4 解. 计算可得

$$\begin{aligned}\int xe^{-x}\,dx &= -\int x\,de^{-x} = -xe^{-x} + \int e^{-x}\,dx \\ &= -xe^{-x} - e^{-x} + C = -(x+1)e^{-x} + C.\end{aligned}$$

■

5 解. 计算可得

$$\begin{aligned}\int x^2 \ln x\,dx &= \int \ln x \cdot x^2\,dx = \int \ln x\,d\frac{x^3}{3} = \frac{x^3\ln x}{3} - \int \frac{x^3}{3}\,d\ln x \\ &= \frac{x^3\ln x}{3} - \int \frac{x^2}{3}\,dx = \frac{x^3\ln x}{3} - \frac{x^3}{9} + C.\end{aligned}$$

■

8 解. 计算可得

$$\begin{aligned}\int x\cos\frac{x}{2}\,dx &= 2\int x\,d\sin\frac{x}{2} = 2x\sin\frac{x}{2} - 2\int \sin\frac{x}{2}\,dx \\ &= 2x\sin\frac{x}{2} + 4\cos\frac{x}{2} + C.\end{aligned}$$

■

1.(10) 解. 计算可得

$$\begin{aligned}\int x\tan^2 x\,dx &= \int x(\sec^2 x - 1)\,dx = \int x\sec^2 x\,dx - \int x\,dx \\ &= \int x\,d\tan x - \frac{x^2}{2} = x\tan x - \int \tan x\,dx - \frac{x^2}{2} \\ &= x\tan x + \ln|\cos x| - \frac{x^2}{2} + C.\end{aligned}$$

■

1.(12) 解. 计算可得

$$\begin{aligned}\int te^{-2t}\,dt &= -\frac{1}{2}\int t\,de^{-2t} = -\frac{1}{2}\left(te^{-2t} - \int e^{-2t}\,dt\right) \\ &= -\frac{1}{2}\left(te^{-2t} + \frac{1}{2}e^{-2t}\right) + C \\ &= -\frac{(2t+1)e^{-2t}}{4} + C.\end{aligned}$$

■

13 解. 计算可得

$$\begin{aligned}\int \ln^2 x\,dx &= x\ln^2 x - \int x\,d\ln^2 x = x\ln^2 x - 2\int \ln x\,dx \\ &= x\ln^2 x - 2\left(x\ln x - \int x\,d\ln x\right) = x\ln^2 x - 2\left(x\ln x - \int 1\,dx\right) \\ &= x\ln^2 x - 2(x\ln x - x) + C = x\ln^2 x - 2x\ln x + 2x + C.\end{aligned}$$

■

16 解. 计算可得

$$\begin{aligned}
\int x\ln(x-1)\,\mathrm{d}x &= \int \ln(x-1)\cdot x\,\mathrm{d}x = \frac{1}{2}\int \ln(x-1)\,\mathrm{d}(x^2)\\
&= \frac{1}{2}\left(x^2\ln(x-1) - \int x^2\,\mathrm{d}\ln(x-1)\right) = \frac{1}{2}\left(x^2\ln(x-1) - \int \frac{x^2}{x-1}\,\mathrm{d}x\right)\\
&= \frac{1}{2}\left(x^2\ln(x-1) - \int \frac{(x^2-1)+1}{x-1}\,\mathrm{d}x\right)\\
&= \frac{1}{2}\left(x^2\ln(x-1) - \int \left(x+1+\frac{1}{x-1}\right)\mathrm{d}x\right)\\
&= \frac{1}{2}\left(x^2\ln(x-1) - \frac{x^2}{2} - x - \ln(x-1)\right) + C\\
&= \frac{1}{2}\left((x^2-1)\ln(x-1) - \frac{x^2}{2} - x\right) + C.
\end{aligned}$$

■