

PRE029006 - PROCESSOS ESTOCÁSTICOS (2024 .2 - T01)

Avaliação 6

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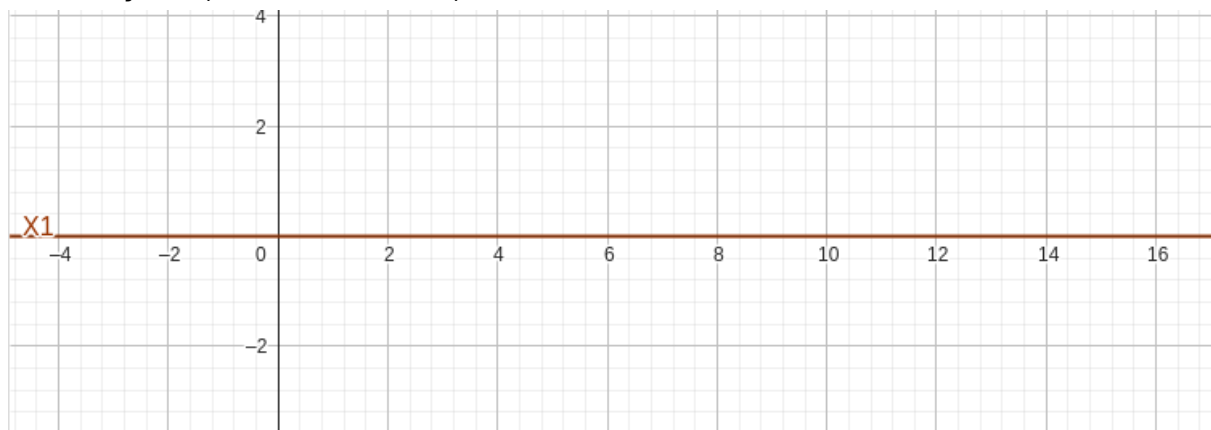
3. Sejam $A, B, C \sim \text{iid Bernoulli}(1/4)$. Considere o processo estocástico $X(t)$ definido por

$$X(t) = A \text{rect}(t - 2/4) + B \text{rect}(t - 4/8) + C \text{rect}(t - 6/12).$$

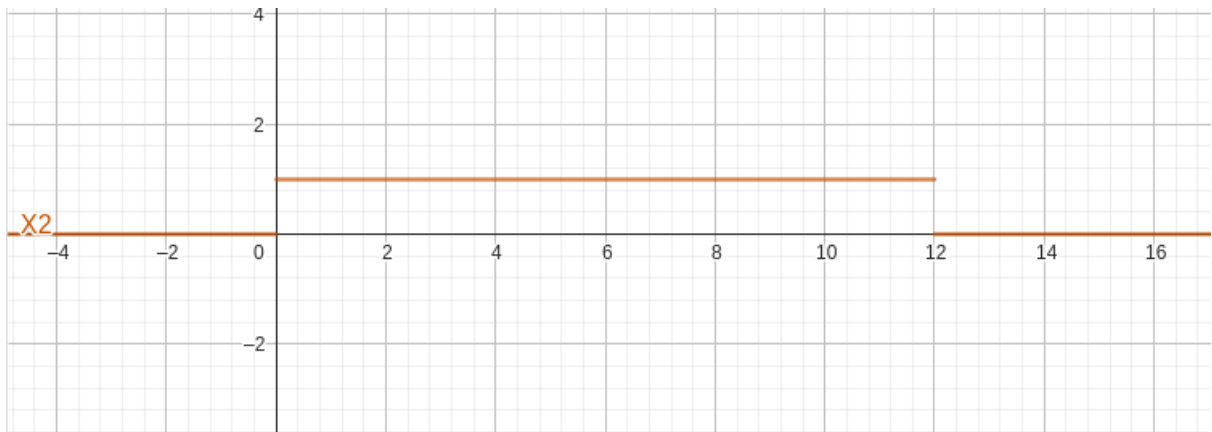
(a) Determine e esboce todas as possíveis funções-amostra de $X(t)$.

Combinação	$X(t)$	A	B	C
1	0	0	0	0
2	$\text{rect}((t-6)/12)$	0	0	1
3	$\text{rect}((t-4)/8)$	0	1	0
4	$\text{rect}((t-4)/8) + \text{rect}((t-6)/12)$	0	1	1
5	$\text{rect}((t-2)/4)$	1	0	0
6	$\text{rect}((t-2)/4) + \text{rect}((t-6)/12)$	1	0	1
7	$\text{rect}((t-2)/4) + \text{rect}((t-4)/8)$	1	1	0
8	$\text{rect}((t-2)/4) + \text{rect}((t-4)/8) + \text{rect}((t-6)/12)$	1	1	1

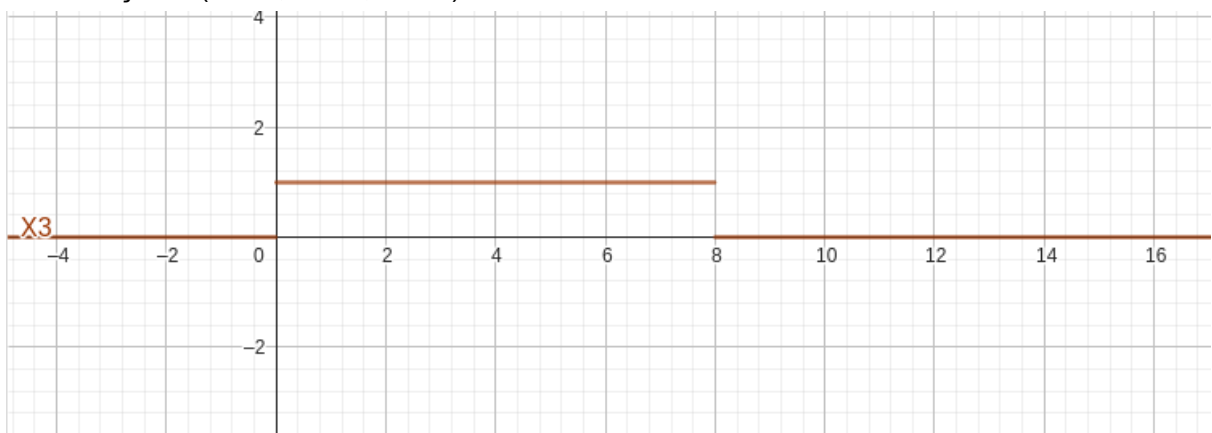
Combinação 1 ($A = 0, B = 0, C = 0$):



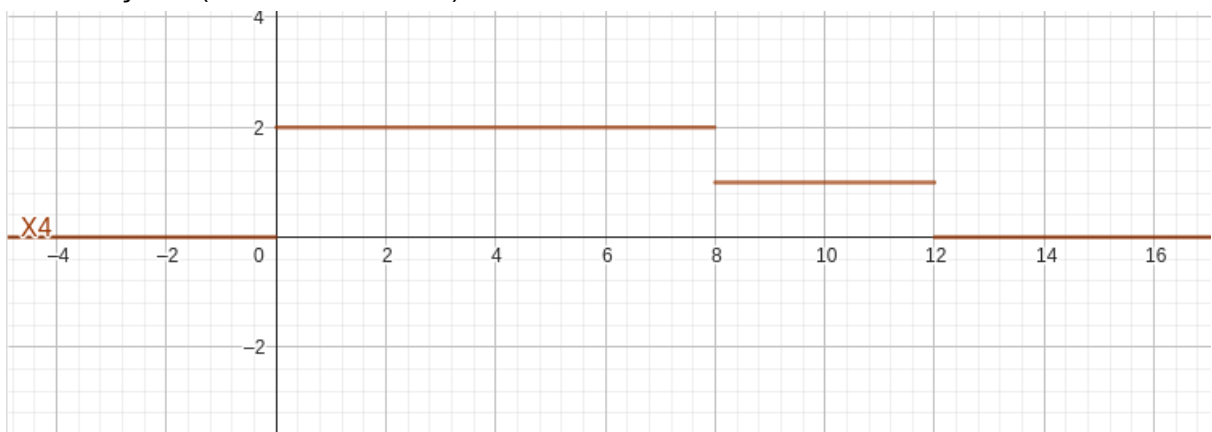
Combinação 2 ($A = 0, B = 0, C = 1$):



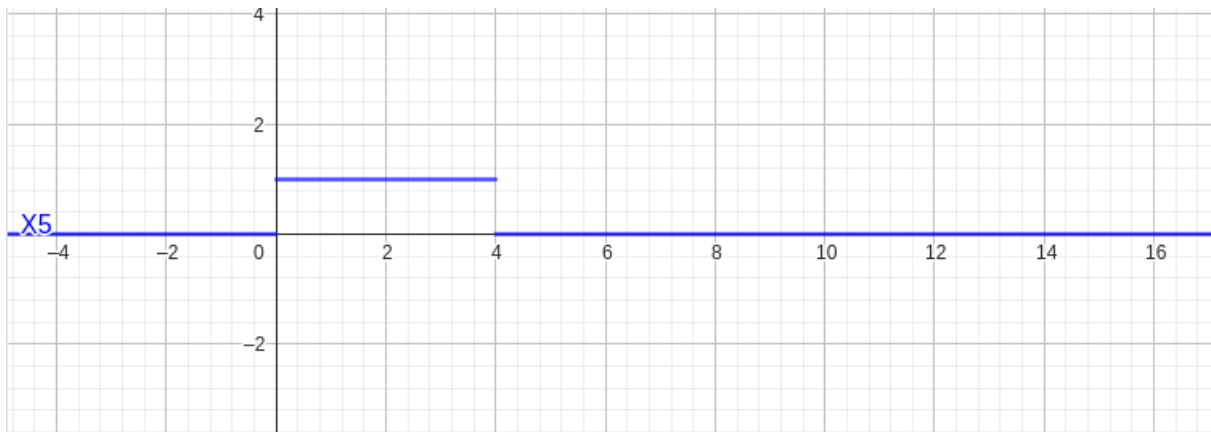
Combinação 3 ($A = 0$, $B = 1$, $C = 0$):



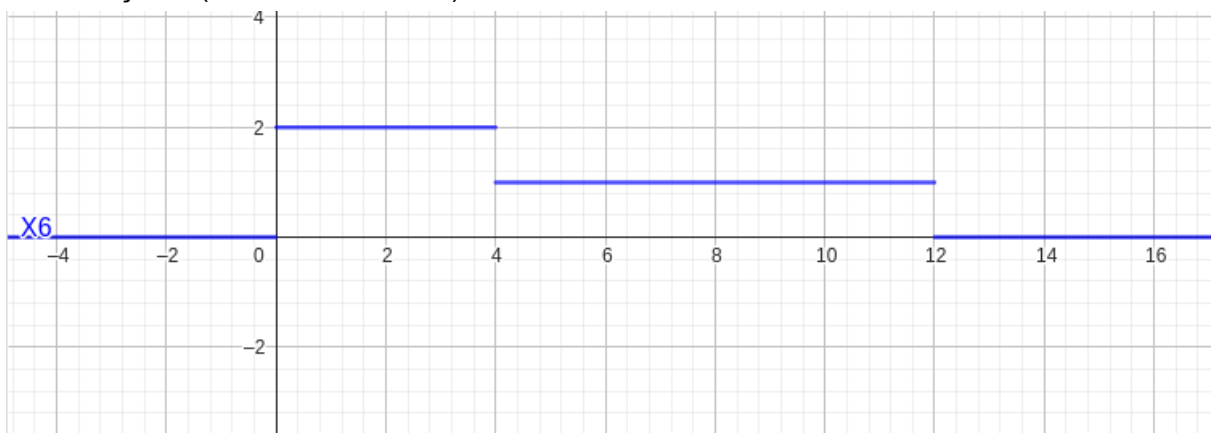
Combinação 4 ($A = 0$, $B = 1$, $C = 1$):



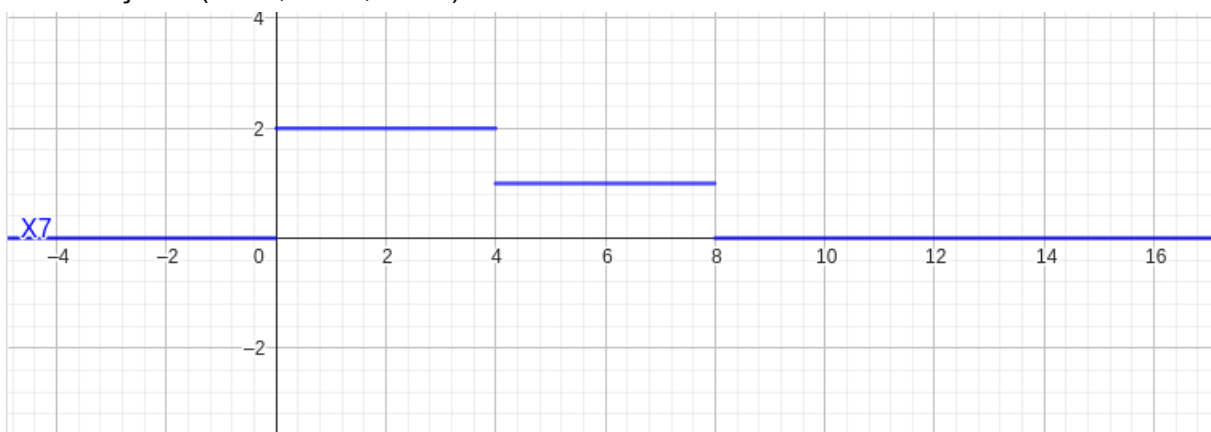
Combinação 5 ($A = 1$, $B = 0$, $C = 0$):



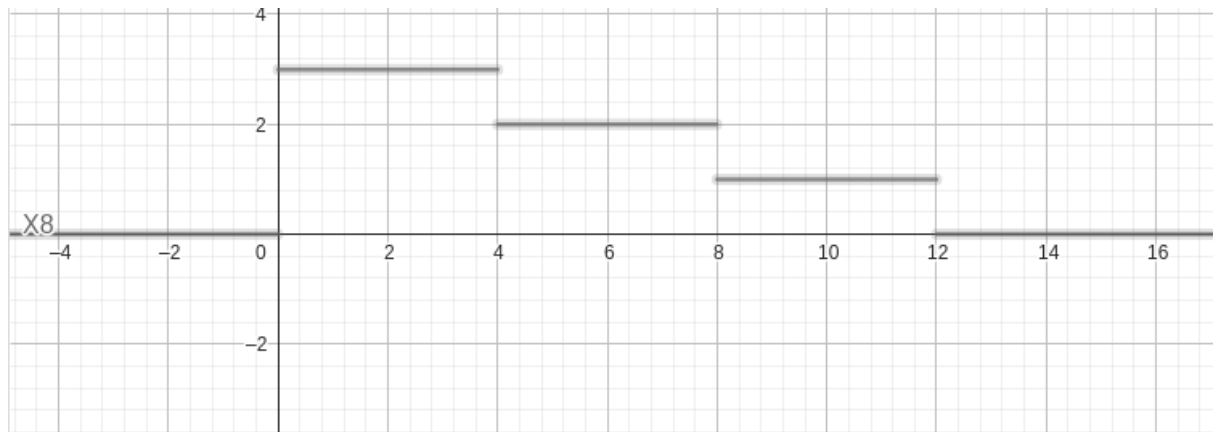
Combinação 6 ($A = 1, B = 0, C = 1$):



Combinação 7 ($A = 1, B = 1, C = 0$):



Combinação 8 ($A = 1, B = 1, C = 1$):



Fonte: <https://www.geogebra.org/classic/yfxgwvxp>

(b) Determine e esboce a função densidade de probabilidade de primeira ordem de $X(t)$.
(Deve haver um esboço para cada intervalo de tempo relevante.)

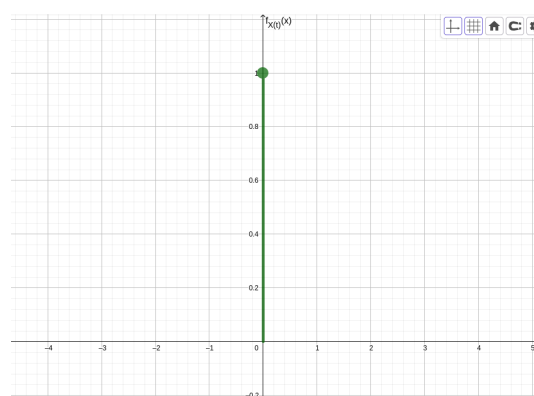
$$P(A=1) = P(B=1) = P(C=1) = 1/4$$

$$P(A=0) = P(B=0) = P(C=0) = 3/4$$

Caso $t < 0$:	$x(t) = 0$
Caso $0 < t < 4$;	$x(t) = A + B + C$
Caso $4 < t < 8$	$x(t) = B + C$
Caso $8 < t < 12$	$x(t) = C$
Caso $t > 12$	$x(t) = 0$

Caso $t < 0$ & $t > 12$: $x(t) = 0$

$PX(t)(x) = \delta[x]$



Caso $0 < t < 4$; $x(t) = A + B + C$

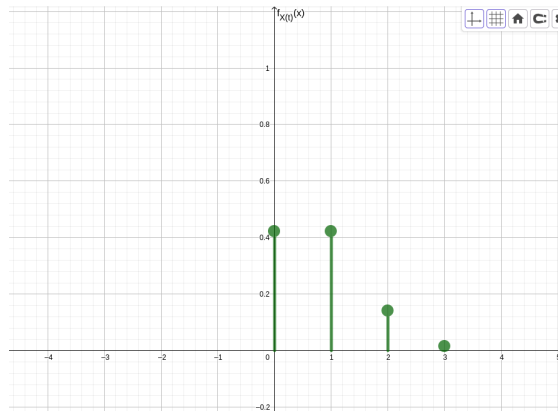
Valores possíveis de $X(t)$:

0: $[0 \ 0 \ 0] = \frac{3}{4} \cdot \frac{3}{4} \cdot \frac{3}{4} = 27/64$

1: $[1 \ 0 \ 0] = \frac{1}{4} \cdot \frac{3}{4} \cdot \frac{3}{4} = 27/64$

2: $[1 \ 1 \ 0] = \frac{1}{4} \cdot \frac{1}{4} \cdot \frac{3}{4} = 9/64$

3: $[1 \ 1 \ 1] = \frac{1}{4} \cdot \frac{1}{4} \cdot \frac{1}{4} = 1/64$



$$P_X(t)(x) = 27/64 \cdot \delta[x] + 27/64 \cdot \delta[x - 1] + 9/64 \cdot \delta[x - 2] + 1/64 \cdot \delta[x - 3]$$

Caso $4 < t < 8$ $x(t) = B + C$

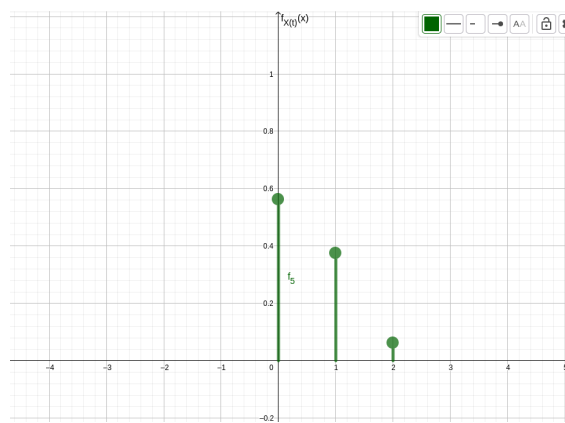
Valores possíveis de $X(t)$:

0: $[0 \ 0] = \frac{3}{4} \cdot \frac{3}{4} = 9/16$

1: $[1 \ 0]$ ou $[0 \ 1] = 2 \cdot \frac{1}{4} \cdot \frac{3}{4} = 6/16$

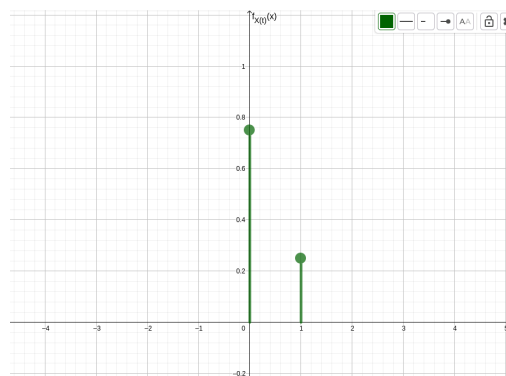
2: $[1 \ 1] = \frac{1}{4} \cdot \frac{1}{4} = 1/16$

$$P_X(t)(x) = 9/16 \cdot \delta[x] + 6/16 \cdot \delta[x - 1] + 1/16 \cdot \delta[x - 2]$$



Caso $8 < t < 12$ $x(t) = C$

$$P_X(t)(x) = 3/4 \cdot \delta[x] + 1/4 \cdot \delta[x - 1]$$



Caso $t < 0$:	$x(t) = 0$	$PX(t)(x) = \delta[x]$
Caso $0 < t < 4$;	$x(t) = A + B + C$	$PX(t)(x) = 27/64 \cdot \delta[x] + 27/64 \cdot \delta[x - 1] + 9/64 \cdot \delta[x - 2] + 1/64 \cdot \delta[x - 3]$
Caso $4 < t < 8$	$x(t) = B + C$	$PX(t)(x) = 9/16 \cdot \delta[x] + 6/16 \cdot \delta[x - 1] + 1/16 \cdot \delta[x - 2]$
Caso $8 < t < 12$	$x(t) = C$	$PX(t)(x) = 3/4 \cdot \delta[x] + 1/4 \cdot \delta[x - 1]$
Caso $t > 12$,	$x(t) = 0$	$PX(t)(x) = \delta[x]$

(c) Determine e esboce a função média de $X(t)$.

Caso $t < 0$ e $t > 12$:

$uX(t) = 0$, pois $X(t) = 0$

Caso $0 < t < 4$:

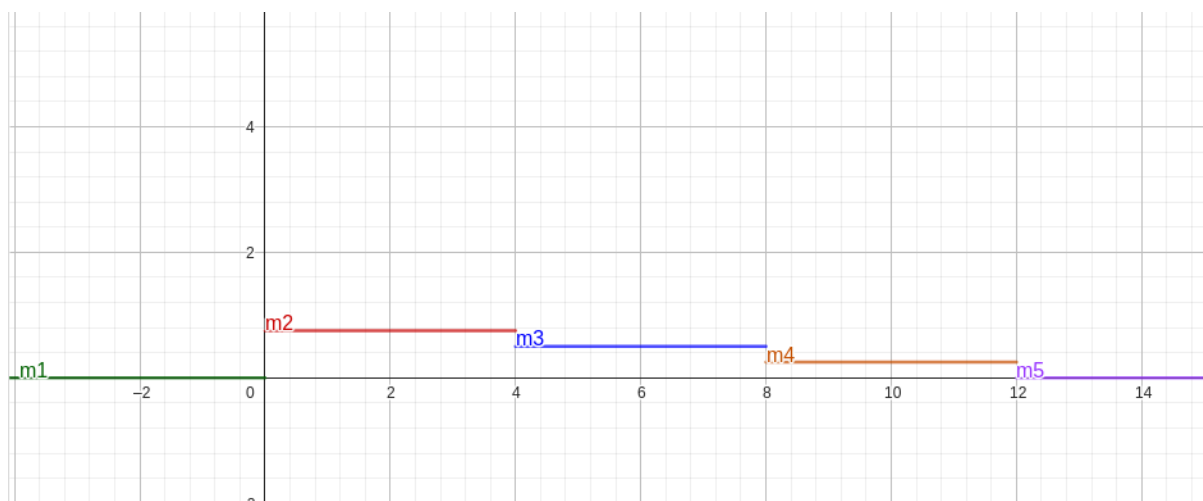
$uX(t) = 0 \cdot 27/64 + 1 \cdot 27/64 + 2 \cdot 9/64 + 3 \cdot 1/64 = 24/64 + 18/64 + 3/64 = 45/64 = 3/4$

Caso $4 < t < 8$:

$uX(t) = 0 \cdot 9/16 + 1 \cdot 6/16 + 2 \cdot 1/16 = 0 + 6/16 + 2/16 = 1/2$

Caso $8 < t < 12$:

$uX(t) = 0 \cdot 3/4 + 1 \cdot 1/4 = 0 + 1/4 = 1/4$



Fonte: <https://www.geogebra.org/classic/rv6npdrj>

(d) Determine a função densidade de probabilidade de segunda ordem de $X(t)$, considerando apenas valores t_1 e t_2 satisfazendo de $0 < t_1 < 4$ e $-\infty < t_2 < \infty$. (Não é necessário esboçar.)

Caso $0 < t_1 < 4$ e $0 < t_2 < 4$, $x(t) = A + B + C$

Valores possíveis de $X(t)$:

0: $[0 \ 0 \ 0] = \frac{3}{4} \cdot \frac{3}{4} \cdot \frac{3}{4} = 27/64$

1: $[1 \ 0 \ 0] = \frac{1}{4} \cdot \frac{3}{4} \cdot \frac{3}{4} = 27/64$

2: $[1 \ 1 \ 0] = \frac{1}{4} \cdot \frac{1}{4} \cdot \frac{3}{4} = 9/64$

3: $[1 \ 1 \ 1] = \frac{1}{4} \cdot \frac{1}{4} \cdot \frac{1}{4} = 1/64$

t_1, t_2	$x_2 = 0$	$x_2 = 1$	$x_2 = 2$	$x_2 = 3$
$x_1 = 0$	$(27/64) \cdot (27/64) = 729/4096$	$(27/64) \cdot (27/64) = 729/4096$	$(27/64) \cdot (9/64) = 243/4096$	$(27/64) \cdot (1/64) = 27/4096$
$x_1 = 1$	$(27/64) \cdot (27/64) = 729/4096$	$(27/64) \cdot (27/64) = 729/4096$	$(27/64) \cdot (9/64) = 243/4096$	$(27/64) \cdot (1/64) = 27/4096$
$x_1 = 2$	$(9/64) \cdot (27/64) = 243/4096$	$(9/64) \cdot (27/64) = 243/4096$	$(9/64) \cdot (9/64) = 81/4096$	$(9/64) \cdot (1/64) = 9/4096$
$x_1 = 3$	$(1/64) \cdot (27/64) = 27/4096$	$(1/64) \cdot (27/64) = 27/4096$	$(1/64) \cdot (9/64) = 9/4096$	$(1/64) \cdot (1/64) = 1/4096$

Soma = $4096/4096 = 1$, [Resultado WolframAlfa](#)

Caso $4 < t_1 < 8$ e $0 < t_2 < 4$, $x(t_1) = B + C$ e $x(t_2) = A + B + C$

Valores possíveis de $X(t_1)$:

0: $[0 \ 0] = \frac{3}{4} \cdot \frac{3}{4} = 9/16$

1: $[1 \ 0]$ ou $[0 \ 1] = 2 \cdot \frac{1}{4} \cdot \frac{3}{4} = 6/16$

2: $[1 \ 1] = \frac{1}{4} \cdot \frac{1}{4} = 1/16$

Valores possíveis de $X(t_2)$:

0: $[0 \ 0 \ 0] = \frac{3}{4} \cdot \frac{3}{4} \cdot \frac{3}{4} = 27/64$

1: $[1 \ 0 \ 0] = \frac{1}{4} \cdot \frac{3}{4} \cdot \frac{3}{4} = 27/64$

2: $[1 \ 1 \ 0] = \frac{1}{4} \cdot \frac{1}{4} \cdot \frac{3}{4} = 9/64$

3: $[1 \ 1 \ 1] = \frac{1}{4} \cdot \frac{1}{4} \cdot \frac{1}{4} = 1/64$

t_1, t_2	$x_2 = 0$	$x_2 = 1$	$x_2 = 2$	$x_2 = 3$
$x_1 = 0$	$(9/16) \cdot (27/64) = 243/1024$	$(9/16) \cdot (27/64) = 243/1024$	$(9/16) \cdot (9/64) = 81/1024$	$(9/16) \cdot (1/64) = 9/1024$
$x_1 = 1$	$(6/16) \cdot (27/64) = 162/1024$	$(6/16) \cdot (27/64) = 162/1024$	$(6/16) \cdot (9/64) = 54/1024$	$(6/16) \cdot (1/64) = 6/1024$
$x_1 = 2$	$(1/16) \cdot (27/64) = 27/1024$	$(1/16) \cdot (27/64) = 27/1024$	$(1/16) \cdot (9/64) = 9/1024$	$(1/16) \cdot (1/64) = 1/1024$

$x_1 = 3$	0	0	0	0
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Soma = $1024/1024 = 1$, [Resultado WolframAlpha](#)

Caso $8 < t_1 < 12$ e $0 < t_2 < 4$, $x(t_1) = C$ e $X(t_2) = A + B + C$

Valores possíveis de $X(t_1)$:

0: $[0 \ 0] = \frac{3}{4}$

1: $[1 \ 0]$ ou $[0 \ 1] = \frac{1}{4}$

Valores possíveis de $X(t_2)$:

0: $[0 \ 0 \ 0] = \frac{3}{4} \cdot \frac{3}{4} \cdot \frac{3}{4} = 27/64$

1: $[1 \ 0 \ 0] = \frac{1}{4} \cdot \frac{3}{4} \cdot \frac{3}{4} = 27/64$

2: $[1 \ 1 \ 0] = \frac{1}{4} \cdot \frac{1}{4} \cdot \frac{3}{4} = 9/64$

3: $[1 \ 1 \ 1] = \frac{1}{4} \cdot \frac{1}{4} \cdot \frac{1}{4} = 1/64$

t_1, t_2	$x_2 = 0$	$x_2 = 1$	$x_2 = 2$	$x_2 = 3$
$x_1 = 0$	$(3/4) \cdot (27/64) = 81/256$	$(3/4) \cdot (27/64) = 81/256$	$(3/4) \cdot (9/64) = 27/256$	$(3/4) \cdot (1/64) = 3/256$
$x_1 = 1$	$(1/4) \cdot (27/64) = 27/256$	$(1/4) \cdot (27/64) = 27/256$	$(1/4) \cdot (9/64) = 9/256$	$(1/4) \cdot (1/64) = 1/256$
$x_1 = 2$	0	0	0	0
$x_1 = 3$	0	0	0	0

Soma = $256/256 = 1$, [Resultado WolframAlpha](#)

Caso $t_1 > 12$ e $t_2 > 12$, $x(t) = 0$

$P(X(t_1)=0, X(t_2)=0) = 1$

todas as outras = 0

t_1, t_2	$x_2 = 0$	$x_2 = 1$	$x_2 = 2$	$x_2 = 3$
$x_1 = 0$	1	0	0	0
$x_1 = 1$	0	0	0	0
$x_1 = 2$	0	0	0	0
$x_1 = 3$	0	0	0	0

Soma = 1