AVALIAÇÃO 01 - PRE - WAGNER SANTOS

1)

a) $u \to 1,\ X = -5,\ P_1(X=-5) = 1/6,\ F_x(x) = \frac{1}{6}\delta(x+5)$

$u \to 2 \text{ ou } 3,\ X \sim \text{UNIF}([-8,2]) = 1/3,\ F_x(x) = 1/10$

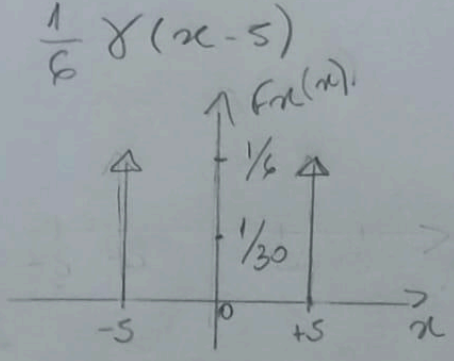
$u \to 4 \text{ ou } 5,\ X \sim \text{UNIF}([-2,8]) = 1/3,\ F_x(x) = 1/10$

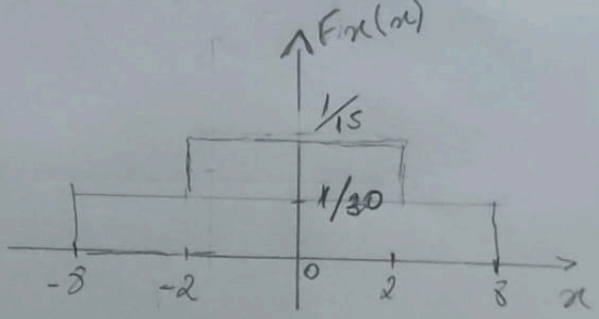
$u \to 6,\ X = 5,\ P_1(X=5) = 1/6,\ F_x(x) = \frac{1}{6}\delta(x-5)$

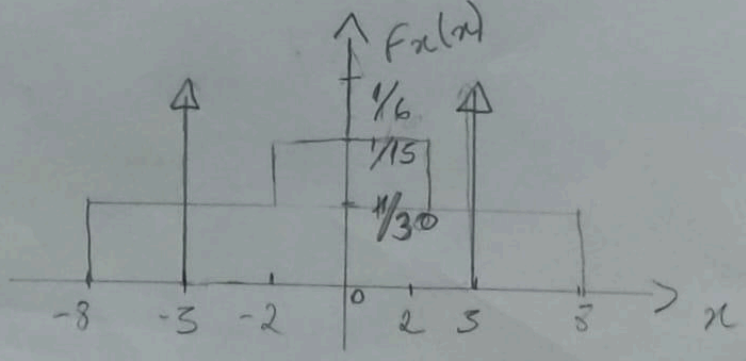
...

$$F_x(x) = \frac{1}{6}\gamma(x+5) + \frac{1}{10}\cdot\frac{1}{3}[-8 \le x \le 2] + \frac{1}{10}\cdot\frac{1}{3}[-2 \le x \le 8] + \frac{1}{6}\gamma(x-5)$$

$$F_x(x) = \frac{1}{6}\gamma(x+5) + \frac{1}{30}[-8 \le x \le 2] + \frac{1}{30}[-2 \le x \le 8] + \frac{1}{6}\gamma(x-5)$$







b)

caso $x < -8$

$$F_X(x) = \int_{-\infty}^{x} 0 \, du = 0 \;/$$

caso $-8 \leq x < -5$

$$F_X(x) = \int_{-8}^{x} 1/30 \, du = 1/30 \int_{-8}^{x} 1 \, du = \frac{1}{30} \Big|_{-8}^{x} x - (-8)$$

$$F_X(x) = \frac{1}{30} \cdot (x + 8) = \frac{x+8}{30} \;/$$

caso $x = -5$

$$F_X(x) = \int_{(-5)^-}^{(-5)^+} 1/6 \, \delta(u - 5) \, du = 1/6$$

caso $-5 < x \leq 2$

$$F_X(x) = \int_{(-5)^+}^{x} 1/30 \, du = 1/30 \int_{-5}^{x} 1 \, du = \frac{1}{30} \Big|_{-5}^{x} x - (-5)$$

$$F_X(x) = \frac{1}{30} \cdot (x + 5) = \frac{x+5}{30}$$

$$F_X(x) = \int \delta(x + 5) = 1/6$$

caso $-2 \leq x < 2$

$$F_x(x) = \int_{-2}^{x} 1/15 \, du = \frac{1}{15} \Big|_{-2}^{x} 1 \, du = \frac{1}{15} \Big|_{-2}^{x} x-(-2)$$

$$F_x(x) = \frac{1}{15} \cdot (x+2) = \frac{x+2}{15}$$

caso $2 \leq x < 5$

$$F_x(x) = \int_{2}^{x} \frac{1}{30} \, du = \frac{1}{30} \Big|_{2}^{x} 1 \, du = \frac{1}{30} \Big|_{2}^{x} x-2$$

$$F_x(x) = \frac{1}{30} \cdot (x-2) = \frac{x-2}{30}$$

caso $x = 5$

$$F_x(x) = \int_{5}^{5} \frac{1}{6} \gamma(x+5) \, du = \frac{1}{6}$$

caso $5 < x < 8$

$$F_x(x) = \int_{5}^{x} \frac{1}{30} \, du = \frac{1}{30} \Big|_{5}^{x} du = \frac{1}{30} \Big|_{5}^{x} x-5$$

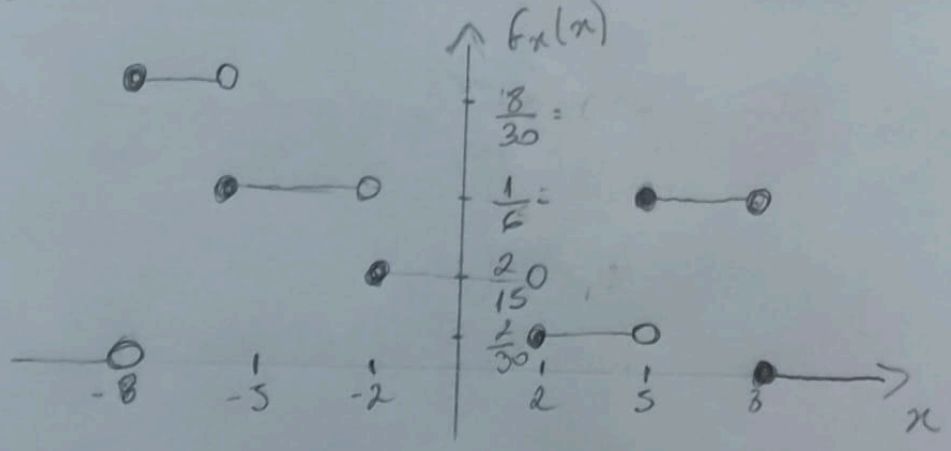
$$F_x(x) = \frac{1}{30} \cdot (x-5) = \frac{x-5}{30}$$

caso $x \geq 8$

$$F_x(x) = \int_8^x 0 \, du = \emptyset$$

$$F_x(x) = \begin{cases} 0, & x < -8 \\ \frac{x+8}{30}, & -8 \leq x < -5 \\ \frac{1}{6}, & x = -5 \\ \frac{x+5}{30}, & -5 < x < -2 \\ \frac{x+2}{15}, & -2 \leq x < 2 \\ \frac{x-2}{30}, & 2 \leq x < 5 \\ \frac{1}{6}, & x = 5 \\ \frac{x-5}{30}, & 5 < x < 8 \\ 0, & x \geq 8 \end{cases}$$

$$F_x(x) = \begin{cases} 0, & x < -8 \\ \frac{x+8}{30}, & -8 \le x < -5 \\ \frac{1}{6} + \frac{x+5}{30}, & -5 \le x < -2 \\ \frac{x+2}{13}, & -2 \le x < 2 \\ \frac{x-2}{30}, & 2 \le x < 5 \\ \frac{1}{6} + \frac{x-5}{30}, & 5 \le x < 8 \\ 0, & x \ge 8 \end{cases}$$



c) $E[X] = \int_{-8}^{-5} x\,\frac{1}{30}\,dx + \int_{-5}^{-2} x\,\frac{1}{6}\,\gamma(x-5)\,dx + \int_{-2}^{2} x\,\frac{1}{30}\,dx$

$+ \int_{-2}^{2} x\,\frac{1}{15}\,dx + \int_{2}^{5} x\,\frac{1}{30}\,dx + \int_{5}^{5} \frac{1}{6}\,\gamma(x+5)$

$+ \int_{5}^{8} x\,\frac{1}{30}\,dx$

$E[X] = \frac{1}{30}\left[\frac{x^2}{2}\right]_{-8}^{-5} + 0 + \frac{1}{30}\left[\frac{x^2}{2}\right]_{-5}^{-2} + \frac{1}{15}\left[\frac{x^2}{2}\right]_{-2}^{2} +$

$+ \frac{1}{30}\left[\frac{x^2}{2}\right]_{2}^{5} + 0 + \frac{1}{30}\left[\frac{x^2}{2}\right]_{5}^{8}$

$E[x] = \left(-\frac{13}{20}\right) + 0 + \left(-\frac{7}{20}\right) + 0 + \frac{7}{20} + \frac{13}{20} = \emptyset$

d) $P_1[5 \leq x \leq 10]$

TÁ SOBEMOS QUE:

$P_1(X = 5) = 1/6$

$P_1(5 < X < 8) = 1/10$

FORA DO INTERVALO É $\emptyset$

ENTÃO $\frac{1}{6} + \frac{1}{10} = \frac{5+3}{30} = \frac{8}{30} = \frac{4}{15}$