

**FACULTY OF MANAGEMENT STUDIES  
UNIVERSITY OF DELHI  
MBA FT - November 2015**  
**Paper 6102: Quantitative Methods for Management**

**Time: 3 hours**

**Max. Marks: 50**

**The question paper contains Seven questions in seven pages. Answer any FIVE questions selecting at least TWO questions from each section**

**Section A**

1. A farm owner is interested in determining how to divide the farmland among four different types of crops. The farmer owns two farms in separate locations and has decided to plant the following four types of crops in these farms: corn, wheat, bean, and cotton. The first farm consists of 1,450 acres of land, while the second farm consists of 850 acres of land. Any of the four crops may be planted on either farm. However, after a survey of the land, based on the characteristics of the farmlands, the following Table shows the maximum acreage restrictions the farmer has placed for each crop.

**Table: Max Acreage restrictions for Agricultural Problem**

Crop					
Farm	Corn	Wheat	Bean	Cotton	
Farm1	550	450	350	400	
Farm2	250	300	200	350	

The revenue per acre for each crop is estimated as follows:

Crop	Revenue/acre	Crop	Revenue/acre
Corn	\$500	Bean	\$300
Wheat	\$400	Cotton	\$350

In determining the optimal cultivation of land, the farmer has to account for the cost of fertilizer estimated for each acre of land. Due to the different terrain and soil, the two farms have different costs of fertilizers per acre.

Farm	Cost of Fertilizer/Acre
Farm 1	\$100
Farm 2	\$70

Seasonal demand for the four crops is given in the following table:

Crop	Seasonal demand (Acres' worth)
Corn	450
Wheat	550
Bean	400
Cotton	600

The farmer has a storage facility that can store 100 acres' worth of the excess supply of different types of crops. In addition, the farmer wants to ensure that total wheat and bean cultivation must be proportionally equal to the maximum acreage restriction of both farms. In other words, the farm owner wants the same proportion of wheat and beans in both farms. The farmer's objective is to determine how much of each crop to plant on each farm in order to maximize profit and satisfy seasonal demand. Formulate the above problem as a linear programming problem. You are not required to find out solution to the above problem.

(10)

2. (a) Consider the following two LP formulations. Using a graphical approach in each case, determine  
 (i) Which formulation does have more than one optimal solution?  
 (ii) Which formulation has an unbounded solution? In which direction the solution is unbounded?

**Formulation 1**

$$\begin{aligned} \text{Max } z &= 2x_1 + 4x_2 \\ \text{Subject to} \\ x_1 + 2x_2 &\leq 5 \\ x_1 + x_2 &\leq 4 \\ x_1, x_2 &\geq 0 \end{aligned}$$

**Formulation 2**

$$\begin{aligned} \text{Max } z &= 2x_1 + x_2 \\ \text{Subject to} \\ x_1 - x_2 &\leq 10 \\ 2x_1 &\leq 40 \\ x_1, x_2 &\geq 0 \end{aligned}$$

- (b) Write the dual of the following problem:

$$\begin{aligned} \text{Max } Z &= 5X_1 + 6X_2 \\ \text{Subject to} \end{aligned}$$

$$\begin{aligned} X_1 + 2X_2 &= 5 \\ -X_1 + 5X_2 &\geq 3 \\ 4X_1 + 7X_2 &\leq 8 \\ X_1 \text{ unrestricted}, X_2 &\geq 0 \end{aligned}$$

(6+4=10)

- 3(a) A production manager wants to determine how many units of each product to produce weekly to maximize weekly profits. Production requirements for the products are shown in the following table:

<u>Product</u>	<u>Material 1 (lbs.)</u>	<u>Material 2 (lbs.)</u>	<u>Labor (hours)</u>
A	3	2	4
B	1	4	2
C	5	none	3.5

Material 1 costs \$7 a pound, material 2 costs \$5 a pound, and labor costs \$15 per hour. Product A sells for \$101 a unit, product B sells for \$67 a unit, and product C sells for \$97.50 a unit. Each week there are

300 pounds of material 1; 400 pounds of material 2; and 200 hours of labor. Moreover, there is a standing order of 10 units of product C each week.

### Sensitivity Report

#### Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
\$B\$4	Optimal Values: A	0	-10	10	10	1E+30
\$C\$4	Optimal Values: B	82.5	0	10	1E+30	4.285
\$D\$4	Optimal Values: C	10	0	10	7.5	1E+30

#### Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H.S. Side	Allowable Increase	Allowable Decrease
\$E\$7	Constraint 1	132.5	0	300	1E+30	167.5
\$E\$8	Constraint 2	330	0	400	1E+30	70
\$E\$9	Constraint 3	200	5	200	35	165
\$E\$10	Constraint 4	10	-7.5	10	47.142	10

Constraint 1: Material 1

Constraint 2: Material 2

Constraint 3: Labour hours

Constraint 4: Standing order of C

Answer the following questions:

- i. What is the optimal value of the objective function?
  - ii. Suppose that we force the production of one unit of product A. What will be the value of new objective function?
  - iii. Suppose that the production manager procures an additional 10 labor hours. What impact will this have on the current optimal objective function value?
  - iv. Suppose the standing order of C increases by 40 units? What impact it will have on the objective function value?
- (b) What do you understand by reduced cost as provided in excel solver output?

(8+2 = 10)

- 4(a) A telecommunications company is considering expanding its cable and internet service operations into a new area. The area is divided into 10 neighbourhoods. The company is considering 7 location nodes to reach all 10 neighbourhoods. Of course, it costs a significant amount of money to open a node or make a node operational. The company would like to minimize total cost but at the same time reach all of the neighbourhoods. The cost of opening a node differs based on the characteristics of the land and the technical aspects of setting up a node.

The costs of opening seven nodes are as follows:

Node 1	Node 2	Node 3	Node 4	Node 5	Node 6	Node 7
125	85	70	60	90	100	110

The seven nodes can reach or provide internet/cable coverage to the following areas:

Node 1: Neighbourhoods 1, 3, 4, 6, 9, 10

Node 2: Neighbourhoods 2, 4, 6, 8

Node 3: Neighbourhoods 1, 2, 5

Node 4: Neighbourhoods 3, 6, 7, 10

Node 5: Neighbourhoods 2, 3, 7, 9

Node 6: Neighbourhoods 4, 5, 8, 10

Node 7: Neighbourhoods 1, 5, 7, 8, 9

Which nodes should be opened to provide coverage to all neighborhoods at the minimum cost? Formulate the problem.

(b)

A company is considering several investment opportunities, each of which differs in the initial capital required. The accounting department has done a thorough analysis of each of these investments and has estimated the long-term profit of each. The initial capital required and estimated profits (in lacs of Rs.) are summarized in the following table. Investments 1 and 4 are high-risk investments and management has decided to invest in at most one of these. In addition, investment 6 is contingent upon investing in investment 3. If Rs. 120 lacs of initial capital is available, formulate an integer programming model to determine the optimal investment strategy.

Investment	Initial capital	Estimated profit
1	26	18
2	34	12
3	18	7
4	45	24
5	31	11
6	39	15
7	23	9
8	13	6

(6+4 = 10)

## Section B

- 5(a) Morgan Stanley use three methods to select stocks for their portfolio. Use Kruskal-Wallis test, at 5% level of significance, to determine if there is a significant difference in the rates of return for each method based on the following data:

Portfolio	Rate of return						
	29	17	14	28	27	22	22
B	27	20	13	15	22	21	-
C	38	28	27	34	32	---	---

- (b) An investment bank wants to study, "is there any relationship between cost per share and earning per share" at NYSE? The bank studied a random sample of 14 stocks. For each stock, the bank found the cost per share (in dollars) and ranked each of the stock according to cost. After three months, the bank found earnings per share on each stock (in dollars) and ranked each of the stock based on earnings. Higher ranks means more cost or more earnings.

Stock	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Cost ranking per share	5	2	4	7	11	8	12	3	13	14	10	1	9	6
Earnings ranking per share	5	13	1	10	7	3	14	6	4	12	8	2	11	9

At 5% level of significance, what is your conclusion?

( 10 )

- 6(a) Use suitable tests to determine whether there is a difference in the average income of families who make purchases from Wal-Mart retail store and families who do not make purchases from Wal-Mart retail store, at 5% level of significance.

Annual income of families making purchases from Wal-Mart (in dollars)	Annual income of families who do not make purchases from Wal-Mart (in dollars)
\$24500	\$41000
39400	32500
36800	33000
43000	21000
57960	40500
32000	32400
61000	16000
34000	21500
43500	39500
55000	27600
39000	43500
62500	51900
61400	27800
53000	----

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- 14  
 (b) The variance in production process is an important measure of the quality process. A large variance often signals an opportunity for improvement in the process by finding ways to reduce the process variance. Conduct a statistical test to determine whether there is a significant difference between the variances in the bag weights for the two machines. Use a 10% level of significance. What is your conclusion? Which machine, if either, provides the greater opportunity for quality improvements?

	No of Observations	Mean	Standard Deviation
Machine 1	25	5.9	2
Machine 2	22	6.3	1.9

(10)

7(a)

In a study of brand loyalty in the automotive industry, new-car customers were asked whether the make of their new car was the same as the make of their previous car. The break down of 600 responses shows the brand loyalty for domestic, European and American cars.

Purchaser	Domestic	European	American
Same make:	125	55	68
Different make:	140	105	107

Test a hypothesis to determine. Whether brand loyalty is independent of the manufacturer. Use level of significance 5%. What is your conclusion? If a significant difference is found, which manufacturer appears to have the greatest brand loyalty?

- (b) A consulting firm is preparing a study on consumer behavior. The company collected the following data in thousand dollars to determine whether there is a relationship between consumer income and consumption levels:

Consumer No:	1	2	3	4	5	6	7	8	9	10	11	12
Income:	24.3	12.5	31.2	28.0	35.1	10.5	23.2	10.0	8.5	15.9	14.7	15
Consumption:	16.2	8.5	15	17	24.2	11.2	15	7.1	3.5	11.5	10.7	9.2

Compute and interpret the regression model. What does this model tell about the relationship between consumption and income? What consumption would the model predict for someone who earns \$27500?

(10)