Evaluation of Jarvis' March and Graham's Scan

Introduction

This is the evaluation on two algorithms Jarvis' March and Grahams's scan which are used to find a convex hull. By evaluating through different cases of input and count basic operation. It is intended to find the time complexity for different type of input for those two algorithms.

Method

Set the basic operation of Jarvis' March to Comparison between angles of points.

Set the basic operation of Grahams' Scan to Comparison during sort.

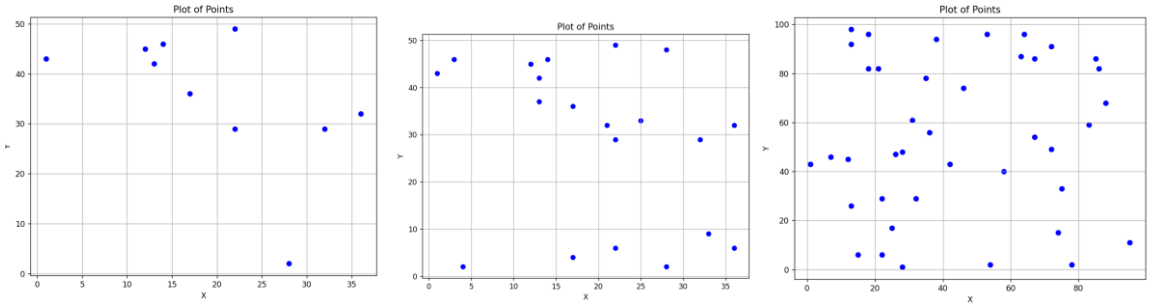
Test through three cases each with three different size (10 20 40).

One is totally random distributed point. One is all the point form convex hull, and the third one is four points form a convex hull with random points inside.

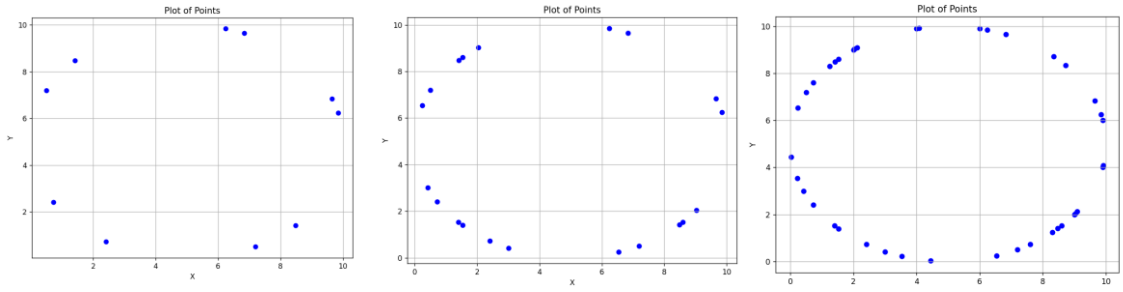
Test each algorithm and record the operation count

Graph of each point in each case:

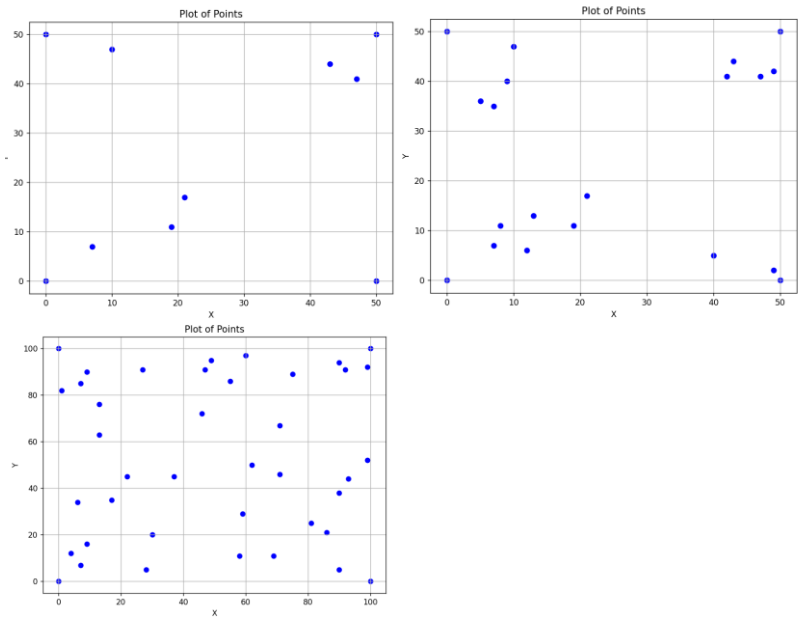
Case 1:



Case 2:



Case 3:



Result

Jarvis march:

Input size\test case	Case 1(all random)	Case 2(All convex hull)	Case 3(Four point convex hull)
10	36	89	36
20	152	379	76
40	390	1559	156

Graham's Scan

Input size\test case	Case 1(all random)	Case 2(All convex hull)	Case 3(Four point convex hull)
10	22	24	23
20	62	67	68
40	166	164	165

Discussion

Data Structure

In both implantations, double linked list is used to store the convex hull point. Using double linked list make the requirement to print point clockwise and counterclockwise

easier to implement. For Graham's Scan, there are function delete at end and insert at end for the double linked list. This make the double linked list a stack which allow it to track the recent newly added point. This is needed for the stage in graham scan after sorts all the point.

Jarvis March

The Jarvis March is highly affected by the number of points in the convex hull. It is input sensitive. It requires each point in convex hull to compare angles with all the other points. The time complexity of the Jarvis March is

$$Let\ n\ be\ the\ number\ of\ point\ and\ c\ be\ the\ number\ of\ point\ in\ convex\ hull$$

$$Number\ of\ operation = (n-1) * c$$

$$O(nc)$$

In case 2 when all points are convex hull the increase is quadratic. Since c = n Big O = $O(n^2)$. And in case 3, the increase is linear because c is a constant in this case big O = $O(n)$. It is hard to find the increase in case 1 since the number of convex hull points varies due to randomness. But the time complexity is $O(n^2)$ if we take the average

The space complexity for Jarvis March is O(1)

Graham Scan

The push and pop stage of Graham scan is linear time complexity. The basic operation in the test is angle comparation in sorting. Merge sort is used in the implementation. The time complexity for merge sort is $O(nlogn)$. Merge sort is input insensitive. The operation should be consistent for all different input. And from the test result, the number of operation counts is similar for all the cases with same input size. However, The space complexity for merge sort space is O(n).

Conclusion

In general Graham Scan is a better algorithm than Jarvis March in time complexity. Even though graham scan has a linear component after the sort. It is quicker in most cases. However Jarvis March is better in time complexity if the number of points is much smaller than the total number of points c < log(n). And when the space is restricted.