

FACULTY OF MANAGEMENT STUDIES
UNIVERSITY OF DELHI
MBA Iyr 1st Semester Examination, November 2014
MBAFT 6102: Quantitative methods

Time: 3 hours

Maximum marks: 50

Answer any **FIVE** questions.

- 1.(a) A company has installed five intrusion-detection devices in a downtown branch of a private bank. The devices are very sensitive, on any given night each one has 5% chance of mistakenly sounding an alarm when no intruder is present. If two or more of the devices are activated on the same night, the intrusion system automatically sends a signal to the local police. If no intruder is present on a given night, what is the probability that the system will make the mistake of calling the police station?
- (b) The *ABC cereal company* (ABC) packages and distributes *FruitOCrunch* Cereal. Each box is "guaranteed" to contain 500 grams of cereal. They also sell cereal in "10s-pack," where each pack contains 10 boxes of cereal. The 10-packs are "guaranteed" to have a total weight of 5,000 grams. ABC can choose μ , the mean amount of cereal to put in each of the boxes, but their filling process has some inaccuracy. Regardless of the amount μ that they select (typically between 500 grams and 550 grams), the amount of cereal placed in the box by their filling process is Normally distributed with mean μ and standard deviation 20 grams. (The mean μ is the same for all of the boxes.)
- (i) Suppose that ABC selects $\mu = 510$. What is the expected total weight of the 10 boxes in a randomly selected 10s-pack? What is the standard deviation of the total weight of the 10 boxes in a 10s-pack? What is the distribution of the total weight? (You may assume in this part and in part (ii) below, that the boxes in a 10-pack are selected independently. That is, the weights of boxes in a 10-pack are independent and identically distributed.)
- (ii) Suppose that $\mu = 510$ grams. What is the probability that the total weight of the 10 boxes in a randomly selected 10s-pack is less than 5,000 grams?
- 2.(a) A telephone survey is to be conducted to estimate the proportion of members of a book club who would be inclined to also enroll in a new service specializing in audio recordings of books. The book club would like to estimate this proportion to within plus or minus 5 percentage points at a confidence level of 95%.

(a) What is the size of the sample that would need to be surveyed?

(b) Suppose that there is a 10% chance that a telephone call is unanswered and also assume that the sample size is 500. Let X denote the number of members who answer the telephone call. What is the distribution of X ? (Characterize its relevant parameters like mean and standard deviation).

(b) In a survey to understand the primary source of news for households in a locality, a survey was conducted. The results, presented in the table below, indicates where different age groups primarily get their news:

Media	Age group		
	Under-35	35-50	50+
Local TV	107	119	133
National TV	73	102	127
Radio	75	97	109
Local Newspaper	52	79	107
Internet	95	83	76

At 0.05 level of significance, is there evidence of a significant relationship between the age group and where people primarily get their news? If so, explain the relationship.

3.(a) A retail store is trying to decide the optimal order quantity for brown sugar. During the last seven months demand for brown sugar at the store was as given in the following table.

Demand quantity	10kg	20kg	30kg	40kg	50kg	60kg
Frequency observed	30	40	30	50	30	30

Verify whether the demand for brown sugar is uniformly distributed between 10 and 60 kilograms. Use 0.01 level of significance.

- (b) There are several methods for calculating the fuel economy. The table below shows the mileage as calculated by owner company and by Automobile association standards.

Vehicle	Owner	Automobile association
2012 model Brand A	14.3	16.8
2012 model Brand B	15.0	17.8
2012 model Brand C	27.8	26.2
2012 model Brand D	27.9	34.2
2012 model Brand E	48.8	47.6
2013 model Brand F	16.8	18.3
2013 model Brand G	23.7	28.5
2013 model Brand H	32.8	33.1
2013 model Brand I	37.3	56.0

- (i) Construct a scatter plot
- (ii) Compute the coefficient of correlation
- (iii) Assuming linear relationship, use the least-squares method to determine the regression coefficients.
- (iv) Interpret the meaning of the slope.

- 4.(a) Hong Kong bank of Commerce and Industry is a busy bank that has requirements of 10 to 18 tellers depending on the time of a day. The afternoon time, from 12.00 noon to 3.00 pm is usually the heaviest. The following table indicates the number of workers needed at various hours that the bank is open.

Time period	Number required
9 am – 10 am	10
10 am – 11 am	12
11 am – 12 noon	14
12 noon – 1.00 pm	16
1.00 pm – 2.00 pm	18
2.00 pm – 3.00 pm	17
3.00 pm – 4.00 pm	15
4.00 pm – 5.00 pm	10

The bank employs 12 full-time tellers but has also several part-time employees available in its roster. A part-time employee must put in exactly 4 hours per day but can start anytime between 9.00 am and 1.00 pm. Part-timers are a fairly inexpensive labour pool because no extra benefits are provided to them. Full-timers, on the other hand, work from 9.00 am to 5.00 pm and are quite expensive. The bank's corporate policy limits part-time hours to a maximum of 50% of the day's total requirement. Part-timers earn Rs.70 per hour (or Rs.280 per day) on average while full-timers earn Rs.900 per day in salary and benefits, on average. The bank would like to set a schedule that would minimize its total personnel costs.

Formulate the above problem as an optimization model.

- (b) A manufacturer of travel pillows must determine the production plan for the next production cycle. It wishes to make at least 300 of each of the three models that his firm offers and no more than 1200 of any one model. The specifics of each model are shown in the following table.

Pillow model	Selling price	Required time (in hours)			
		Cutting	Sewing	Finishing	Packing
Junior travel pillow	Rs.575	0.10	0.05	0.18	0.20
Travel pillow	Rs.695	0.15	0.12	0.24	0.20
Deluxe travel pillow	Rs.750	0.20	0.18	0.20	0.20
Available hours		450	550	600	450
Cost per hour		Rs.700	Rs.900	Rs.850	Rs.725

Formulate the above production planning problem as an optimization model.

- 5.(a) An investment company currently has \$1 million available for investment in five different stocks. The company wants to maximize the interest earned over the next year. The five investment possibilities along with the expected interest earned are shown below. To manage risk, the investment firm wishes to have at least 35% of the investment in stocks A and B. Furthermore, no more than 15% of the investment may be in stock E.

Investment Expected Interest Earned (%)

Stock A	7
Stock B	9
Stock C	8
Stock D	10
Stock E	11

An optimization model was solved using a computer program and following is an excerpt from the solution report.

Sensitivity Report

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
\$B\$4	Stock A	0	-0.02	0.07	0.02	1E+30
\$C\$4	Stock B	350000	0	0.09	0.01	0.02
\$D\$4	Stock C	0	-0.02	0.08	0.02	1E+30
\$E\$4	Stock D	500000	0	0.1	0.01	0.01
\$F\$4	Stock E	150000	0	0.11	1E+30	0.01

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
\$G\$7	Constraint 1	1000000	0.1	1000000	1E+30	500000
\$G\$8	Constraint 2	350000	-0.01	350000	500000	350000
\$G\$9	Constraint 3	150000	0.01	150000	500000	150000

Answer the following questions:

- What is the optimal total expected interest earned for next year?
- Which constraints are binding? Which constraints are not binding?
- What would be the impact on the optimal allocation if the expected interest earned on stock A decreases to 6%?
- What would be the impact on the optimal allocation if the expected interest earned on stock A increases to 10%?
- Suppose that total investment in stocks A and B must be at least 40% of the total amount available for investment (i.e., \$400,000). What impact would this have on the current optimal objective function value?

$$4x_1 + 5x_2$$

$$-4x_1 + 2x_2 = 266$$

$$w = 2y_1 + 5y_2$$

$$-y_1 + 2y_2 \geq 66$$

$$y_1 + 3y_2 \geq -22$$

(b) Write the dual for the following primal problem:

$$\text{Maximize } Z = 66 X_1 - 22 X_2$$

Subject to

$$-X_1 + X_2 \leq -2$$

$$2 X_1 + 3 X_2 \leq 5$$

$$X_1, X_2 \geq 0$$

$$A = \left[\begin{array}{cc|c} -1 & 1 & -2 \\ 2 & 3 & 5 \\ \hline 66 & -22 & 0 \end{array} \right]$$

$$A^T = \left[\begin{array}{ccc} -1 & 2 & 66 \\ 1 & 3 & -22 \\ -2 & 5 & 0 \end{array} \right]$$

6.(a) A Houston based investment firm specializes in recommending oil stock portfolios for wealthy clients. One such client has up to \$3 million available for investments and insists on purchasing large blocks of shares of each company in which he invests. The following table details various companies that are under consideration.

Company (Location)	Expected annual return ('000)	Cost per block of shares ('000)
A (Texas)	\$ 50	\$ 480
B (Foreign)	\$ 80	\$540
C (Foreign)	\$ 90	\$ 680
D (Texas)	\$ 120	\$ 1,000
E (Texas)	\$ 110	\$ 700
F (California)	\$ 40	\$ 510
G (California)	\$ 75	\$ 900

The objective is to maximize annual return on investment subject to the following specifications made by the client:

- At least two Texas companies must be in the portfolios.
- No more than one investment can be made in foreign companies.
- Exactly one of the two California companies must be included.
- If stock B is included in the portfolio, then stock A must also be included.

Formulate an optimization model for the problem.

(b) The owner of a new cab company wants to determine locations for cab waiting points. He has gathered the following information:

Location	Can serve areas	Cost per day
1	A,E	Rs.400
2	A,C,D	Rs.500
3	B,C,E	Rs.450
4	B,D	Rs.440
5	D,E	Rs.430

The owner wants to be sure that all areas can be served, yet he wants this done for as little cost as possible. Formulate the above problem as an optimization model