

1.(1) 解. 当  $\omega \neq 0$  时, 计算可得

$$\lim_{x \rightarrow 0} \frac{\sin \omega x}{x} = \lim_{x \rightarrow 0} \omega \frac{\sin \omega x}{\omega x} = \omega \lim_{x \rightarrow 0} \frac{\sin \omega x}{\omega x} = \omega.$$

当  $\omega = 0$  时

$$\lim_{x \rightarrow 0} \frac{\sin \omega x}{x} = \lim_{x \rightarrow 0} \frac{\sin(0 \cdot x)}{x} = \lim_{x \rightarrow 0} \frac{0}{x} = \lim_{x \rightarrow 0} 0 = 0 = \omega.$$

综合可得  $\lim_{x \rightarrow 0} \frac{\sin \omega x}{x} = \omega$ . ■

⊙ 注意计算过程要分  $\omega = 0$  和  $\omega \neq 0$  两种情况。

⊙ 极限计算过程中不能随便把极限运算符  $\lim_{x \rightarrow 0}$  去掉。

1.(3) 解. 计算可得

$$\lim_{x \rightarrow 0} \frac{\sin 2x}{\sin 5x} = \lim_{x \rightarrow 0} \frac{2 \cdot \frac{\sin 2x}{2x}}{5 \cdot \frac{\sin 5x}{5x}} = \frac{2}{5} \frac{\lim_{x \rightarrow 0} \frac{\sin 2x}{2x}}{\lim_{x \rightarrow 0} \frac{\sin 5x}{5x}} = \frac{2}{5}. \quad \blacksquare$$

⊙ 没有  $\sin ax = a \sin x$  这样的公式。

1.(5) 解. 计算可得

$$\lim_{x \rightarrow 0} \frac{1 - \cos 2x}{x \sin x} = \lim_{x \rightarrow 0} \frac{\frac{1 - \cos 2x}{x^2}}{\frac{\sin x}{x}} = \frac{\lim_{x \rightarrow 0} \frac{1 - \cos 2x}{x^2}}{\lim_{x \rightarrow 0} \frac{\sin x}{x}} = \frac{1}{2}. \quad \blacksquare$$

⊙ 符号  $\sin x^2$  的意思是  $\sin(x^2)$ , 符号  $\sin^2 x$  的意思是  $(\sin x)^2$ .

2.(1) 解. 计算可得

$$\lim_{x \rightarrow 0} (1 - x)^{\frac{1}{x}} = \lim_{x \rightarrow 0} \left( (1 + (-x))^{\frac{1}{-x}} \right)^{-1} = \left( \lim_{x \rightarrow 0} (1 + (-x))^{\frac{1}{-x}} \right)^{-1} = e^{-1}. \quad \blacksquare$$

2.(3) 解. 计算可得

$$\lim_{x \rightarrow \infty} \left( \frac{1+x}{x} \right)^{2x} = \lim_{x \rightarrow \infty} \left( 1 + \frac{1}{x} \right)^{2x} = \lim_{x \rightarrow \infty} \left( \left( 1 + \frac{1}{x} \right)^x \right)^2 = \left( \lim_{x \rightarrow \infty} \left( 1 + \frac{1}{x} \right)^x \right)^2 = e^2. \quad \blacksquare$$