

Compressible Fluid Dynamics

Practical 1: The advection equation

Following the structure of the code shown in lectures (or otherwise), implement a numerical solver for the advection equation

$$\frac{\partial u}{\partial t} + a \frac{\partial u}{\partial x} = 0$$

Unless otherwise stated, throughout this practical, chosen values for this equation and its numerical solution are:

- $x \in [0 : 1]$
- $t \in [0 : 1]$
- $a = 1$
- 100 points in the domain, spacing $\Delta x = 1/100$

Two different functions should be considered for initial data:

$$u_0(x) = \sin(2\pi x)$$

or

$$u_0(x) = \begin{cases} 1 & 0.25 \leq x \leq 0.75 \\ 0 & \text{otherwise} \end{cases}$$

Your code should output both data at both the initial and final time of the simulation (at least - produce more output to better visualise what is happening). This data should be plotted and stored in a document. Any plotting software is fine, all my plots are produced in gnuplot, which is freely available and runs well on linux machines. If you are new to plotting software, the command:

```
plot 'advectionResults.dat' with linespoints
```

will plot data (as output in the lecture slides) from the appropriately named file.

Exercises:

1. Test a code with the spatial derivative approximated through forward, backward and centred differences, plot the results and check that you get the expected stability

2. Now set $a = -1$ and repeat the tests for the three derivatives, again checking for expected results.
3. Now try $a = 0.5$ and $a = 2$ (you only need to try this for backward difference). Check again that you get either the expected smearing or lack of convergence. What happens if you try $a = 1.05$?
4. Adjust t and Δt such that you get the same results for these tests as you did for the first? Recalling the physical quantities in the origin of the advection equation, and using dimensional analysis, may help.
5. Now return to $a = 1$, choose an appropriate differencing scheme, and set $\Delta t = 0.9\Delta x$ and investigate the level of diffusion at different final times, for example, consider times 1,2,5 and 10
6. Repeat the previous test, but keep the final time fixed, and vary the number of points used instead, considering e.g. 200, 400, 1000 points - what is happening here, and can you quantify it?