# <2T>

# Binary Search Tree Description

A binary search tree is organized, as the name suggests, in a binary tree. We can represent such a tree by a linked data structure in which each node is an object. In addition to a key and satellite data, each node contains attributes left, right ,and p that point to the nodes corresponding to its left child, its right child, and its parent, respectively. If a child or the parent is missing, the appropriate attribute contains the value NIL. The root node is the only node in the tree whose parent is NIL.

The keys in a binary search tree are always stored in such a way as to satisfy the

binary-search-tree property:

Let x be a node in a binary search tree. If y is a node in the left subtree of x, then y.key <= x.key. If y is a node in the right subtree of x, then y.key => x.key.

Searching

We use the following procedure to search for a node w ith a given key in a binary

search tree. Given a pointer to the root of the tree and a key k, TREE-SEARCH returns a pointer to a node with key k if one exists; otherwise, it returns NIL.

The procedure begins its search at the root and traces a simple path downward in

the tree, as shown in Figure 12.2. For each node x it encounters, it compares the key

k with x.key .If the two keys are equal, the search terminates. If k is smaller than x.key , the search continues in the left subtree of x, since the binary-search-tree property implies that k

could not be stored in the right subtree. Symmetrically, if k is larger than x.key , the search continues in the right subtree. The nodes encountered during the recursion form a simple path downward from the root of the tree, and thus the running time of TREE-SEARCH is O(h)

, where h is the height of the tree.

We can rewrite this procedure in an iterative fashion by “unrolling” the recursion into a

While loop. On most computers, the iterative version is more efficient.

Minimum and maximum

We can always find an element in a binary search tree whose key is a minimum by

Following left child pointers from the root until we encounter a NIL. If a node x has no left subtree, then since every key in the right subtree of x is at least as large as x.key, the minimum key in the subtree rooted at x is x.key. If node x has a left subtree, then since no key in the right subtree is smaller than x.key and every key in the left subtree is not larger than x.key, the minimum key in the subtree rooted at x resides in the subtree rooted at x.left. Similar logic follows for maximum and the right most child.

# Binary Search Tree Pseudo-Code

Inorder-Tree-Walk(x)  
1 Inorder-Tree-Walk(x.left)  
2 print x.key  
3 Inorder-tree-walk(x.right)

Tree-Search(x, k)  
1 if x == NIL or k == x.key  
2 return x  
3 if k < x.key  
4 return Tree-Search(x.left, k)  
5 else return Tree-Search(x.right, k)

Iterative-Tree\_Search(x, k)  
1 while x != NIL and k != x.key  
2 if k < x.key  
3 x = x.left  
4 else x = x.right  
5 return x

Tree-Minimum(x)  
1 while x.left != NIL  
2 x = x.left  
3 return x  
  
  
Tree-Maximum(x)  
1 while x.right != NIL  
2 x = x.right  
3 return x  
  
Tree-Successor(x)  
1 if x.right != NIL  
2 return Tree-Minimum(x.right)  
3 y = x.p  
4 while y != NIL and x == y.right  
5 x = y  
6 y = y.p  
7 return y

Tree-Insert(T, z)  
1 y = NIL  
2 x = T.root  
3 while x != NIL  
4 y = x  
5 if z.key < x.key  
6 x = x.left  
7 else x = x.right  
8 z.p = y  
9 if y == NIL  
10 T.root = z //tree T was empty  
11 elseif z.key < y.key  
12 y.left = z  
13 else y.right = z

Transplant(T, u, v)   
1 if u.p == NIL  
2 T.root = v  
3 elseif u == u.p.left  
4 u.p.left = v  
5 else u.p.right = v  
6 if v != NIL  
7 v.p = u.p

Tree-Delete(T, z)  
1 if z.left == NIL  
2 Transplant(T, z, z.right)  
3 elseif z.right == NIL  
4 Transplant(T, z, z.left(  
5 else y = Tree-minimum(z.right)  
6 if y.p != z  
7 Transplant(T, y, y.right)  
8 y.right = z.right  
9 y.right.p = y  
10 Transplant(T, z, y)  
11 y.left = z.left  
12 y.left.p = y

# Binary Search Tree Code

class TreeNode:

def \_\_init\_\_(self,key,val,left=None,right=None,parent=None):

self.key = key

self.payload = val

self.leftChild = left

self.rightChild = right

self.parent = parent

def hasLeftChild(self):

return self.leftChild

def hasRightChild(self):

return self.rightChild

def isLeftChild(self):

return self.parent and self.parent.leftChild == self

def isRightChild(self):

return self.parent and self.parent.rightChild == self

def isRoot(self):

return not self.parent

def isLeaf(self):

return not (self.rightChild or self.leftChild)

def hasAnyChildren(self):

return self.rightChild or self.leftChild

def hasBothChildren(self):

return self.rightChild and self.leftChild

def replaceNodeData(self,key,value,lc,rc):

self.key = key

self.payload = value

self.leftChild = lc

self.rightChild = rc

if self.hasLeftChild():

self.leftChild.parent = self

if self.hasRightChild():

self.rightChild.parent = self

class BinarySearchTree:

def \_\_init\_\_(self):

self.root = None

self.size = 0

def length(self):

return self.size

def \_\_len\_\_(self):

return self.size

def put(self,key,val):

if self.root:

self.\_put(key,val,self.root)

else:

self.root = TreeNode(key,val)

self.size = self.size + 1

def \_put(self,key,val,currentNode):

if key < currentNode.key:

if currentNode.hasLeftChild():

self.\_put(key,val,currentNode.leftChild)

else:

currentNode.leftChild = TreeNode(key,val,parent=currentNode)

else:

if currentNode.hasRightChild():

self.\_put(key,val,currentNode.rightChild)

else:

currentNode.rightChild = TreeNode(key,val,parent=currentNode)

def \_\_setitem\_\_(self,k,v):

self.put(k,v)

def get(self,key):

if self.root:

res = self.\_get(key,self.root)

if res:

return res.payload

else:

return None

else:

return None

def \_get(self,key,currentNode):

if not currentNode:

return None

elif currentNode.key == key:

return currentNode

elif key < currentNode.key:

return self.\_get(key,currentNode.leftChild)

else:

return self.\_get(key,currentNode.rightChild)

def \_\_getitem\_\_(self,key):

return self.get(key)

def \_\_contains\_\_(self,key):

if self.\_get(key,self.root):

return True

else:

return False

def delete(self,key):

if self.size > 1:

nodeToRemove = self.\_get(key,self.root)

if nodeToRemove:

self.remove(nodeToRemove)

self.size = self.size-1

else:

raise KeyError('Error, key not in tree')

elif self.size == 1 and self.root.key == key:

self.root = None

self.size = self.size - 1

else:

raise KeyError('Error, key not in tree')

def \_\_delitem\_\_(self,key):

self.delete(key)

def spliceOut(self):

if self.isLeaf():

if self.isLeftChild():

self.parent.leftChild = None

else:

self.parent.rightChild = None

elif self.hasAnyChildren():

if self.hasLeftChild():

if self.isLeftChild():

self.parent.leftChild = self.leftChild

else:

self.parent.rightChild = self.leftChild

self.leftChild.parent = self.parent

else:

if self.isLeftChild():

self.parent.leftChild = self.rightChild

else:

self.parent.rightChild = self.rightChild

self.rightChild.parent = self.parent

def findSuccessor(self):

succ = None

if self.hasRightChild():

succ = self.rightChild.findMin()

else:

if self.parent:

if self.isLeftChild():

succ = self.parent

else:

self.parent.rightChild = None

succ = self.parent.findSuccessor()

self.parent.rightChild = self

return succ

def findMin(self):

current = self

while current.hasLeftChild():

current = current.leftChild

return current

def remove(self,currentNode):

if currentNode.isLeaf(): #leaf

if currentNode == currentNode.parent.leftChild:

currentNode.parent.leftChild = None

else:

currentNode.parent.rightChild = None

elif currentNode.hasBothChildren(): #interior

succ = currentNode.findSuccessor()

succ.spliceOut()

currentNode.key = succ.key

currentNode.payload = succ.payload

else: # this node has one child

if currentNode.hasLeftChild():

if currentNode.isLeftChild():

currentNode.leftChild.parent = currentNode.parent

currentNode.parent.leftChild = currentNode.leftChild

elif currentNode.isRightChild():

currentNode.leftChild.parent = currentNode.parent

currentNode.parent.rightChild = currentNode.leftChild

else:

currentNode.replaceNodeData(currentNode.leftChild.key,

currentNode.leftChild.payload,

currentNode.leftChild.leftChild,

currentNode.leftChild.rightChild)

else:

if currentNode.isLeftChild():

currentNode.rightChild.parent = currentNode.parent

currentNode.parent.leftChild = currentNode.rightChild

elif currentNode.isRightChild():

currentNode.rightChild.parent = currentNode.parent

currentNode.parent.rightChild = currentNode.rightChild

else:

currentNode.replaceNodeData(currentNode.rightChild.key,

currentNode.rightChild.payload,

currentNode.rightChild.leftChild,

currentNode.rightChild.rightChild)

mytree = BinarySearchTree()

mytree[3]="red"

mytree[4]="blue"

mytree[6]="yellow"

mytree[2]="at"

print(mytree[6])

print(mytree[2])