INSTRUCTIONS

This is a Close-book examination. You are allowed to carry probability distribution tables. Electronic calculator is allowed.
Attempt any **FIVE** questions. All questions carry equal marks.

1. A consulting firm is preparing a study on consumer behavior. The company collected the following data in thousand dollars to determine whether there is a relationship between consumer income and consumption levels:

Consumer No:	1	2	3	4	5	6	7	8	9	10	11	12
Income:	24.3	12.5	31.2	28.0	35.1	10.5	23.2	10.0	8.5	15.9	14.7	15
Consumption:	16.2	8.5	15	17	24.2	11.2	15	7.1	3.5	11.5	10.7	9.2

Compute and interpret the regression model. What does this model tell about the relationship between consumption and income? What consumption would the model predict for someone who earns $27500?

(14)

2. (a) Before the Sub-prime crisis in the United States, the nation's 40000 mortgage brokerages were some of the most profitable small businesses in the country. These low profile companies found loans for customers in exchange of commissions. Using Mortgage Banker's data, a sample of 270 loans made in 2006 provided a mean loan $1750000. The Data for 2005 showed a sample of 250 loans made with mean loan of $1650000. Based on historical data, the population standard deviation for the loan amounts can be assumed known at $55000, in 2006 and $50000 in 2005. Do the sample data indicate an increase in the mean loan amount between 2005 and 2006? Use a 5% level of significance.

(b) The variance in production process is an important measure of the quality process. A large variance often signals an opportunity for improvement in the process by finding ways to reduce the process variance. Conduct a statistical test to determine whether there is a significant difference between the

variances in the bag weights for the two machines. Use a 10% level of significance. What is your conclusion? Which machine, if either, provides the greater opportunity for quality improvements?

	No of Observations	Mean	Standard Deviation
Machine 1	25	5.9	2
Machine 2	22	6.3	1.9

(7+7 = 14)

3. (a) In a study of brand loyalty in the automotive industry, new-car customers were asked whether the make of their new car was the same as the make of their previous car. The breakdown of 600 responses shows the brand loyalty for domestic, European and American cars.

Purchaser	Domestic	European	American
Same make:	125	55	68
Different make:	140	105	107

Test a hypothesis to determine whether brand loyalty is independent of the manufacturer. Use 5% level of significance. What is your conclusion? If a significant difference is found, which manufacturer appears to have the greatest brand loyalty?

(b) Your Managing Director asked you to assist in the analysis of effectiveness of three advertising displays. Five consumers are randomly selected. Each is shown an advertisement and asked to rate it. The result is shown in the following table:

Consumer	Advertisement Display		
	1	2	3
1	50	45	45
2	45	30	35
3	30	25	20
4	45	35	40
5	40	30	35

Is there a significant difference in effectiveness of three advertising displays? Use level of significance 5%.

(7+7 = 14)

4. (a) The owner of a private freighter is trying to decide which cargo he should carry on his next trip. He has two choices of cargo, which he can agree to carry in any combination. He has two choices of cargo, which he can agree in any combination. He may carry up to 15 tons of cargo A, which takes up 675 cubic feet per ton and earns revenue of Rs. 6350 per ton. Or he may carry up to 54 tons of cargo B, with a volume of 450 cubic feet per ton and earns revenue of Rs. 5900 per ton.

The freighter is divided into two holds, starboard and port. The starboard hold has a volume of 14,000 cubic feet and a weight capacity of 26 tons. The port hold has a volume of 15,400 cubic feet and a weight capacity of 32 tons. For steering reasons, it is necessary that the weight be distributed equally between the two sides of the freighter. However, the freighter engines and captain's bridge, which together weigh 6 tons, are on the starboard side of the freighter. This means that the port side is usually loaded with 6 tons more cargo to equalize the weight. The owner may carry any combination of the two cargoes in the same hold without any problem. Formulate the above problem to maximize revenue. You are not required to solve the problem.

(7)

(b) A real estate developer is planning to build an office complex. Currently, there are three office sizes under consideration: small, medium, and large. Small offices can be rented for $600 per month, medium offices can be rented for $750 per month, and large offices can be rented for $1000 per month. Each small office requires 600 square feet, each medium office requires 800 square feet, and each large office requires 1000 square feet. The current plot of land available to the developer is 100,000 square feet. The developer wants to ensure that the office complex has at least 3 units of each office size. Moreover, zoning restrictions limit the total number of offices to 50. Formulate the above problem.

The sensitivity report of the problem is provided in the following screenshot.

Sensitivity Report

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
B4	Optimal Values Small	3	0	600	400	1E+30
C4	Optimal Values Medium	3	0	750	250	1E+30
D4	Optimal Values Large	44	0	1000	1E+30	250

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
E8	Square footage	48200	0	100000	1E+30	51800
E9	Minimum no. of small	3	-400	3	41	3
E10	Minimum no. of medium	3	-250	3	41	3
E11	Minimum no. of large	44	0	3	41	1E+30
E12	Total no. of offices	50	1000	50	51.8	41

Use the Sensitivity Report to answer the following questions:

(i) What is the total optimal monthly revenue?

(ii) How much square footage would remain unused if the developer implements the optimal solution?

(iii) What would be the impact on the optimal allocation of offices and the objective function value if small offices can be rented for $800 per month rather than $600 per month?

(iv) What would be the impact on the optimal allocation of offices if medium offices can be rented for $1100.00 rather than $750 per month?

(v) Is the solution to the problem unique or are there alternate optimal solutions?

(2+5 = 7)

5. (a) A manufacturer must produce a certain product in sufficient quantity to meet contracted sales in the next three months. The product may be produced in one month and then held for sale in a later month, but at a storage cost of Rs. 1 per unit per month. No storage cost is incurred for goods sold in the same month in which they are produced. There is presently no inventory of this product and none is desired at the end of four months. The production can be in regular time or using overtime. Regular time cost to produce a unit is Rs. 10 while overtime cost is Rs. 15. You cannot meet the demand of a month by producing in a subsequent month. Regular time capacity is 300 units/month and overtime capacity is 100 units/month. The demand for the three months is 200, 400 and 300 units respectively. Find out the initial basic feasible solution by applying Vogel's Approximation Method.

(7)

(b) Consider the following two LP formulations. Using a graphical approach in each case, determine

     (i)   Which formulation has more than one optimal solution? Find out optimal solution from two corner points.

     (ii)   Which formulation has an unbounded solution? In which direction the solution is unbounded?

Formulation 1	**Formulation 2**
Maximize: 3X + 6Y	Maximize: 2X + 3Y
Subject to	Subject to
7X + 6Y <=42	X + 2Y >= 12
X + 2Y <= 10	8X + 7Y >= 56
X <= 4	2Y >= 5
2Y <=9	X <= 9
X, Y >= 0	X, Y >= 0

                                                           (7)

6. (a) A certain geographical region has got six neighbourhoods that need to be served by fire stations. The number of minutes it takes to travel between the communities is shown in the following table. The region would like to establish minimum number of fire stations so that each region can get a response in five minutes or less. Formulate the above problem so that it is possible to determine the number of stations and also the neighbourhoods served by each station.

	A	B	C	D	E	F
A	-	4	6	3	5	8
B		-	4	10	6	5
C			-	9	3	5
D				-	6	3
E					-	10

                                                           (7)

(b) A Methods Engineer wants to assign four new methods to three new work centres. The assignment of the new methods will increase production which is given below. If only one method can be assigned to a work centre, determine optimal assignment.

Methods	Work Centres with increase in production (units)		
	A	B	C
1	10	7	8
2	8	9	7
3	7	12	6
4	10	10	8

(7)