# Appendix

## A Proof of Properties

**Proposition 1 (Task Reduction is a Non-Strict Partial Ordering Relation).** *Suppose that* $\forall\ (\tau_\xi, \tau_\zeta) \in \mathcal{T}^2$, $H_{\xi,\zeta}$ *and* $G_{\zeta,\xi}$ *include the identity and are closed under composition on* $\mathcal{T}$. *Then, task reductions satisfy the following properties and thus define a non-strict partial ordering relation.*

**Property 1.a**. *Reflexivity:* $\tau_1 \preceq \tau_1$.

**Property 1.b**. *Antisymmetry:* $\tau_1 \prec \tau_2 \implies \neg(\tau_2 \preceq \tau_1)$, *where* $\tau_1 \prec \tau_2$ *is defined as* $(\tau_1 \preceq \tau_2) \wedge \neg(\tau_1 \equiv \tau_2)$.

**Property 1.c**. *Transitivity:* $(\tau_1 \preceq \tau_2) \wedge (\tau_2 \preceq \tau_3) \implies \tau_1 \preceq \tau_3$.

*Proof.* **Property 1.a:** $\tau_1 \preceq \tau_1 \implies \exists\ g \in G_{1,1},\ h \in H_{1,1}$ such that

$$g \circ \pi_1^\star \circ h \in \Pi_1^\star. \tag{6}$$

If $g$ and $h$ are the identity function, then $g \circ \pi_1^\star \circ h = \pi_1^\star\ \forall\ \pi_1^\star \in \Pi_1^\star$. Thus, $\tau_1 \preceq \tau_1$ when $G_{1,1}$ and $H_{1,1}$ include their respective identity functions.

**Property 1.b:** Suppose $\tau_1 \prec \tau_2$ and thus $(\tau_1 \preceq \tau_2) \wedge \neg(\tau_1 \equiv \tau_2)$. Note that $\neg(\tau_1 \equiv \tau_2) \implies \neg\big((\tau_1 \preceq \tau_2) \wedge (\tau_2 \preceq \tau_1)\big) \implies \neg(\tau_1 \preceq \tau_2) \vee \neg(\tau_2 \preceq \tau_1)$. We assumed $(\tau_1 \preceq \tau_2)$, so we must have that $\neg(\tau_2 \preceq \tau_1)$.

**Property 1.c:** Suppose $\tau_1 \preceq \tau_2$ and $\tau_2 \preceq \tau_3$. By Definition 4, $\exists\ g_1 \in G_{2,1}$, $g_2 \in G_{3,2}$, $h_1 \in H_{1,2}$, and $h_2 \in H_{2,3}$ such that $g_1 \circ \pi_2^\star \circ h_1 \in \Pi_1^\star\ \forall\ \pi_2^\star \in \Pi_2^\star$ and $g_2 \circ \pi_3^\star \circ h_2 \in \Pi_2^\star\ \forall\ \pi_3^\star \in \Pi_3^\star$. Consider

$$\underbrace{g_1 \circ \overbrace{g_2 \circ \pi_3^\star \circ h_2}^{\in \Pi_2^\star} \circ h_1}_{\in \Pi_1^\star} \tag{7}$$

for all $\pi_3^\star \in \Pi_3^\star$. Let $g_3 := g_1 \circ g_2$ and $h_3 := h_2 \circ h_1$ so that $g_3 \circ \pi_3^\star \circ h_3 \in \Pi_1^\star$ for all $\pi_3^\star \in \Pi_3^\star$. If $G_{3,1}$ and $H_{1,3}$ are closed under composition on $\mathcal{T}$, then $g_3 \in G_{3,1}$ and $h_3 \in H_{1,3}$ and $\tau_1 \preceq \tau_3$. Thus, task reductions are transitive if $G_{3,1}$ and $H_{1,3}$ are closed under composition on $\mathcal{T}$. □

**Proposition 6 (Strict Task Reduction is a Strict Partial Ordering Relation).** *Suppose that* $\forall\ (\tau_\xi, \tau_\zeta) \in \mathcal{T}^2$, $H_{\xi,\zeta}$ *and* $G_{\zeta,\xi}$ *include the identity and are closed under composition on* $\mathcal{T}$. *Then, strict task reductions satisfy the following properties and thus define a strict partial ordering relation.*

**Property 6.a**. *Irreflexivity:* $\neg(\tau_1 \prec \tau_1)$.

**Property 6.b**. *Asymmetry:* $\tau_1 \prec \tau_2 \implies \neg(\tau_2 \prec \tau_1)$.

**Property 6.c**. *Transitivity:* $\tau_1 \prec \tau_2 \wedge \tau_2 \prec \tau_3 \implies \tau_1 \prec \tau_3$.

*Proof.* **Property 6.a:** Suppose $\tau_1 \prec \tau_1 \implies \tau_1 \preceq \tau_1 \wedge \neg(\tau_1 \equiv \tau_1)$. $\tau_1 \equiv \tau_1$ by Property 2.b. $\Rightarrow\!\Leftarrow \implies \neg(\tau_1 \prec \tau_1)$ when $H_{1,1}$ and $G_{1,1}$ include their respective identity functions.

**Property 6.b:** $\tau_1 \prec \tau_2 \implies \neg(\tau_2 \preceq \tau_1)$ by Property 1.b, since $\tau_1 \prec \tau_2 \implies \neg(\tau_1 \equiv \tau_2)$. $\neg(\tau_2 \preceq \tau_1) \iff \neg(\tau_2 \preceq \tau_1) \vee \tau_2 \equiv \tau_1 \implies \neg\big(\tau_2 \preceq \tau_1 \wedge \neg(\tau_2 \equiv \tau_1)\big) \implies \neg(\tau_2 \prec \tau_1)$.

**Property 6.c:** $\tau_1 \prec \tau_2 \wedge \tau_2 \prec \tau_3 \implies \tau_1 \preceq \tau_2 \wedge \tau_2 \preceq \tau_3 \wedge \neg(\tau_1 \equiv \tau_2) \wedge \neg(\tau_2 \equiv \tau_3) \implies \tau_1 \preceq \tau_3 \wedge \neg(\tau_1 \equiv \tau_3)$ by Properties 1.c and 2.c. $\implies \tau_1 \prec \tau_3$ when $H_{1,3}$ and $G_{3,1}$ are closed under composition on $\mathcal{T}$. □

**Proposition 2 (Task Equivalence is an Equivalence Relation).** *Suppose that* $\forall\ (\tau_\xi, \tau_\zeta) \in \mathcal{T}^2$, $H_{\xi,\zeta}$ *and* $G_{\zeta,\xi}$ *include the identity and are closed under composition on* $\mathcal{T}$. *Then, task equivalence satisfies the following properties and thus defines an equivalence relation.*

**Property 2.a**. *Reflexivity:* $\tau_1 \equiv \tau_1$.

**Property 2.b**. *Symmetry:* $\tau_1 \equiv \tau_2 \implies \tau_2 \equiv \tau_1$.

**Property 2.c**. *Transitivity:* $\tau_1 \equiv \tau_2 \wedge \tau_2 \equiv \tau_3 \implies \tau_1 \equiv \tau_3$.

*Proof.* **Property 2.a:** $\tau_1 \equiv \tau_1 \implies \tau_1 \preceq \tau_1$ by Property 1.a when $G_{1,1}$ and $H_{1,1}$ include the identity. Thus, task equivalence is reflexive if $G_{1,1}$ and $H_{1,1}$ include the identity.

**Property 2.b:** $\tau_1 \equiv \tau_2 \implies (\tau_1 \preceq \tau_2) \wedge (\tau_2 \preceq \tau_1)$ by Definition 5 $\implies (\tau_2 \preceq \tau_1) \wedge (\tau_1 \preceq \tau_2) \implies \tau_2 \equiv \tau_1$.

**Property 2.c:** $(\tau_1 \equiv \tau_2) \wedge (\tau_2 \equiv \tau_3) \implies (\tau_1 \preceq \tau_2) \wedge (\tau_2 \preceq \tau_3) \wedge (\tau_3 \preceq \tau_2) \wedge (\tau_2 \preceq \tau_1)$ by Definition 5. $(\tau_3 \preceq \tau_2) \wedge (\tau_2 \preceq \tau_1) \implies (\tau_3 \preceq \tau_1)$ by Property 1.c when $G_{3,1}$ and $H_{1,3}$ are closed under composition on $\mathcal{T}$. Similarly, $(\tau_1 \preceq \tau_2) \wedge (\tau_2 \preceq \tau_3) \implies (\tau_1 \preceq \tau_3)$. Thus $(\tau_1 \preceq \tau_3) \wedge (\tau_3 \preceq \tau_1) \implies \tau_1 \equiv \tau_3$. Thus task equivalence is transitive if $G_{3,1}$ and $H_{1,3}$ are closed under composition on $\mathcal{T}$. □

**Proposition 5 (Properties of the Relative Complexity).** *Relative Complexity satisfies the following properties:*

**Property 5.a**. *Nonnegativity and boundedness:* $C_{\tau_1/\tau_2} \in [0, 1]$.

**Property 5.b**. *Monotonicity with respect to* $H$ *and* $G$*: If* $H \subseteq H'$ *and* $G \subseteq G'$*, then* $C_{\tau_1/\tau_2}(H', G') \preceq C_{\tau_1/\tau_2}(H, G)$.

*Assume that the supremum and infimum in Definition 7 are attained by functions in* $\Pi_2^\star, H, G$. *Then:*

**Property 5.c**. *Equivalence between reduction and* 0 *relative complexity:* $C_{\tau_1/\tau_2} = 0 \iff \tau_1 \preceq \tau_2$.

**Property 5.d**. *Equivalence between no reduction and positive relative complexity:* $C_{\tau_1/\tau_2} \in (0, 1] \iff \neg(\tau_1 \preceq \tau_2)$.

*Proof.* **Property 5.a:** $R_1(g \circ \pi_2^\star \circ h) \in [0, R_1^\star]$. Therefore, $R_1(g \circ \pi_2^\star \circ h)/R_1^\star \in [0, 1] \implies C_{\tau_1/\tau_2} \in [0, 1]$ for any $H, G$.

**Property 5.b:** Consider $H, H'$ such that $H \subseteq H'$ and $G, G'$ such that $G \subseteq G'$. For any function $f$, the following is true $\forall \pi_2^\star$:

$$\inf_{h \in H', g \in G'} f(h, g, \pi_2^\star) \leq \inf_{h \in H, g \in G} f(h, g, \pi_2^\star). \tag{8}$$

This implies the following:

$$\sup_{\pi_2 \in \Pi_2^\star} \inf_{h \in H', g \in G'} \left[1 - \frac{R_1(g \circ \pi_2^\star \circ h)}{R_1^\star}\right] \leq \sup_{\pi_2^\star \in \Pi_2^\star} \inf_{h \in H, g \in G} \left[1 - \frac{R_1(g \circ \pi_2^\star \circ h)}{R_1^\star}\right]. \tag{9}$$

**Property 5.c:** Assume $C_{\tau_1/\tau_2} = 0$ for some $H$ and $G$ $\iff$ for any $\pi_2^\star \in \Pi_2^\star\ \exists\, g \in G$ and $h \in H$ such that $R_1(g \circ \pi_2^\star \circ h) = R_1^\star$. $R_1(\pi_1) = R_1^\star \iff \pi_1 \in \Pi_1^\star$. Thus, for all $\pi_2^\star \in \Pi_2^\star\ \exists\, g \in G$ and $h \in H$ such that $g \circ \pi_2^\star \circ h \in \Pi_1^\star \iff \tau_1 \preceq \tau_2$.

**Property 5.d:** The contrapositive of Property 5.c is $\neg(\tau_1 \preceq \tau_2) \iff C_{\tau_1/\tau_2} \neq 0$. By Property 5.a, the complexity measure is $C_{\tau_1/\tau_2} \in [0, 1]$, therefore, $C_{\tau_1/\tau_2} \in (0, 1] \iff C_{\tau_1/\tau_2} \neq 0$. Thus $C_{\tau_1/\tau_2} \in (0, 1] \iff \neg(\tau_1 \preceq \tau_2)$. □

## B Additional Experimental Details

**Approximating Relative Complexity using Q-learning.** We apply Q-learning to Algorithm 1 by letting the loss functions $L_1$ and $L_2$ correspond to a Q-learning loss: $L_\xi(\pi_\xi) = -\frac{1}{B}\sum_{b=1}^{B}[Q^{\pi_\xi}(s_b, a_b)\log p(a_b)]$, where $p(a_b)$ corresponds to the probability of an action for policy $\pi_\xi$ (which may be a transformation of another policy such as $\pi_\xi = g \circ \pi_\zeta \circ h$), $Q^{\pi_\xi}(s_b, a_b)$ are the Q-values, and $B$ is the batch size. We run Algorithm 1 for 1000 iterations and use a batch size $B$ of 1000 transitions.

**Approximating Relative Complexity using SAC.** We modify Algorithm 1 to use SAC for approximating the relative complexity. Let $Q_2^{\pi_2}$ be a critic of $\pi_2$ on task $\tau_2$ and $Q_1^{g\circ\pi_2\circ h}$ be a critic of $g \circ \pi_2 \circ h$ on task $\tau_1$. We add an additional step to the algorithm for updating the critics on task $\tau_1$ and $\tau_2$. The critics are then used in the updates for the policy $\pi_2$ and the encoder/decoder. The resulting method is presented in Algorithm 2. We run Algorithm 2 for 50,000 iterations and use a batch size of 200 transitions.

**Algorithm 2** Approximating Relative Complexity using SAC

**Input:** Learning rates $\lambda_1, \lambda_2$, adversarial tuning parameter $\alpha$
**Input:** Function spaces $H, G$
**Input:** Q-functions $Q, Q$ loss functions for $\tau_1, \tau_2$ respectively
**Output:** Approximate relative complexity $\tilde{C}_{\tau_1/\tau_2} \approx C_{\tau_1/\tau_2}$
**while** $\neg$(converged $\wedge$ $R_2(\pi_2) = R_2^\star$) **do**
    **Step 0: critic update**
    Update critic $Q_2^{\pi_2}$
    Update critic $Q_1^{g\circ\pi_2\circ h}$
    **Step 1: $\pi_2$ update**
    $\pi_2 \leftarrow \pi_2 + \lambda_1 \nabla_{\pi_2}[Q_2^{\pi_2} - \alpha Q_1^{g\circ\pi_2\circ h}]$
    **Step 2: encoder/decoder update**
    $[h, g] \leftarrow [h, g] + \lambda_2 \nabla_{[h,g]}[Q_1^{g\circ\pi_2\circ h}]$
**end while**
$\tilde{C}_{\tau_1/\tau_2} \leftarrow \left[1 - \frac{R_1(g\circ\pi_2^\star\circ h)}{R_1^\star}\right]$