

1.(2) 解. 计算可得

$$\lim_{x \rightarrow 0} \frac{e^x - e^{-x}}{\sin x} = \lim_{x \rightarrow 0} \frac{e^x + e^{-x}}{\cos x} = 2. \quad \blacksquare$$

1.(3) 解一. 计算可得

$$\lim_{x \rightarrow 0} \frac{\tan x - x}{x - \sin x} = \lim_{x \rightarrow 0} \frac{\sec^2 x - 1}{1 - \cos x} = \lim_{x \rightarrow 0} \frac{\tan^2 x}{1 - \cos x} = \lim_{x \rightarrow 0} \frac{x^2}{\frac{1}{2}x^2} = 2. \quad \blacksquare$$

1.(3) 解二. 计算可得

$$\lim_{x \rightarrow 0} \frac{\tan x - x}{x - \sin x} = \lim_{x \rightarrow 0} \frac{\sec^2 x - 1}{1 - \cos x} = \lim_{x \rightarrow 0} \frac{1 + \cos x}{\cos^2 x} = 2. \quad \blacksquare$$

1.(8) 解. 计算可得

$$\lim_{x \rightarrow \frac{\pi}{2}} \frac{\tan x}{\tan 3x} = \lim_{x \rightarrow \frac{\pi}{2}} \left(\frac{\sin x}{\sin 3x} \cdot \frac{\cos 3x}{\cos x} \right) = -1 \cdot \lim_{x \rightarrow \frac{\pi}{2}} \frac{\cos 3x}{\cos x} = -1 \cdot \lim_{x \rightarrow \frac{\pi}{2}} \frac{-3 \sin 3x}{-\sin x} = 3. \quad \blacksquare$$

1.(13) 解. 计算可得

$$\lim_{x \rightarrow 1} \left(\frac{2}{x^2 - 1} - \frac{1}{x - 1} \right) = \lim_{x \rightarrow 1} \frac{2 - (x + 1)}{x^2 - 1} = \lim_{x \rightarrow 1} \frac{1 - x}{x^2 - 1} = \lim_{x \rightarrow 1} \frac{-1}{x + 1} = -\frac{1}{2}. \quad \blacksquare$$

1.(15) 解. 因为

$$\lim_{x \rightarrow 0} (\sin x \ln x) = \lim_{x \rightarrow 0} (x \ln x) = \lim_{x \rightarrow 0} \frac{\ln x}{\frac{1}{x}} = \lim_{x \rightarrow 0} \frac{\frac{1}{x}}{-\frac{1}{x^2}} = \lim_{x \rightarrow 0} (-x) = 0.$$

所以

$$\lim_{x \rightarrow 0} x^{\sin x} = \lim_{x \rightarrow 0} e^{\sin x \ln x} = e^{\lim_{x \rightarrow 0} (\sin x \ln x)} = e^0 = 1. \quad \blacksquare$$