1.(3) 解. 方程两端同时对 x 求导可得

$$y + x\frac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{e}^{x+y}\frac{\mathrm{d}}{\mathrm{d}x}(x+y) = \mathrm{e}^{x+y}\left(1 + \frac{\mathrm{d}y}{\mathrm{d}x}\right),$$

解 $\dfrac{\mathrm{d}y}{\mathrm{d}x}$, 并化简可得

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{e}^{x+y} - y}{x - \mathrm{e}^{x+y}} = \frac{xy - y}{x - xy}.$$ ■

4.(1) 解. 对函数两端同时求对数可得

$$\ln y = x \ln \frac{x}{1+x}$$

方程两边同时对 x 求导可得

$$\begin{aligned}\frac{1}{y}\frac{\mathrm{d}y}{\mathrm{d}x} &= \ln\frac{x}{1+x} + x \cdot \frac{1}{\frac{x}{1+x}}\left(\frac{x}{1+x}\right)' = \ln\frac{x}{1+x} + (1+x)\left(1 - \frac{1}{1+x}\right)' \\ &= \ln\frac{x}{1+x} + (1+x)\cdot\frac{1}{(1+x)^2} \\ &= \ln\frac{x}{1+x} + \frac{1}{1+x}.\end{aligned}$$

从而

$$\frac{\mathrm{d}y}{\mathrm{d}x} = y\left(\ln\frac{x}{1+x} + \frac{1}{1+x}\right) = \left(\frac{x}{1+x}\right)^x\left(\ln\frac{x}{1+x} + \frac{1}{1+x}\right).$$ ■

5.(2) 解. 计算可得

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\big(\theta\cos\theta\big)'}{\big(\theta(1-\sin\theta)\big)'} = \frac{\cos\theta - \theta\sin\theta}{(1-\sin\theta) + \theta(-\cos\theta)} = \frac{\cos\theta - \theta\sin\theta}{1 - \sin\theta - \theta\cos\theta}.$$ ■