1 Equações de Navier-Stokes Aplicadas em Escoamentos Monofásicos

1.1 Descrição do Problema

Escrever Texto sobre o problema.

Serão utilizadas as seguintes hipóteses:

1. Escoamento monofásico;
2. Fluido monocomponente;
3. Não há trocas de calor entre o meio e a tubulação;
4. Escoamento isotérmico;
5. Sem atrito com as paredes do tubo (Equação de Euler);

1.2 Equações Governantes

A equação de Navier-Stokes unidimensional pode ser escrita como:

$$\frac{\partial}{\partial t}\left(\rho\right) + \nabla\cdot\left(\rho\vec{v}\right) = S_m \tag{1.1}$$

$$\frac{\partial}{\partial t}\left(\rho\vec{v}\right) + \nabla\cdot\left(\rho\vec{v}\vec{v}\right) = -\nabla p + \rho g sen\left(\theta\right) \tag{1.2}$$

A Eq. 1.1 representa a equação da conservação da massa sendo uma equação puramente convectiva. A eq. 1.2 é a conservação da quantidade de movimento sendo

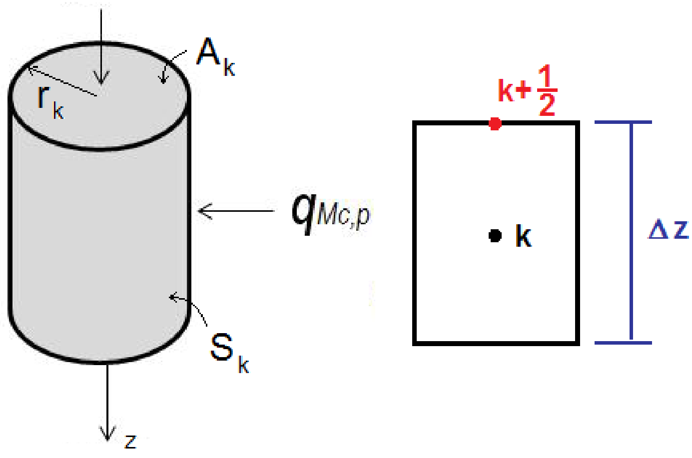

Figura 1: Volume de Controle Unidimensional.

uma equação difusiva e convectiva. Observa-se dois fatos que tornam a solução numérica desse conjunto de equações dificil:

1. A Eq. da quantidade de movimento é não linear pois apresenta o produto de velocidades na derivada.

2. As equações estão acopladas entre sí, o que torna inviável a solução delas de forma separada.

1.3 Volume de Controle

A Fig. 1, ilustra o volume de controle que será adotado para a discretização das equações médias de transporte. O subíndice k representa o elemento discreto de uma tubulação na direção z. As variáveis A_k, S_k e r_k representam, respectimvamente, a área transversal ao escomanto, a superfície lateral e o raio do elemento discreto. O termo $q_{m_{c,p}}$ representa a vazão mássica do componente c na fase p por unidade de volume.

De acordo com a Fig. 1, é possível definir os seguintes termos:

$$dV_k = \pi r_k^2 \Delta z_k \tag{1.3}$$

$$A_k = \pi r_k^2 \tag{1.4}$$

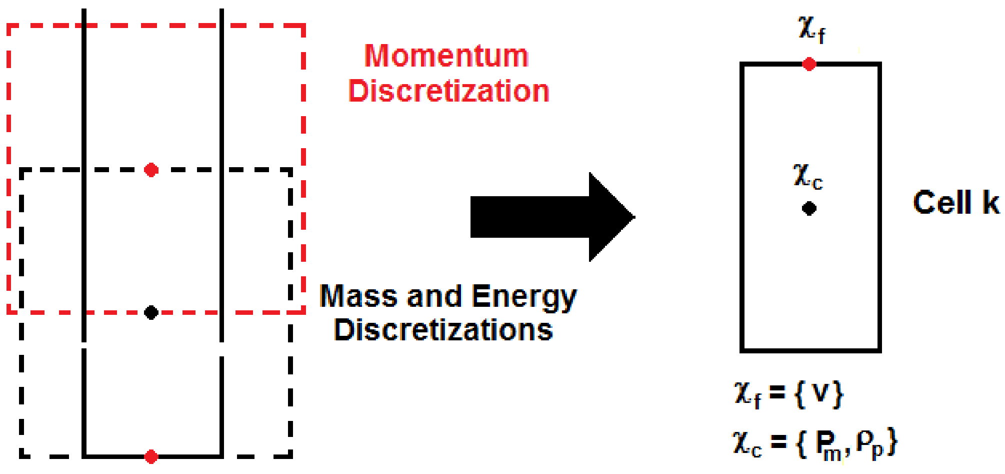

Figura 2: Malha deslocada para solução das equações de Navier-Stokes unidimensionais.

$$dS_k = 2\pi r_k \Delta z_k \tag{1.5}$$

Para facilitar a solução das equações, será adotado o esquema de malhas deslocadas, mostrado na Figura 2. Nesse esquema a pressão é armazenada no centro da célula e a velocidade na face da mesma.

1.4 Equações de Navier-Stokes discretizadas

Será aplicado o método dos volumes finitos na equação da difusão como descrito no capítulo 4 e 8 de **??**) .

1.4.1 Conservação da Massa

Utilizando o método dos volumes finitos, a equação da conservação da massa pode ser escrita como:

$$\int_{t}^{t+\Delta t} \iiint_{\text{V.C.}} \frac{\partial}{\partial t}(\rho)\, dV dt + \int_{t}^{t+\Delta t} \iiint_{\text{V.C.}} \nabla \cdot (\rho \vec{v})\, dV dt = \int_{t}^{t+\Delta t} \iiint_{\text{V.C.}} S_m dV dt \tag{1.6}$$

Aplicando a regra de Leibiniz no primeiro termo:

$$\iiint_{\text{V.C.}} \int_{t}^{t+\Delta t} \left[\frac{\partial}{\partial t}(\rho)\right] dt dV + \int_{t}^{t+\Delta t} \iiint_{\text{V.C.}} \nabla \cdot (\rho \vec{v})\, dV dt = \int_{t}^{t+\Delta t} \iiint_{\text{V.C.}} S_m dV dt \tag{1.7}$$

Aplicando o teorema de Gauss:

$$\iiint_{\text{V.C.}} \int_{t}^{t+\Delta t} \left[\frac{\partial}{\partial t}(\rho)\right] dt dV + \int_{t}^{t+\Delta t} \iiint_{\text{S.C.}} (\rho \vec{v}) \cdot \vec{n} dS dt = \int_{t}^{t+\Delta t} \iiint_{\text{V.C.}} S_m dV dt \tag{1.8}$$

Avaliando a integral no tempo do primeiro termo da equação:

$$\int_{t}^{t+\Delta t} \left[\frac{\partial}{\partial t}(\rho)\right] dt = \rho^{n+1} - \rho^{n} \tag{1.9}$$

Para se avaliar as integrais no tempo das outras variáveis, será utilizado um parametro θ que pondera quanta informação deve ser utilizada do passo de tempo anterior, assim:

$$\int_{t}^{t+\Delta t} f(x,\, t)\, dt = \left[\theta f^{n+1} + (1-\theta) f^{n}\right] \Delta t \tag{1.10}$$

Como será utilizado um esquema totalmente implícito, $\theta = 1$, assim:

$$\int_{t}^{t+\Delta t} f(x,\, t)\, dt = f^{n+1} \Delta t \tag{1.11}$$

Substituindo na Equação 1.8 e utilizando o volume de controle unidimensional (Figura 1):

$$(A\Delta x)_k \frac{(\rho^{n+1} - \rho^{n})_k}{\Delta t} + (A\rho v)_{k+\frac{1}{2}}^{n+1} - (A\rho v)_{k-\frac{1}{2}}^{n+1} = (AS_m\Delta x)_k^{n+1} \tag{1.12}$$

1.4.2 Conservação da Quantidade de Movimento

A equação da quantidade de movimento pode ser escrita como:

$$\int_{t}^{t+\Delta t}\iiint_{\mathrm{V.C.}}\frac{\partial}{\partial t}\left(\rho\vec{v}\right)dVdt+\int_{t}^{t+\Delta t}\iiint_{\mathrm{V.C.}}\nabla\cdot\left(\rho\vec{v}\vec{v}\right)dVdt=-\int_{t}^{t+\Delta t}\iiint_{\mathrm{V.C.}}\nabla pdVdt+\int_{t}^{t+\Delta t}\iiint_{\mathrm{V.C.}}\rho gsen\left(\theta\right)dVdt \tag{1.13}$$

Aplicando o teorema de Gauss e a regra de Leibiniz:

$$\iiint_{\mathrm{V.C.}}\int_{t}^{t+\Delta t}\left[\frac{\partial}{\partial t}\left(\rho\vec{v}\right)\right]dtdV+\int_{t}^{t+\Delta t}\iiint_{\mathrm{S.C.}}\left(\rho\vec{v}\vec{v}\right)\cdot\vec{n}dSdt=-\int_{t}^{t+\Delta t}\iiint_{\mathrm{V.C.}}\nabla pdVdt+\int_{t}^{t+\Delta t}\iiint_{\mathrm{V.C.}}\rho gsen\left(\theta\right)dVdt \tag{1.14}$$

Discretizando implicitamente no tempo, e utilizando o volume de controle unidimensional:

$$\iiint_{\mathrm{V.C.}}\left[\left(\rho\vec{v}\right)^{n+1}-\left(\rho\vec{v}\right)^{n}\right]dV+\iiint_{\mathrm{S.C.}}\left(\rho\vec{v}\vec{v}\right)^{n+1}\cdot\vec{n}dS\Delta t=-\iiint_{\mathrm{V.C.}}\nabla p^{n+1}dV\Delta t+\iiint_{\mathrm{V.C.}}\rho^{n+1}gsen\left(\theta\right)dV\Delta t \tag{1.15}$$

$$\begin{aligned}\left(A\Delta x\right)_{k+\frac{1}{2}}\frac{\left[\left(\rho v\right)^{n+1}-\left(\rho v\right)^{n}\right]_{k+\frac{1}{2}}}{\Delta t}+\left[\left(A\rho vv\right)_{k+1}^{n+1}-\left(A\rho vv\right)_{k}^{n+1}\right]=&\\-\left(A\Delta x\frac{\partial p}{\partial x}\right)_{k+\frac{1}{2}}^{n+1}+\left(A\Delta x\rho gsen\left(\theta\right)\right)_{k+\frac{1}{2}}^{n+1}&\end{aligned} \tag{1.16}$$

Utilizando diferenças finitas para aproximar a derivada da pressão:

$$\begin{aligned}\left(A\Delta x\right)_{k+\frac{1}{2}}\frac{\left[\left(\rho v\right)^{n+1}-\left(\rho v\right)^{n}\right]_{k+\frac{1}{2}}}{\Delta t}+\left[\left(A\rho vv\right)_{k+1}^{n+1}-\left(A\rho vv\right)_{k}^{n+1}\right]=&\\-\left(A\Delta x\right)_{k+\frac{1}{2}}\left(\frac{p_{k+1}-p_{k}}{\frac{1}{2}\left(\Delta x_{k+1}+\Delta x_{k}\right)}\right)^{n+1}+\left(A\Delta x\rho gsen\left(\theta\right)\right)_{k+\frac{1}{2}}^{n+1}&\end{aligned} \tag{1.17}$$

Será utilizado o esquema de interpolação Donor-Cell para se avaliar a velocidade no centro das células.

1.5 Algorítimo COUPLE com formulação Full-Implicit

1.5.1 Método de Newton-Rapson

A formulação implícita do problema será realizada utilizando-se o método de Newton-Raphson, descrito por:

$$R_k^{\nu} + \sum_{j}^{N\ Equations} \frac{\partial R_k^{\nu}}{\partial x_j} \left(x_j^{(\nu+1)} - x_j^{(\nu)} \right) = 0 \tag{1.18}$$

ou em notação matricial:

$$J^{\nu} \delta x^{\nu+1} = -R^{\nu} \tag{1.19}$$

onde J é a matriz Jacobiano do problema, R é o vetor de resíduos e δx é a variação da solução do problema. Observa-se que todas as propriedades serão avaliadas no tempo $n+1$.

Os resíduos para a equação da conservação da massa são dados por:

$$R_k^m = (A\Delta x)_k \frac{(\rho^{n+1} - \rho^n)_k}{\Delta t} + (A\rho v)_{k+\frac{1}{2}}^{n+1} - (A\rho v)_{k-\frac{1}{2}}^{n+1} - (AS_m \Delta x)_k^{n+1} \tag{1.20}$$

Os resíduos para a equação da quantidade de movimento desprezando termos fonte, são dados por:

$$\begin{aligned} R_k^p = (A\Delta x)_{k+\frac{1}{2}} \frac{\left[(\rho v)^{n+1} - (\rho v)^n \right]_{k+\frac{1}{2}}}{\Delta t} + \left[(A\rho vv)_{k+1}^{n+1} - (A\rho vv)_k^{n+1} \right] \\ + (A\Delta x)_{k+\frac{1}{2}} \left(\frac{p_{k+1} - p_k}{\frac{1}{2}(\Delta x_{k+1} + \Delta x_k)} \right)^{n+1} - (A\Delta x \rho g sen\,(\theta))_{k+\frac{1}{2}}^{n+1} \end{aligned} \tag{1.21}$$

Dessa forma a Equação 1.19 pode ser escrita como:

$$
\begin{bmatrix}
\frac{\partial R_1^p}{\partial p_1} & \frac{\partial R_1^p}{\partial v_1} & \frac{\partial R_1^p}{\partial p_2} & \frac{\partial R_1^p}{\partial v_2} & \frac{\partial R_1^p}{\partial p_3} & \frac{\partial R_1^p}{\partial v_3} & \cdots & \frac{\partial R_1^p}{\partial p_n} & \frac{\partial R_1^p}{\partial v_n} \\
\frac{\partial R_1^m}{\partial p_1} & \frac{\partial R_1^m}{\partial v_1} & \frac{\partial R_1^m}{\partial p_2} & \frac{\partial R_1^m}{\partial v_2} & \frac{\partial R_1^m}{\partial p_3} & \frac{\partial R_1^m}{\partial v_3} & \cdots & \frac{\partial R_1^m}{\partial p_n} & \frac{\partial R_1^m}{\partial v_n} \\
\frac{\partial R_2^p}{\partial p_1} & \frac{\partial R_2^p}{\partial v_1} & \frac{\partial R_2^p}{\partial p_2} & \frac{\partial R_2^p}{\partial v_2} & \frac{\partial R_2^p}{\partial p_3} & \frac{\partial R_2^p}{\partial v_3} & \cdots & \frac{\partial R_2^p}{\partial p_n} & \frac{\partial R_2^p}{\partial v_n} \\
\frac{\partial R_2^m}{\partial p_1} & \frac{\partial R_2^m}{\partial v_1} & \frac{\partial R_2^m}{\partial p_2} & \frac{\partial R_2^m}{\partial v_2} & \frac{\partial R_2^m}{\partial p_3} & \frac{\partial R_2^m}{\partial v_3} & \cdots & \frac{\partial R_2^m}{\partial p_n} & \frac{\partial R_2^m}{\partial v_n} \\
\frac{\partial R_3^p}{\partial p_1} & \frac{\partial R_3^p}{\partial v_1} & \frac{\partial R_3^p}{\partial p_2} & \frac{\partial R_3^p}{\partial v_2} & \frac{\partial R_3^p}{\partial p_3} & \frac{\partial R_3^p}{\partial v_3} & \cdots & \frac{\partial R_3^p}{\partial p_n} & \frac{\partial R_3^p}{\partial v_n} \\
\frac{\partial R_3^m}{\partial p_1} & \frac{\partial R_3^m}{\partial v_1} & \frac{\partial R_3^m}{\partial p_2} & \frac{\partial R_3^m}{\partial v_2} & \frac{\partial R_3^m}{\partial p_3} & \frac{\partial R_3^m}{\partial v_3} & \cdots & \frac{\partial R_3^m}{\partial p_n} & \frac{\partial R_3^m}{\partial v_n} \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
\frac{\partial R_n^p}{\partial p_1} & \frac{\partial R_n^p}{\partial v_1} & \frac{\partial R_n^p}{\partial p_2} & \frac{\partial R_n^p}{\partial v_2} & \frac{\partial R_n^p}{\partial p_3} & \frac{\partial R_n^p}{\partial v_3} & \cdots & \frac{\partial R_n^p}{\partial p_n} & \frac{\partial R_n^p}{\partial v_n} \\
\frac{\partial R_n^m}{\partial p_1} & \frac{\partial R_n^m}{\partial v_1} & \frac{\partial R_n^m}{\partial p_2} & \frac{\partial R_n^m}{\partial v_2} & \frac{\partial R_n^m}{\partial p_3} & \frac{\partial R_n^m}{\partial v_3} & \cdots & \frac{\partial R_n^m}{\partial p_n} & \frac{\partial R_n^m}{\partial v_n}
\end{bmatrix}
\begin{bmatrix} \delta p_1 \\ \delta v_1 \\ \delta p_2 \\ \delta v_2 \\ \delta p_3 \\ \delta v_3 \\ \vdots \\ \delta p_n \\ \delta v_n \end{bmatrix}
= -
\begin{bmatrix} R_1^p \\ R_1^m \\ R_2^p \\ R_2^m \\ R_3^p \\ R_3^m \\ \vdots \\ R_n^p \\ R_n^m \end{bmatrix}
\tag{1.22}
$$

Observa-se que as funções de resíduo (Equações 1.20 e 1.21) para a célula k só dependem dos vizinhos a esquerda e a direita. Logo:

$$
\begin{bmatrix}
\frac{\partial R_1^p}{\partial p_1} & \frac{\partial R_1^p}{\partial v_1} & \frac{\partial R_1^p}{\partial p_2} & \frac{\partial R_1^p}{\partial v_2} & 0 & 0 & \cdots & 0 & 0 \\
\frac{\partial R_1^m}{\partial p_1} & \frac{\partial R_1^m}{\partial v_1} & \frac{\partial R_1^m}{\partial p_2} & \frac{\partial R_1^m}{\partial v_2} & 0 & 0 & \cdots & 0 & 0 \\
\frac{\partial R_2^p}{\partial p_1} & \frac{\partial R_2^p}{\partial v_1} & \frac{\partial R_2^p}{\partial p_2} & \frac{\partial R_2^p}{\partial v_2} & \frac{\partial R_2^p}{\partial p_3} & \frac{\partial R_2^p}{\partial v_3} & \cdots & 0 & 0 \\
\frac{\partial R_2^m}{\partial p_1} & \frac{\partial R_2^m}{\partial v_1} & \frac{\partial R_2^m}{\partial p_2} & \frac{\partial R_2^m}{\partial v_2} & \frac{\partial R_2^m}{\partial p_3} & \frac{\partial R_2^m}{\partial v_3} & \cdots & 0 & 0 \\
0 & 0 & \frac{\partial R_3^p}{\partial p_2} & \frac{\partial R_3^p}{\partial v_2} & \frac{\partial R_3^p}{\partial p_3} & \frac{\partial R_3^p}{\partial v_3} & \cdots & 0 & 0 \\
0 & 0 & \frac{\partial R_3^m}{\partial p_2} & \frac{\partial R_3^m}{\partial v_2} & \frac{\partial R_3^m}{\partial p_3} & \frac{\partial R_3^m}{\partial v_3} & \cdots & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & 0 & 0 & 0 & 0 & \cdots & \frac{\partial R_n^p}{\partial p_n} & \frac{\partial R_n^p}{\partial v_n} \\
0 & 0 & 0 & 0 & 0 & 0 & \cdots & \frac{\partial R_n^m}{\partial p_n} & \frac{\partial R_n^m}{\partial v_n}
\end{bmatrix}
\begin{bmatrix} \delta p_1 \\ \delta v_1 \\ \delta p_2 \\ \delta v_2 \\ \delta p_3 \\ \delta v_3 \\ \vdots \\ \delta p_n \\ \delta v_n \end{bmatrix}
= -
\begin{bmatrix} R_1^p \\ R_1^m \\ R_2^p \\ R_2^m \\ R_3^p \\ R_3^m \\ \vdots \\ R_n^p \\ R_n^m \end{bmatrix}
\tag{1.23}
$$

1.6 Derivadas Parciais

1.6.1 Derivada do resíduo da massa em relação a pressão

1.6.1.1 Bloco à esquerda:

$$
\frac{\partial R_k^m}{\partial p_{k-1}} = \frac{\partial}{\partial p_{k-1}} \left[(A\Delta x)_k \frac{(\rho^{n+1} - \rho^n)_k}{\Delta t} + (A\rho v)_{k+\frac{1}{2}}^{n+1} - (A\rho v)_{k-\frac{1}{2}}^{n+1} - (AS_m \Delta x)_k^{n+1} \right] \tag{1.24}
$$

$$\frac{\partial R_k^m}{\partial p_{k-1}} = \frac{\partial}{\partial p_{k-1}}\left((A\Delta x)_k \frac{(\rho^{n+1}-\rho^n)_k}{\Delta t}\right) + \frac{\partial}{\partial p_{k-1}}(A\rho v)_{k+\frac{1}{2}}^{n+1} - \frac{\partial}{\partial p_{k-1}}(A\rho v)_{k-\frac{1}{2}}^{n+1} - \frac{\partial}{\partial p_{k-1}}(AS_m\Delta x)_k^{n+1} \tag{1.25}$$

$$\frac{\partial R_k^m}{\partial p_{k-1}} = -(Av)_{k-\frac{1}{2}}^{n+1}\frac{\partial}{\partial p_{k-1}}(\rho)_{k-\frac{1}{2}}^{n+1} \tag{1.26}$$

como

$$\rho_{k-\frac{1}{2}}^{n+1} = \frac{\rho_{k-1}^{n+1}+\rho_k^{n+1}}{2}$$

então:

$$\frac{\partial R_k^m}{\partial p_{k-1}} = -\frac{(Av)_{k-\frac{1}{2}}^{n+1}}{2}\frac{\partial\left(\rho_{k-1}^{n+1}\right)}{\partial p_{k-1}} \tag{1.27}$$

1.6.1.2 Bloco central:

$$\frac{\partial R_k^m}{\partial p_k} = \frac{\partial}{\partial p_k}\left[(A\Delta x)_k \frac{(\rho^{n+1}-\rho^n)_k}{\Delta t} + (A\rho v)_{k+\frac{1}{2}}^{n+1} - (A\rho v)_{k-\frac{1}{2}}^{n+1} - (AS_m\Delta x)_k^{n+1}\right] \tag{1.28}$$

$$\frac{\partial R_k^m}{\partial p_k} = \frac{\partial}{\partial p_k}\left((A\Delta x)_k \frac{(\rho^{n+1}-\rho^n)_k}{\Delta t}\right) + \frac{\partial}{\partial p_k}(A\rho v)_{k+\frac{1}{2}}^{n+1} - \frac{\partial}{\partial p_k}(A\rho v)_{k-\frac{1}{2}}^{n+1} - \frac{\partial}{\partial p_k}(AS_m\Delta x)_k^{n+1} \tag{1.29}$$

$$\frac{\partial R_k^m}{\partial p_k} = \frac{(A\Delta x)_k}{\Delta t}\frac{\partial\left(\rho_k^{n+1}\right)}{\partial p_k} + (Av)_{k+\frac{1}{2}}^{n+1}\frac{\partial}{\partial p_k}\rho_{k+\frac{1}{2}}^{n+1} - (Av)_{k-\frac{1}{2}}^{n+1}\frac{\partial}{\partial p_k}\rho_{k-\frac{1}{2}}^{n+1} - (A\Delta x)_k^{n+1}\frac{\partial}{\partial p_k}{S_m}_k^{n+1} \tag{1.30}$$

como

$$\rho_{k-\frac{1}{2}}^{n+1} = \frac{\rho_{k-1}^{n+1}+\rho_k^{n+1}}{2}$$

$$\rho_{k+\frac{1}{2}}^{n+1} = \frac{\rho_k^{n+1}+\rho_{k+1}^{n+1}}{2}$$

então:

$$\frac{\partial R_k^m}{\partial p_k} = \frac{(A\Delta x)_k}{\Delta t}\frac{\partial\left(\rho_k^{n+1}\right)}{\partial p_k} + \frac{(Av)_{k+\frac{1}{2}}^{n+1}}{2}\frac{\partial\left(\rho_k^{n+1}\right)}{\partial p_k} - \frac{(Av)_{k-\frac{1}{2}}^{n+1}}{2}\frac{\partial\left(\rho_k^{n+1}\right)}{\partial p_k} - (A\Delta x)_k^{n+1}\frac{\partial\left({S_m}_k^{n+1}\right)}{\partial p_k} \tag{1.31}$$

$$\frac{\partial R_k^m}{\partial p_k} = \left[\frac{(A\Delta x)_k}{\Delta t} + \frac{(Av)_{k+\frac{1}{2}}^{n+1}}{2} - \frac{(Av)_{k-\frac{1}{2}}^{n+1}}{2}\right]\frac{\partial\left(\rho_k^{n+1}\right)}{\partial p_k} - (A\Delta x)_k^{n+1}\frac{\partial\left({S_m}_k^{n+1}\right)}{\partial p_k} \tag{1.32}$$

1.6.1.3 Bloco à direita:

$$\frac{\partial R_k^m}{\partial p_{k+1}} = \frac{\partial}{\partial p_{k+1}}\left[(A\Delta x)_k\frac{(\rho^{n+1}-\rho^n)_k}{\Delta t} + (A\rho v)_{k+\frac{1}{2}}^{n+1} - (A\rho v)_{k-\frac{1}{2}}^{n+1} - (AS_m\Delta x)_k^{n+1}\right] \tag{1.33}$$

$$\frac{\partial R_k^m}{\partial p_{k+1}} = \frac{\partial}{\partial p_{k+1}}\left((A\Delta x)_k\frac{(\rho^{n+1}-\rho^n)_k}{\Delta t}\right) + \frac{\partial}{\partial p_{k+1}}(A\rho v)_{k+\frac{1}{2}}^{n+1} - \frac{\partial}{\partial p_{k+1}}(A\rho v)_{k-\frac{1}{2}}^{n+1} - \frac{\partial}{\partial p_{k+1}}(AS_m\Delta x)_k^{n+1} \tag{1.34}$$

$$\frac{\partial R_k^m}{\partial p_{k+1}} = \frac{\partial}{\partial p_{k+1}}(A\rho v)_{k+\frac{1}{2}}^{n+1} \tag{1.35}$$

como

$$\rho_{k+\frac{1}{2}}^{n+1} = \frac{\rho_k^{n+1} + \rho_{k+1}^{n+1}}{2}$$

então:

$$\frac{\partial R_k^m}{\partial p_{k+1}} = \frac{(Av)_{k+\frac{1}{2}}^{n+1}}{2}\frac{\partial\left(\rho_{k+1}^{n+1}\right)}{\partial p_{k+1}} \tag{1.36}$$

1.6.2 Derivada do resíduo da massa em relação a velocidade

1.6.2.1 Bloco à esquerda:

$$\frac{\partial R_k^m}{\partial v_{k-\frac{1}{2}}} = \frac{\partial}{\partial v_{k-\frac{1}{2}}}\left[(A\Delta x)_k\frac{(\rho^{n+1}-\rho^n)_k}{\Delta t} + (A\rho v)_{k+\frac{1}{2}}^{n+1} - (A\rho v)_{k-\frac{1}{2}}^{n+1} - (AS_m\Delta x)_k^{n+1}\right] \tag{1.37}$$

$$\frac{\partial R_k^m}{\partial v_{k-\frac{1}{2}}} = \frac{\partial}{\partial v_{k-\frac{1}{2}}}\left((A\Delta x)_k \frac{(\rho^{n+1}-\rho^n)_k}{\Delta t}\right) + \frac{\partial}{\partial v_{k-\frac{1}{2}}}(A\rho v)_{k+\frac{1}{2}}^{n+1} - \frac{\partial}{\partial v_{k-\frac{1}{2}}}(A\rho v)_{k-\frac{1}{2}}^{n+1} - \frac{\partial}{\partial v_{k-\frac{1}{2}}}(AS_m\Delta x)_k^{n+1} \tag{1.38}$$

$$\frac{\partial R_k^m}{\partial v_{k-\frac{1}{2}}} = -\frac{\partial}{\partial v_{k-\frac{1}{2}}}(A\rho v)_{k-\frac{1}{2}}^{n+1} \tag{1.39}$$

$$\frac{\partial R_k^m}{\partial v_{k-\frac{1}{2}}} = -(A\rho)_{k-\frac{1}{2}}^{n+1} \tag{1.40}$$

1.6.2.2 Bloco central:

$$\frac{\partial R_k^m}{\partial v_{k+\frac{1}{2}}} = \frac{\partial}{\partial v_{k+\frac{1}{2}}}\left[(A\Delta x)_k \frac{(\rho^{n+1}-\rho^n)_k}{\Delta t} + (A\rho v)_{k+\frac{1}{2}}^{n+1} - (A\rho v)_{k-\frac{1}{2}}^{n+1} - (AS_m\Delta x)_k^{n+1}\right] \tag{1.41}$$

$$\frac{\partial R_k^m}{\partial v_{k+\frac{1}{2}}} = \frac{\partial}{\partial v_{k+\frac{1}{2}}}\left((A\Delta x)_k \frac{(\rho^{n+1}-\rho^n)_k}{\Delta t}\right) + \frac{\partial}{\partial v_{k+\frac{1}{2}}}(A\rho v)_{k+\frac{1}{2}}^{n+1} - \frac{\partial}{\partial v_{k+\frac{1}{2}}}(A\rho v)_{k-\frac{1}{2}}^{n+1} - \frac{\partial}{\partial v_{k+\frac{1}{2}}}(AS_m\Delta x)_k^{n+1} \tag{1.42}$$

$$\frac{\partial R_k^m}{\partial v_{k+\frac{1}{2}}} = \frac{\partial}{\partial v_{k+\frac{1}{2}}}(A\rho v)_{k+\frac{1}{2}}^{n+1} \tag{1.43}$$

$$\frac{\partial R_k^m}{\partial v_{k+\frac{1}{2}}} = (A\rho)_{k+\frac{1}{2}}^{n+1} \tag{1.44}$$

1.6.2.3 Bloco à direita:

$$\frac{\partial R_k^m}{\partial v_{k+\frac{3}{2}}} = \frac{\partial}{\partial v_{k+\frac{3}{2}}}\left[(A\Delta x)_k \frac{(\rho^{n+1}-\rho^n)_k}{\Delta t} + (A\rho v)_{k+\frac{1}{2}}^{n+1} - (A\rho v)_{k-\frac{1}{2}}^{n+1} - (AS_m\Delta x)_k^{n+1}\right] \tag{1.45}$$

$$\frac{\partial R_k^m}{\partial v_{k+\frac{3}{2}}} = \frac{\partial}{\partial v_{k+\frac{3}{2}}}\left((A\Delta x)_k \frac{(\rho^{n+1}-\rho^n)_k}{\Delta t}\right) + \frac{\partial}{\partial v_{k+\frac{3}{2}}}(A\rho v)_{k+\frac{1}{2}}^{n+1} - \frac{\partial}{\partial v_{k+\frac{3}{2}}}(A\rho v)_{k-\frac{1}{2}}^{n+1} - \frac{\partial}{\partial v_{k+\frac{3}{2}}}(AS_m\Delta x)_k^{n+1} \tag{1.46}$$

$$\frac{\partial R_k^m}{\partial v_{k+\frac{3}{2}}} = 0 \tag{1.47}$$

1.6.3 Derivada do resíduo do momentum em relação a pressão

1.6.3.1 Bloco à esquerda:

$$\frac{\partial R_k^p}{\partial p_{k-1}} = \frac{\partial}{\partial p_{k-1}} \Bigg[(A\Delta x)_{k+\frac{1}{2}} \frac{\left[(\rho v)^{n+1} - (\rho v)^n\right]_{k+\frac{1}{2}}}{\Delta t} + \left[(A\rho vv)_{k+1}^{n+1} - (A\rho vv)_k^{n+1}\right] + (A\Delta x)_{k+\frac{1}{2}} \left(\frac{p_{k+1} - p_k}{\frac{1}{2}(\Delta x_{k+1} + \Delta x_k)}\right)^{n+1} - (A\Delta x\rho gsen\,(\theta))_{k+\frac{1}{2}}^{n+1} \Bigg] \tag{1.48}$$

$$\frac{\partial R_k^p}{\partial p_{k-1}} = \frac{\partial}{\partial p_{k-1}} \left((A\Delta x)_{k+\frac{1}{2}} \frac{\left[(\rho v)^{n+1} - (\rho v)^n\right]_{k+\frac{1}{2}}}{\Delta t} \right) + \frac{\partial}{\partial p_{k-1}} (A\rho vv)_{k+1}^{n+1} - \frac{\partial}{\partial p_{k-1}} (A\rho vv)_k^{n+1} + \frac{\partial}{\partial p_{k-1}} \left[(A\Delta x)_{k+\frac{1}{2}} \left(\frac{p_{k+1} - p_k}{\frac{1}{2}(\Delta x_{k+1} + \Delta x_k)}\right)^{n+1} \right] - \frac{\partial}{\partial p_{k-1}} (A\Delta x\rho gsen\,(\theta))_{k+\frac{1}{2}}^{n+1} \tag{1.49}$$

$$\frac{\partial R_k^p}{\partial p_{k-1}} = 0 \tag{1.50}$$

1.6.3.2 Bloco central:

$$\frac{\partial R_k^p}{\partial p_k} = \frac{\partial}{\partial p_k} \Bigg[(A\Delta x)_{k+\frac{1}{2}} \frac{\left[(\rho v)^{n+1} - (\rho v)^n\right]_{k+\frac{1}{2}}}{\Delta t} + \left[(A\rho vv)_{k+1}^{n+1} - (A\rho vv)_k^{n+1}\right] + (A\Delta x)_{k+\frac{1}{2}} \left(\frac{p_{k+1} - p_k}{\frac{1}{2}(\Delta x_{k+1} + \Delta x_k)}\right)^{n+1} - (A\Delta x\rho gsen\,(\theta))_{k+\frac{1}{2}}^{n+1} \Bigg] \tag{1.51}$$

$$\frac{\partial R_k^p}{\partial p_k} = \frac{\partial}{\partial p_k}\left((A\Delta x)_{k+\frac{1}{2}} \frac{\left[(\rho v)^{n+1} - (\rho v)^n \right]_{k+\frac{1}{2}}}{\Delta t} \right) + \frac{\partial}{\partial p_k} (A\rho v v)_{k+1}^{n+1} - \frac{\partial}{\partial p_k} (A\rho v v)_k^{n+1}$$
$$+ \frac{\partial}{\partial p_k} \left[(A\Delta x)_{k+\frac{1}{2}} \left(\frac{p_{k+1} - p_k}{\frac{1}{2}\left(\Delta x_{k+1} + \Delta x_k\right)} \right)^{n+1} \right] - \frac{\partial}{\partial p_k} \left(A\Delta x \rho g sen\left(\theta\right)\right)_{k+\frac{1}{2}}^{n+1} \quad (1.52)$$

$$\frac{\partial R_k^p}{\partial p_k} = \frac{(A\Delta x)_{k+\frac{1}{2}}}{\Delta t} v_{k+\frac{1}{2}}^{n+1} \frac{\partial}{\partial p_k}\left(\rho_{k+\frac{1}{2}}^{n+1}\right) - (Avv)_k^{n+1} \frac{\partial\left(\rho_k^{n+1}\right)}{\partial p_k}$$
$$- A_{k+\frac{1}{2}} - \left(g sen\left(\theta\right) A\Delta x\right)_{k+\frac{1}{2}}^{n+1} \frac{\partial}{\partial p_k}\left(\rho_{k+\frac{1}{2}}^{n+1}\right) \quad (1.53)$$

como

$$\rho_{k+\frac{1}{2}}^{n+1} = \frac{\rho_k^{n+1} + \rho_{k+1}^{n+1}}{2}$$

então:

$$\frac{\partial R_k^p}{\partial p_k} = \left[\frac{(A\Delta x)_{k+\frac{1}{2}}}{2\Delta t} v_{k+\frac{1}{2}}^{n+1} - (Avv)_k^{n+1} - \frac{\left(g sen\left(\theta\right) A\Delta x\right)_{k+\frac{1}{2}}^{n+1}}{2} \right] \frac{\partial\left(\rho_k^{n+1}\right)}{\partial p_k} - A_{k+\frac{1}{2}} \quad (1.54)$$

1.6.3.3 Bloco à direita:

$$\frac{\partial R_k^p}{\partial p_{k+1}} = \frac{\partial}{\partial p_{k+1}} \left[(A\Delta x)_{k+\frac{1}{2}} \frac{\left[(\rho v)^{n+1} - (\rho v)^n \right]_{k+\frac{1}{2}}}{\Delta t} + \left[(A\rho v v)_{k+1}^{n+1} - (A\rho v v)_k^{n+1} \right] \right.$$
$$\left. + (A\Delta x)_{k+\frac{1}{2}} \left(\frac{p_{k+1} - p_k}{\frac{1}{2}\left(\Delta x_{k+1} + \Delta x_k\right)} \right)^{n+1} - \left(A\Delta x \rho g sen\left(\theta\right)\right)_{k+\frac{1}{2}}^{n+1} \right] \quad (1.55)$$

$$\frac{\partial R_k^p}{\partial p_{k+1}} = \frac{\partial}{\partial p_{k+1}}\left((A\Delta x)_{k+\frac{1}{2}} \frac{\left[(\rho v)^{n+1} - (\rho v)^n \right]_{k+\frac{1}{2}}}{\Delta t} \right) + \frac{\partial}{\partial p_{k+1}} (A\rho v v)_{k+1}^{n+1} - \frac{\partial}{\partial p_{k+1}} (A\rho v v)_k^{n+1}$$
$$+ \frac{\partial}{\partial p_{k+1}} \left[(A\Delta x)_{k+\frac{1}{2}} \left(\frac{p_{k+1} - p_k}{\frac{1}{2}\left(\Delta x_{k+1} + \Delta x_k\right)} \right)^{n+1} \right] - \frac{\partial}{\partial p_{k+1}} \left(A\Delta x \rho g sen\left(\theta\right)\right)_{k+\frac{1}{2}}^{n+1} \quad (1.56)$$

$$\frac{\partial R_k^p}{\partial p_{k+1}} = \frac{(A\Delta x)_{k+\frac{1}{2}}}{\Delta t} v_{k+\frac{1}{2}}^{n+1} \frac{\partial}{\partial p_{k+1}} \left(\rho_{k+\frac{1}{2}}^{n+1}\right) + (Avv)_{k+1}^{n+1} \frac{\partial \left(\rho_{k+1}^{n+1}\right)}{\partial p_{k+1}} + A_{k+\frac{1}{2}} - (gsen\,(\theta)\, A\Delta x)_{k+\frac{1}{2}}^{n+1} \frac{\partial}{\partial p_{k+1}} \left(\rho_{k+\frac{1}{2}}^{n+1}\right) \tag{1.57}$$

como

$$\rho_{k+\frac{1}{2}}^{n+1} = \frac{\rho_k^{n+1} + \rho_{k+1}^{n+1}}{2}$$

então:

$$\frac{\partial R_k^p}{\partial p_{k+1}} = \left[\frac{(A\Delta x)_{k+\frac{1}{2}}}{2\Delta t} v_{k+\frac{1}{2}}^{n+1} + (Avv)_{k+1}^{n+1} - \frac{(gsen\,(\theta)\, A\Delta x)_{k+\frac{1}{2}}^{n+1}}{2}\right] \frac{\partial \left(\rho_{k+1}^{n+1}\right)}{\partial p_{k+1}} + A_{k+\frac{1}{2}} \tag{1.58}$$

1.6.4 Derivada do resíduo do momentum em relação a velocidade

1.6.4.1 Bloco à esquerda:

$$\begin{aligned} \frac{\partial R_k^p}{\partial v_{k-\frac{1}{2}}} = \frac{\partial}{\partial v_{k-\frac{1}{2}}} \Bigg[& (A\Delta x)_{k+\frac{1}{2}} \frac{\left[(\rho v)^{n+1} - (\rho v)^{n}\right]_{k+\frac{1}{2}}}{\Delta t} + \left[(A\rho vv)_{k+1}^{n+1} - (A\rho vv)_{k}^{n+1}\right] \\ & + (A\Delta x)_{k+\frac{1}{2}} \left(\frac{p_{k+1} - p_k}{\frac{1}{2}\left(\Delta x_{k+1} + \Delta x_k\right)}\right)^{n+1} - (A\Delta x \rho gsen\,(\theta))_{k+\frac{1}{2}}^{n+1} \Bigg] \end{aligned} \tag{1.59}$$

$$\begin{aligned} \frac{\partial R_k^p}{\partial v_{k-\frac{1}{2}}} = & \frac{\partial}{\partial v_{k-\frac{1}{2}}} \left[(A\Delta x)_{k+\frac{1}{2}} \frac{\left[(\rho v)^{n+1} - (\rho v)^{n}\right]_{k+\frac{1}{2}}}{\Delta t}\right] + \frac{\partial}{\partial v_{k-\frac{1}{2}}} (A\rho vv)_{k+1}^{n+1} - \frac{\partial}{\partial v_{k-\frac{1}{2}}} (A\rho vv)_{k}^{n+1} \\ & + \frac{\partial}{\partial v_{k-\frac{1}{2}}} \left[(A\Delta x)_{k+\frac{1}{2}} \left(\frac{p_{k+1} - p_k}{\frac{1}{2}\left(\Delta x_{k+1} + \Delta x_k\right)}\right)^{n+1}\right] - \frac{\partial}{\partial v_{k-\frac{1}{2}}} (A\Delta x \rho gsen\,(\theta))_{k+\frac{1}{2}}^{n+1} \end{aligned} \tag{1.60}$$

$$\frac{\partial R_k^p}{\partial v_{k-\frac{1}{2}}} = -(A\rho)_k^{n+1} \frac{\partial (vv)_k^{n+1}}{\partial v_{k-\frac{1}{2}}} \tag{1.61}$$

1.6.4.2 Bloco central:

$$\frac{\partial R_k^p}{\partial v_{k+\frac{1}{2}}} = \frac{\partial}{\partial v_{k+\frac{1}{2}}} \left[(A\Delta x)_{k+\frac{1}{2}} \frac{\left[(\rho v)^{n+1} - (\rho v)^n\right]_{k+\frac{1}{2}}}{\Delta t} + \left[(A\rho vv)_{k+1}^{n+1} - (A\rho vv)_k^{n+1}\right] \right.$$
$$\left. + (A\Delta x)_{k+\frac{1}{2}} \left(\frac{p_{k+1} - p_k}{\frac{1}{2}(\Delta x_{k+1} + \Delta x_k)} \right)^{n+1} - (A\Delta x \rho g sen(\theta))_{k+\frac{1}{2}}^{n+1} \right] \quad (1.62)$$

$$\frac{\partial R_k^p}{\partial v_{k+\frac{1}{2}}} = \frac{\partial}{\partial v_{k+\frac{1}{2}}} \left[(A\Delta x)_{k+\frac{1}{2}} \frac{\left[(\rho v)^{n+1} - (\rho v)^n\right]_{k+\frac{1}{2}}}{\Delta t} \right] + \frac{\partial}{\partial v_{k+\frac{1}{2}}} (A\rho vv)_{k+1}^{n+1} - \frac{\partial}{\partial v_{k+\frac{1}{2}}} (A\rho vv)_k^{n+1}$$
$$+ \frac{\partial}{\partial v_{k+\frac{1}{2}}} \left[(A\Delta x)_{k+\frac{1}{2}} \left(\frac{p_{k+1} - p_k}{\frac{1}{2}(\Delta x_{k+1} + \Delta x_k)} \right)^{n+1} \right] - \frac{\partial}{\partial v_{k+\frac{1}{2}}} (A\Delta x \rho g sen(\theta))_{k+\frac{1}{2}}^{n+1} \quad (1.63)$$

$$\frac{\partial R_k^p}{\partial v_{k+\frac{1}{2}}} = \frac{(A\Delta x \rho)_{k+\frac{1}{2}}^{n+1}}{\Delta t} + (A\rho)_{k+1}^{n+1} \frac{\partial (vv)_{k+1}^{n+1}}{\partial v_{k+\frac{1}{2}}} - (A\rho)_k^{n+1} \frac{\partial (vv)_k^{n+1}}{\partial v_{k+\frac{1}{2}}} \quad (1.64)$$

1.6.4.3 Bloco à direita:

$$\frac{\partial R_k^p}{\partial v_{k+\frac{3}{2}}} = \frac{\partial}{\partial v_{k+\frac{3}{2}}} \left[(A\Delta x)_{k+\frac{1}{2}} \frac{\left[(\rho v)^{n+1} - (\rho v)^n\right]_{k+\frac{1}{2}}}{\Delta t} + \left[(A\rho vv)_{k+1}^{n+1} - (A\rho vv)_k^{n+1}\right] \right.$$
$$\left. + (A\Delta x)_{k+\frac{1}{2}} \left(\frac{p_{k+1} - p_k}{\frac{1}{2}(\Delta x_{k+1} + \Delta x_k)} \right)^{n+1} - (A\Delta x \rho g sen(\theta))_{k+\frac{1}{2}}^{n+1} \right] \quad (1.65)$$

$$\frac{\partial R_k^p}{\partial v_{k+\frac{3}{2}}} = \frac{\partial}{\partial v_{k+\frac{3}{2}}} \left[(A\Delta x)_{k+\frac{1}{2}} \frac{\left[(\rho v)^{n+1} - (\rho v)^n\right]_{k+\frac{1}{2}}}{\Delta t} \right] + \frac{\partial}{\partial v_{k+\frac{3}{2}}} (A\rho vv)_{k+1}^{n+1} - \frac{\partial}{\partial v_{k+\frac{3}{2}}} (A\rho vv)_k^{n+1}$$
$$+ \frac{\partial}{\partial v_{k+\frac{3}{2}}} \left[(A\Delta x)_{k+\frac{1}{2}} \left(\frac{p_{k+1} - p_k}{\frac{1}{2}(\Delta x_{k+1} + \Delta x_k)} \right)^{n+1} \right] - \frac{\partial}{\partial v_{k+\frac{3}{2}}} (A\Delta x \rho g sen(\theta))_{k+\frac{1}{2}}^{n+1} \quad (1.66)$$

$$\frac{\partial R_k^p}{\partial v_{k+\frac{3}{2}}} = (A\rho)_{k+1}^{n+1} \frac{\partial (vv)_{k+1}^{n+1}}{\partial v_{k+\frac{3}{2}}} - (A\rho)_k^{n+1} \frac{\partial (vv)_k^{n+1}}{\partial v_{k+\frac{3}{2}}} \quad (1.67)$$

1.7 Interpolação Donor-Cell

As derivadas dos termos $(vv)_k^{n+1}$ não podem ser calculadas pois esses termos não são armazenados na geometria, dessa forma será utilizado o método Donor-Cell (UDS) para realizar essa interpolação. O método Donor-Cell é recomendado para escoamentos com altos valores do número de Peclet, onde utiliza-se a interpolação pos diferenças centradas para $\vec{v}$ e a interpolação pelo esquema Upwind de primeira ordem (UDS) para a propriedade transportada, ϕ. O termo $\phi_{k+\frac{1}{2}}$ pode ser expresso como:

$$\phi_{k+\frac{1}{2}} = \phi_k, \quad v_k \geq 0 \tag{1.68}$$

$$\phi_{k+\frac{1}{2}} = \phi_{k+1}, \quad v_k < 0 \tag{1.69}$$

Já o termo $\phi_{k-\frac{1}{2}}$ pode ser expresso como:

$$\phi_{k-\frac{1}{2}} = \phi_{k-1}, \quad v_k \geq 0 \tag{1.70}$$

$$\phi_{k-\frac{1}{2}} = \phi_k, \quad v_k < 0 \tag{1.71}$$

1.7.1 Derivadas da propriedade transportada ϕ:

Como o sentido do escoamento é importante para o método Donor-Cell, as derivadas serão feitas considerando velocidades positivas e negativas, respectivamente. Assim, as derivadas parciais das velocidades são calculadas por:

1.7.1.1 $v_{k+\frac{1}{2}} \geq 0$:

$$\frac{\partial\,(vv)_k}{\partial v_{k-\frac{1}{2}}} = \frac{\partial\,(vv)_k}{\partial v_k}\frac{\partial v_k}{\partial v_{k-\frac{1}{2}}} = 2v_k\frac{\partial}{\partial v_{k-\frac{1}{2}}}\left[v_{k-\frac{1}{2}}\right] = 2v_k \tag{1.72}$$

$$\frac{\partial\,(vv)_k}{\partial v_{k+\frac{1}{2}}} = \frac{\partial\,(vv)_k}{\partial v_k}\frac{\partial v_k}{\partial v_{k+\frac{1}{2}}} = 2v_k\frac{\partial}{\partial v_{k+\frac{1}{2}}}\left[v_{k-\frac{1}{2}}\right] = 0 \tag{1.73}$$

$$\frac{\partial\left(vv\right)_{k+1}}{\partial v_{k+\frac{1}{2}}}=\frac{\partial\left(vv\right)_{k+1}}{\partial v_{k+1}}\frac{\partial v_{k+1}}{\partial v_{k+\frac{1}{2}}}=2v_{k+1}\frac{\partial}{\partial v_{k+\frac{1}{2}}}\left[v_{k+\frac{1}{2}}\right]=2v_{k+1} \tag{1.74}$$

$$\frac{\partial\left(vv\right)_{k}}{\partial v_{k+\frac{3}{2}}}=\frac{\partial\left(vv\right)_{k}}{\partial v_{k}}\frac{\partial v_{k}}{\partial v_{k+\frac{3}{2}}}=2v_{k}\frac{\partial}{\partial v_{k+\frac{3}{2}}}\left[v_{k-\frac{1}{2}}\right]=0 \tag{1.75}$$

$$\frac{\partial\left(vv\right)_{k+1}}{\partial v_{k+\frac{3}{2}}}=\frac{\partial\left(vv\right)_{k+1}}{\partial v_{k+1}}\frac{\partial v_{k+1}}{\partial v_{k+\frac{3}{2}}}=2v_{k+1}\frac{\partial}{\partial v_{k+\frac{3}{2}}}\left[v_{k+\frac{1}{2}}\right]=0 \tag{1.76}$$

1.7.1.2 $v_{k+\frac{1}{2}}<0$:

$$\frac{\partial\left(vv\right)_{k}}{\partial v_{k-\frac{1}{2}}}=\frac{\partial\left(vv\right)_{k}}{\partial v_{k}}\frac{\partial v_{k}}{\partial v_{k-\frac{1}{2}}}=2v_{k}\frac{\partial}{\partial v_{k-\frac{1}{2}}}\left[v_{k+\frac{1}{2}}\right]=0 \tag{1.77}$$

$$\frac{\partial\left(vv\right)_{k}}{\partial v_{k+\frac{1}{2}}}=\frac{\partial\left(vv\right)_{k}}{\partial v_{k}}\frac{\partial v_{k}}{\partial v_{k+\frac{1}{2}}}=2v_{k}\frac{\partial}{\partial v_{k+\frac{1}{2}}}\left[v_{k+\frac{1}{2}}\right]=2v_{k} \tag{1.78}$$

$$\frac{\partial\left(vv\right)_{k+1}}{\partial v_{k+\frac{1}{2}}}=\frac{\partial\left(vv\right)_{k+1}}{\partial v_{k+1}}\frac{\partial v_{k+1}}{\partial v_{k+\frac{1}{2}}}=2v_{k+1}\frac{\partial}{\partial v_{k+\frac{1}{2}}}\left[v_{k+\frac{3}{2}}\right]=0 \tag{1.79}$$

$$\frac{\partial\left(vv\right)_{k}}{\partial v_{k+\frac{3}{2}}}=\frac{\partial\left(vv\right)_{k}}{\partial v_{k}}\frac{\partial v_{k}}{\partial v_{k+\frac{3}{2}}}=2v_{k}\frac{\partial}{\partial v_{k+\frac{3}{2}}}\left[v_{k-\frac{1}{2}}\right]=0 \tag{1.80}$$

$$\frac{\partial\left(vv\right)_{k+1}}{\partial v_{k+\frac{3}{2}}}=\frac{\partial\left(vv\right)_{k+1}}{\partial v_{k+1}}\frac{\partial v_{k+1}}{\partial v_{k+\frac{3}{2}}}=2v_{k+1}\frac{\partial}{\partial v_{k+\frac{3}{2}}}\left[v_{k+\frac{1}{2}}\right]=2v_{k+1} \tag{1.81}$$

1.8 Modelo de Fluido

O fluido utilizado será um gás ideal tal que a sua massa específica será dada por:

$$\rho=\frac{pM}{RT} \tag{1.82}$$

onde M representa a massa molar do gás e R a constante universal dos gases.

Assim, a derivada da massa específica com a pressão será dada por:

$$\frac{\partial\rho}{\partial p}=\frac{M}{RT} \tag{1.83}$$