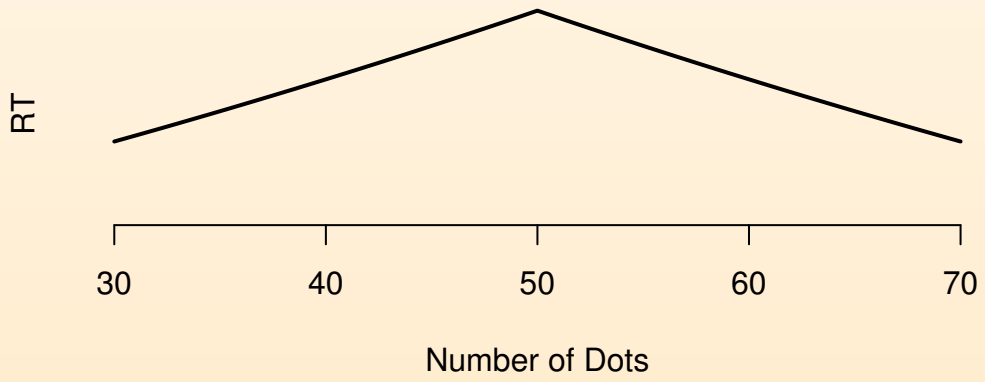
- **For the current data, difficulty may be characterized as number of dots on a given trial (close to 50 = difficult; far from 50 = easy).**
- **General finding: RT decreases with difficulty.**



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- **Both functions can be modified to incorporate difficulty conditions; participant-level effects.**

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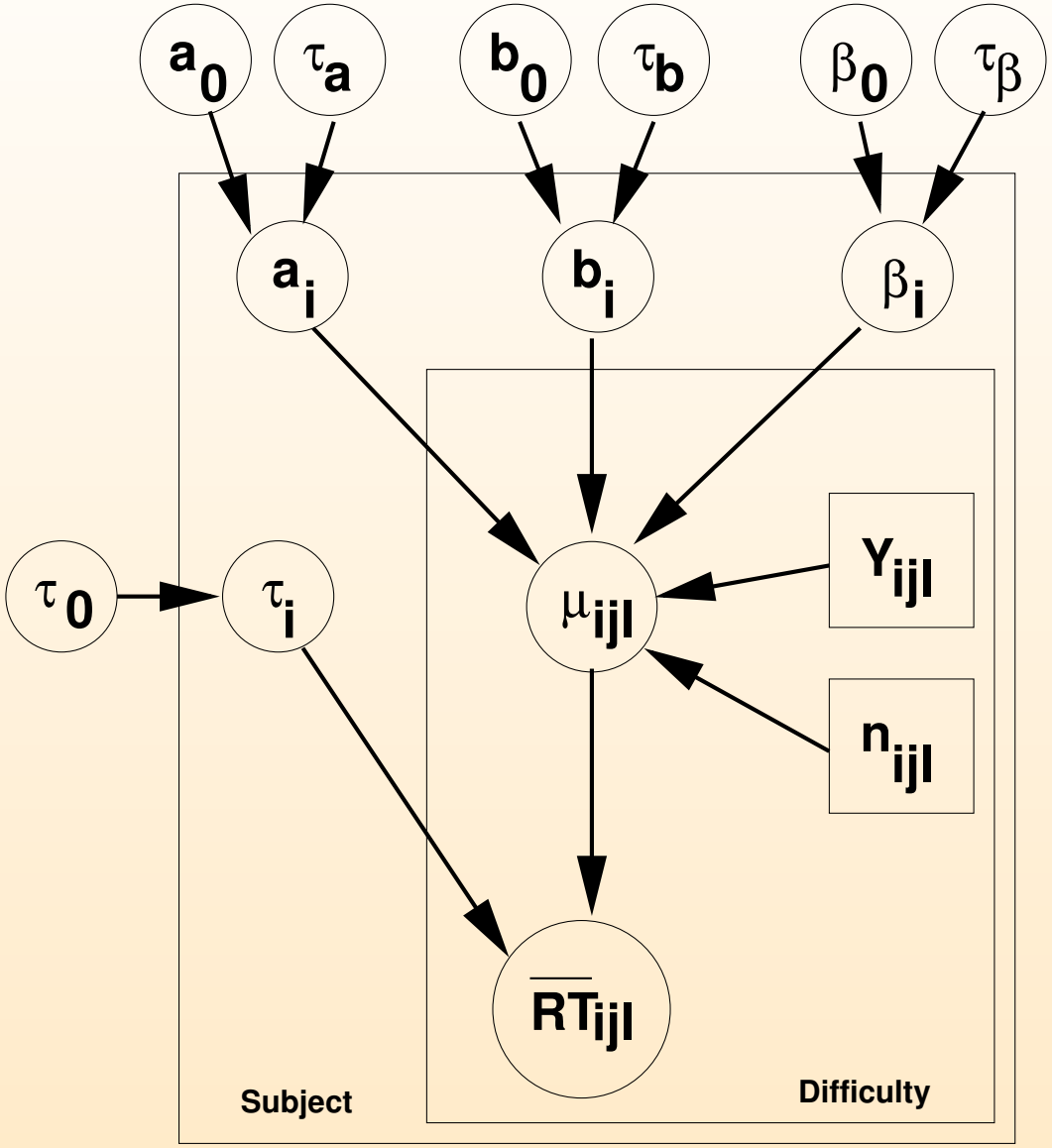
- **Arrive at a model for RT by taking $RT_{ij} \sim N(\mathsf{E}(RT_{ij}), \tau_i)$.**

Linear Model Specification

```
model
{
for (i in 1:N)  {
  for (j in 1:C) {
  for (l in 1:D) {
    mu[i,j,l] <- b[i] + (a[i] + hard[i,j,l]*beta[i]) *
                 (dots[i,j,l]-50.0)
    RT[i,j,l] ~ dnorm(mu[i,j,l], tau[i])
  }
  }
  b[i] ~ dnorm(b.c,tau.b)
  a[i] ~ dnorm(a.c,tau.a)
  beta[i] ~ dnorm(beta.c,tau.beta)
  tau[i] ~ dgamma(1.0,tau.0)
}
b.c ~ dgamma(1.0,1.0E-3)
a.c ~ dnorm(0.0,1.0E-3)
beta.c ~ dnorm(0.0,1.0E-3)
tau.b ~ dgamma(1.0,1.0E-3)
tau.a ~ dgamma(1.0,1.0E-3)
tau.beta ~ dgamma(1.0,1.0E-3)
tau.0 ~ dgamma(1.0,1.0E-3)
}

list(
b.c=5.0,a.c=1.0,beta.c=-0.5,
tau.b=1.0,tau.a=1.0,tau.beta=1.0,
tau.0=1.0
)
```

Model Doodle



Notes on the Model Specification

- **Number of dots have been divided into D=6 groups of difficulty. For example, group 1 contains RTs for 48-52 dots, group 2 contains RTs for 53-57 dots (and for 43-47 dots), ..., group 6 contains RTs for 73-77 dots (and for 23-27 dots). Examination of the data file should make this clearer.**

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- Not all participants have been observed at extreme numbers of dots. Thus, there are some missing values in the response variable RT (denoted by NA). WinBUGS can handle these missing values.

- Sampling from this model is very fast.

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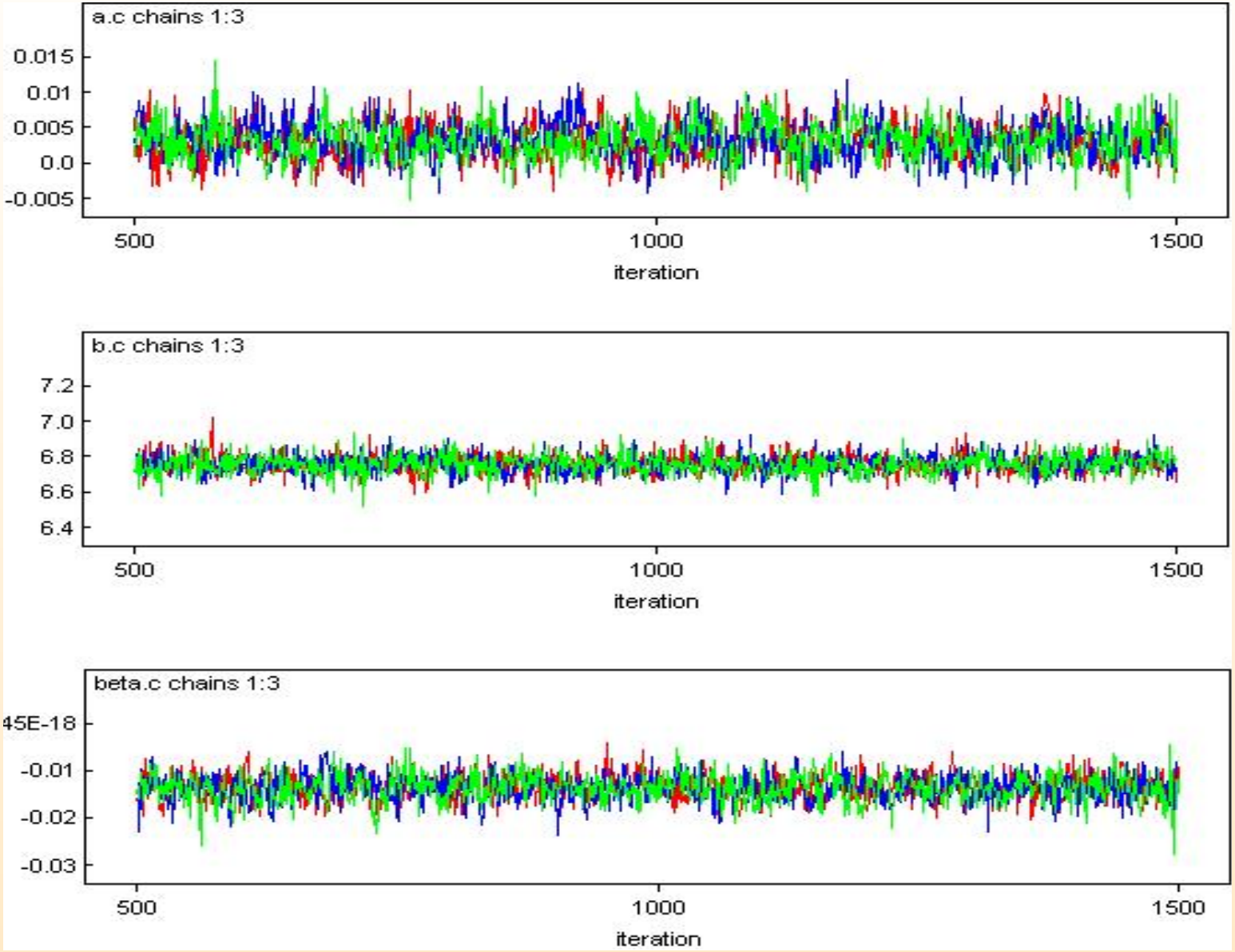

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- **Based on sampled parameters, we obtain 95% posterior intervals:**
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  - **beta.c: (-.019,-.014)**

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  - **a.c: (-.002,.008)**
  - **beta.c: (-.019,-.014)**
- **RTs seem to decrease with difficulty in the hard condition but not the easy condition.**
- **Experiment design responsible for this?**

Exponential Model Specification

```
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for (i in 1:N)  {
  for (j in 1:C) {
  for (l in 1:D) {
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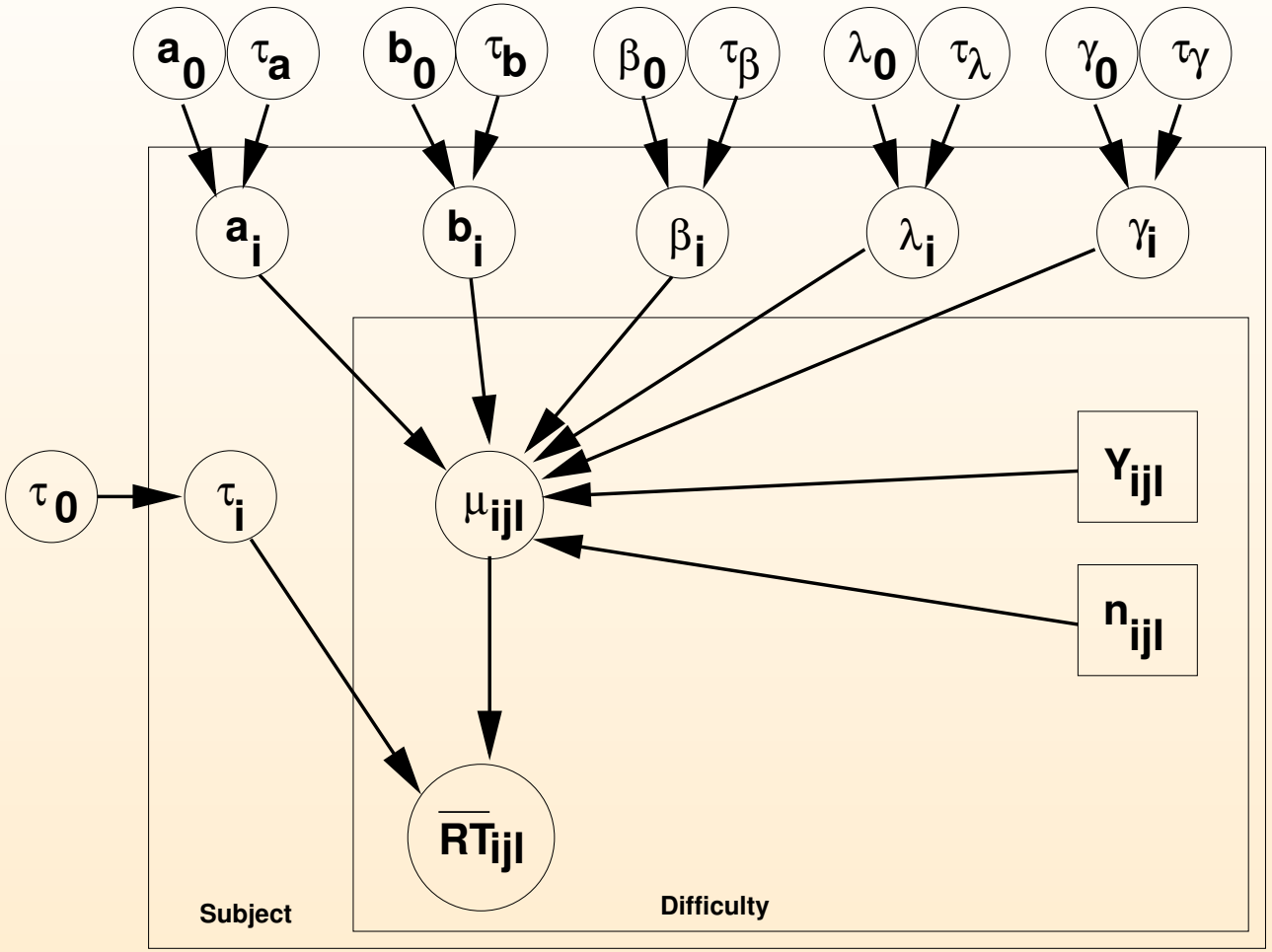
      exp(-(dots[i,j,l]-50.0)*(lambda[i]+hard[i,j,l]*gamma[i]))
    RT[i,j,l] ~ dnorm(mu[i,j,l], tau[i])
  }
  }
  b[i] ~ dnorm(b.c,tau.b)
  a[i] ~ dnorm(a.c,tau.a)
  beta[i] ~ dnorm(beta.c,tau.beta)
  lambda[i] ~ dnorm(lambda.c,tau.lambda)
  gamma[i] ~ dnorm(gamma.c,tau.gamma)
  tau[i] ~ dgamma(1.0,tau.0)
}
b.c ~ dgamma(0.1,0.01)
a.c ~ dnorm(0.0,1.0E-3)
beta.c ~ dnorm(0.0,1.0E-3)
lambda.c ~ dnorm(0.0,1.0E-3)
gamma.c ~ dnorm(0.0,0.01)
tau.b ~ dgamma(0.15,0.01)
tau.a ~ dgamma(1.0,0.01)
tau.beta ~ dgamma(0.3,0.01)
tau.lambda ~ dgamma(4.0,0.01)
tau.gamma ~ dgamma(3.0,0.01)
tau.0 ~ dgamma(1.0,1.0)
}
```

Model Doodle



Starting values

```
list(
b.c=5.0,a.c=1.0,beta.c=-0.5,lambda.c=0.0,gamma.c=0.0,
tau.b=1.0,tau.a=1.0,tau.beta=1.0,tau.lambda=1.0,tau.gamma=1.0,
tau.0=1.0,b = c(5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5,
5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5,
5, 5, 5, 5, 5), a = c(1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1,
1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1,
1, 1, 1, 1, 1, 1), beta = c(-0.5, -0.5, -0.5, -0.5, -0.5, -0.5,
-0.5, -0.5, -0.5, -0.5, -0.5, -0.5, -0.5, -0.5, -0.5, -0.5,
-0.5, -0.5, -0.5, -0.5, -0.5, -0.5, -0.5, -0.5, -0.5, -0.5,
-0.5, -0.5, -0.5, -0.5, -0.5, -0.5, -0.5, -0.5, -0.5, -0.5,
-0.5, -0.5, -0.5, -0.5),
lambda = c(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0), gamma = c(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0), tau = c(1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1,
1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1,
1, 1, 1, 1, 1, 1, 1)
)
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- We have attempted to make the prior distributions for model parameters noninformative, while still keeping the density in the ballpark of the parameter estimates. We obtained similar parameter estimates for a number of noninformative prior distributions.
- The default sampling method for this model requires 4000 iterations before parameter summaries are allowed. After the first 4000 iterations, all WinBUGS tools are available to you.

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- 95% posterior intervals:
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  - a.c: (0.21,0.43)
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- Intervals lead to similar conclusions as for the linear model.

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- **Based on this statistic, we conclude that the exponential model is better.**

Example 3: Estimating RT Distributions

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- **If y_{ij} is participant i's RT on trial j, we can write the density as:**

$$f(y_{ij} \mid \psi_i, \theta_i, \beta_i) = \frac{\beta_i (y_{ij} - \psi_i)^{\beta_i - 1}}{\theta_i^{\beta_i}} \exp\left[-\frac{(y_{ij} - \psi_i)^{\beta_i}}{\theta_i^{\beta_i}} \right], \tag{1}$$

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- **ψ, θ, and β parameters correspond to the distribution's shift, scale, and shape, respectively.**

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- **The resulting model estimates RT distributions at participant level, and hyperparameters can tell us how parameters vary across participants.**

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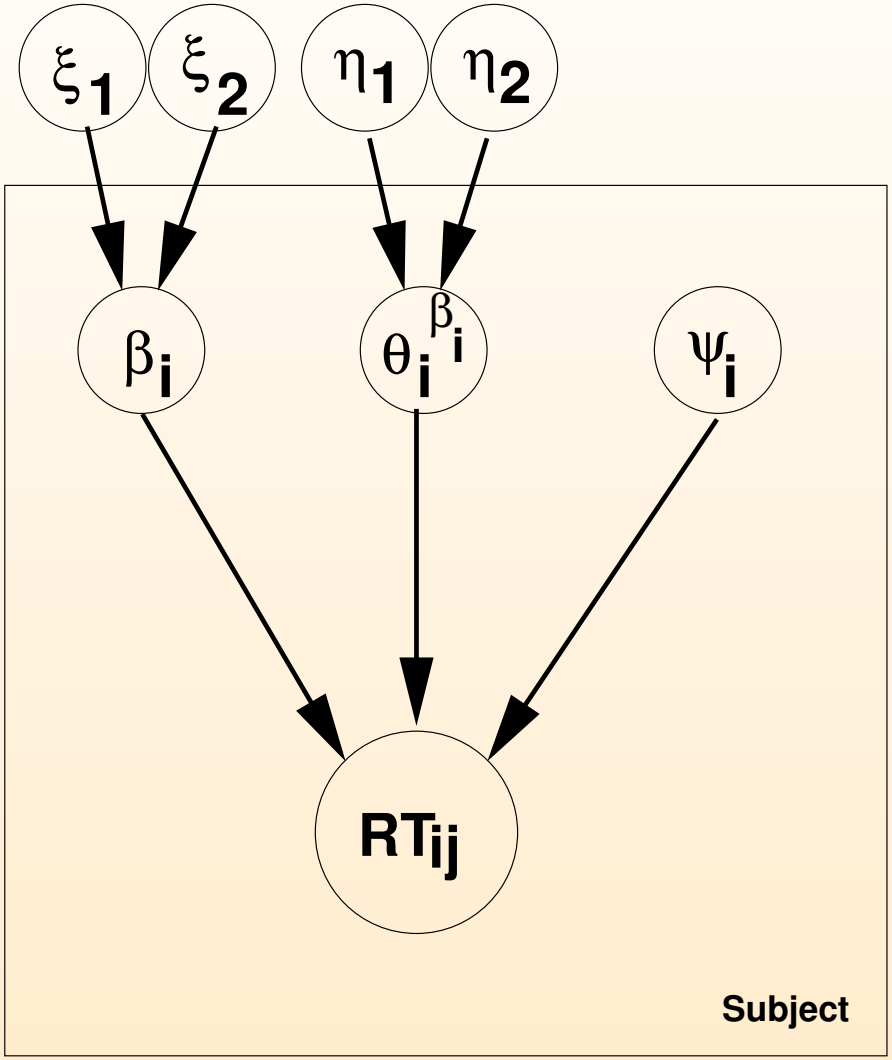
  - **A constant is added to λ to ensure $\lambda > 0$.**

Model Specification

```
model
{
# Works if you change "real nonlinear" to "UpdaterSlice" in methods.odc
# y[i,j] is RT of participant i on trial j
# Only RTs from easy condition are used
# minrt is minimum rt for each participant
C <- 100000000
for (i in 1:N){
  for (j in 1:trials){
    zeros[i,j] <- 0
    term1[i,j] <- beta[i]*log(theta[i]) + pow(y[i,j] - psi[i],
                  beta[i])/pow(theta[i],beta[i])
    term2[i,j] <- log(beta[i]) + (beta[i]-1)*log(y[i,j] -
                  psi[i])
    phi[i,j] <- term1[i,j] - term2[i,j] + C
    zeros[i,j] ~ dpois(phi[i,j])
  }
beta[i] ~ dgamma(eta1,eta2)I(0.01,)
zero[i] <- 0
theta[i] ~ dflat()
phip[i] <- (xi1 + 1)*log(pow(theta[i],beta[i])) +
            xi2/pow(theta[i],beta[i]) + loggam(xi1) - xi1*log(xi2)
zero[i] ~ dpois(phip[i])
psi[i] ~ dunif(0,minrt[i])
}
# Priors as recommended by Rouder et al.
eta1 ~ dgamma(2,0.02)
eta2 ~ dgamma(2,0.04)
xi1 ~ dgamma(2,0.1)
xi2 ~ dgamma(2,2.85)
}
```

Model Doodle



Starting values

```
list(beta = c(1.15,1.3,1.27,1.04,1.16,1.18,1.34,
1.47,1.16,1.22,1.4,1.18,1.08,1.14,1.13,1.13,1.58,1.55,
1.5,1.55,1.3,1.34,1.28,1.22,1.37,0.9,1.26,1.47,1.64,
1.16,1.15,1.39,1.09,1.35,1.69,1.45,1.23), eta1 = 71.5,
eta2 = 56.12, psi = c(542.44,397.88,631.67,397.8,468.62,
654.39,528.16,435.96,562.58,699.9,614.47,400.18,291.57,
651.68,564.97,364.07,416.67,199.92,232.89,525.56,336.01,
421.95,562.76,414.89,397.26,205.09,378.95,309,241.95,
309.44,638.07,234.44,586.83,337.13,346.51,548.37,442.76
), theta = c(1263.77,589.18,763.83,705.66,1655.92,925.8,
1624.29,452.35,650.88,874.17,1164.92,468.03,1221.61,
938.44,740.69,1023,354.17,756.69,565.12,1481.56,1160.22,
917.63,1032.16,1634.01,1118.65,185.36,694.14,381.59,
498.82,755.39,1326.63,1421.59,1096.91,581,490.94,1134.19,
1285.15), xi1 = 0.12, xi2 = 3.92)
```

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- **Running this model using WinBUGS defaults always results in error messages around iteration 1100. We could not figure out why this happens, but the WinBUGS authors suggest that this problem can sometimes be avoided by changing sampling methods. These sampling methods are found in the file Updater/Rsrc/Methods.odc; by changing the method listed in the code, the model usually runs without errors.**

- **The first few (say, 50) iterations take a long time (at least a few minutes) to run, longer if you have multiple chains. The subsequent iterations are faster, taking 3-5 minutes per 1000 iterations.**

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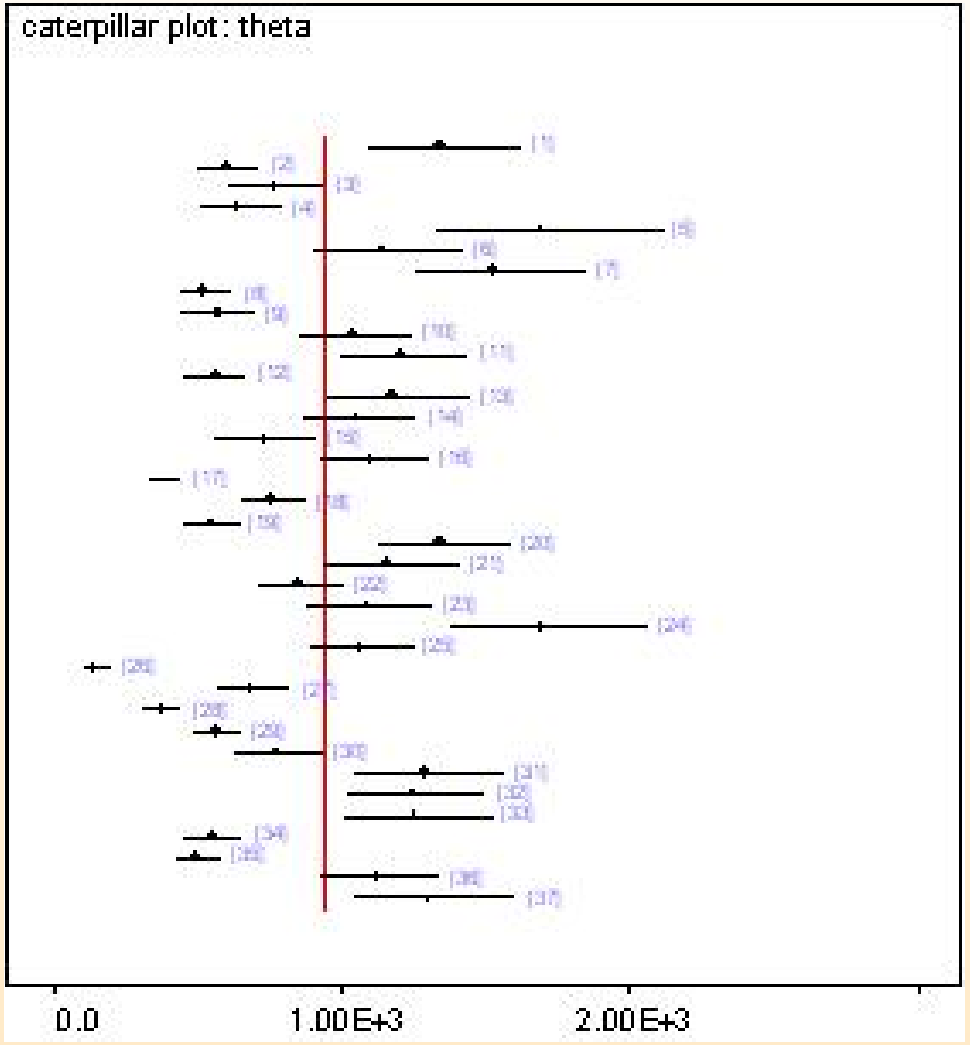
- **Plots/statistics in WinBUGS signify convergence.**

Caterpillar Plots

Participant-level parameters can be visualized using caterpillar plots in the Comparison Tool:



Miscellaneous Notes

- **In the provided set of probability distributions, WinBUGS measures scale parameters by their precision (which equals 1/variance). For example, to assign a normal distribution with mean 1 and variance 2, you would type "dnorm(1,0.5)". It is a good idea to check the parameterization of WinBUGS distributions, which can be found in the help files.**

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- **As parameters are simulated for more iterations, parameter estimates become more accurate. It is often the case, however, that people report parameter estimates to more decimal places than are justified. The "MC error" values give you an idea of how many decimal places you can accurately report.**