

$$\begin{aligned}
 a_0 &= x \\
 \left. \begin{aligned}
 z_i &= W_{i-1} a_{i-1} + b_{i-1} \\
 a_i &= \sigma(z_i) \\
 c &= \frac{1}{n} \|a_L - y\|_2^2
 \end{aligned} \right\} i = 1, 2, 3, 4, \dots, L
 \end{aligned}$$

Numerator layout ~~input~~ ~~output~~ ~~change~~

$$\frac{\partial c}{\partial a_L} = \frac{2}{n} (a_L - y)^T \quad \} \text{MSE}$$

$$\frac{\partial c}{\partial z_i} = \frac{\partial c}{\partial a_i} \text{diag}(\sigma'(z_i)) \quad \} \text{Element-wise activation}$$

$$\left. \begin{aligned}
 \frac{\partial c}{\partial a_{i-1}} &= \frac{\partial c}{\partial z_i} W_{i-1} \\
 \frac{\partial c}{\partial b_{i-1}} &= \frac{\partial c}{\partial z_i} \\
 \frac{\partial c}{\partial W_{i-1}} &= a_{i-1} \frac{\partial c}{\partial z_i}
 \end{aligned} \right\} \text{Linear Layer}$$

Denominator layout

$$\frac{\partial c}{\partial a_L} = \frac{2}{n} (a_L - y) \quad \} \text{MSE}$$

$$\frac{\partial c}{\partial z_i} = \text{diag}(\sigma'(z_i)) \frac{\partial c}{\partial a_i} \quad \} \text{Element-wise activation}$$

$$\left. \begin{aligned}
 \frac{\partial c}{\partial a_{i-1}} &= W_{i-1}^T \frac{\partial c}{\partial z_i} \\
 \frac{\partial c}{\partial b_{i-1}} &= \frac{\partial c}{\partial z_i} \\
 \frac{\partial c}{\partial W_{i-1}} &= \frac{\partial c}{\partial z_i} a_{i-1}^T
 \end{aligned} \right\} \text{Linear layer}$$