Kaialo - Ali

MATHS XII FUNCTIONS AND LIMITS

FUNCTION: A rule that assigns to each element x in set A has a unique element y in set B $f : A \rightarrow B$

Let $A = \{1,3,4\}$ $B = \{2,6,7\}$ then $f = \{(1,2),(3,6),(4,7)\}$ is sub set of $(A \times B)$

Domain of $f : (1,3,6)$ *Range of* f : $(2,6,7)$

Types of Function :

One –One Function (Injetive): A function $f : A \rightarrow B$ is said to be one to one if each element of A has a unique image in B. Eg: A = { 1 , 3 , 4 } B = { 2 , 6 , 7 }, then f = { (1 , 2) , (3 , 6) , (4 , 7) }

Onto Function(Surjective):

A function $f : A \rightarrow B$ *is said to be onto if Range of f is B or y part is repeated .*

Eg: $A = \{1,3,4\}$ $B = \{2,5,\}$, then $f = \{(1,2),(3,5),(4,5)\}$

One – One , Onto Function (Bijective) : A function is said $f : A \rightarrow B$ *is said to be bijective if both one – one and onto take place.* *Eg:* $f = \{(1,2),(3,6),(4,7)\}$ also *Eg:* $f = \{(1,2),(3,5),(4,5)\}$

Into Function: A function is said $f : A \rightarrow B$ *is said to be Into if Range of* $f \subset B$. *Eg:* $A = \{3,4\}$ $B = \{2,6,7\}$ then f = {(3 , 6) , (4 , 7) } is an into function because , Range of $f = \{6,7\} \subset B$

Modulus Function: A function : f = $|x|$ = $f(x) = \begin{cases} -x, & x < 0 \\ x, & x \geq 0 \end{cases}$ graph of Mod: function lies in 1^{st} & 2^{nd} quadrant.

Composite Function: Let $f : A \rightarrow B$, $g : B \rightarrow C$ *then* $h : A \rightarrow C$ defined by h (x) = g o f (x) = g (f (x)) .

Eg: $f(x) = x^2 + 4$ $g(x) = x + 2$, then fog(2) = f (g (x)) = $(x+2)^2 + 4 = (4)^2 + 4 = 20$

Exponential Function. If $y = a^x$ is called exponential function. The inverse of exponential is called Logarithmic Function, written as f (x) = $\log_a x$ with base a . *Eg:* 64 = 2 6 => $\log_2 64 = 6$

Inverse Function: Let $f : A \rightarrow B$ *be a function , then the inverse function of f is* $f^{-1} = B \rightarrow A$ OR $y = f(x)$ then $f^{-1}(y) = x$ Eg: (1) $f(x) = y = \frac{2x+4}{5}$ then $f^{-1}(y) = \frac{5x-4}{2}$

Eg: (2) $f(x) = y = \frac{2x^{\frac{1}{3}}+4}{5}$ then $f^{-1}(y) = (\frac{5x-4}{2})^3$.

Explicit Function : A function $y = f(x) = x^2 + 2x + 4$. separated x and y . ⇒ Contains only one variable.

Implicit Function : A function $f(x,y) = x^2y + 2xy^3 + 4$ together x and y. ⇒ Contains more than one variable

Even and Odd functions: A function f (x) is said to be Even if $f(-x) = f(x)$

A function f (x) is said to be Odd if $f(-x) = -f(x)$ *

If $f(-x) \neq -f(x)$ called Neither even nor odd.

Eg (1) $f(x) = x^4$ or $x^6 + x^{14}$ or $|x|$ or $|-x|$ or Cos x + Sec x even functions

Eg. (2) $f(x) = x^3$ or $x^5 + x^{11}$ or $-|x|$ or or Sin x + Tan x odd functions

Eg. (3) $f(x) = x^4 + x^7$ or $x^7 - x^{14}$ or Cos x + Sin x are neither functions

Rational Functions: $f(x) = \frac{P(x)}{Q(x)}$, $Q(x) \neq 0$

Some Important Limits

1) $\lim_{x \to 2} x^2 + 5x + 3 = 17$

2) $\lim_{x \to 0} \frac{3x^2 + 9x - 4}{7x^2 + 4x + 5} = \frac{-4}{5}$ ⇒ Constants are Answers

3) $\lim_{x \to 2} \frac{x^2-5x+6}{x^2-7x+10} = \frac{(x-3)\cancel{(x-2)}}{\cancel{(x-2)}(x-5)} = \frac{1}{3}$

4) $\lim_{x \to \infty} \frac{3x^2 + 9x - 4}{7x^2 + 4x + 5} = \frac{3}{7}$ ⇒ co-efficient

5) $\lim_{x \to \infty} \frac{3x^6+9x-4}{7x^2+4x+5} = \infty$ ⇒ if there is highest Power in Numenetor then answer will be Infinity ($\propto$) -

6) $\lim_{x \to \infty} \frac{3x^2 + 9x - 4}{7x^5 + 4x + 5} = 0$ opposite

$\because 1/0 = \propto$
$\because 1/\propto = 0$

3) $\lim_{x \to 2} \frac{x^2-5x+6}{x^2-7x+10}$

$\Rightarrow \frac{\cancel{x^2}(1-5/x+6/x^2)}{\cancel{x^2}(1-7/x+10/x^2)}$

$\Rightarrow \frac{1-5/x+6/x^2}{1-7/x+10/x^2}$

$\Rightarrow \frac{1-5/2+5/2}{1-7/2+5/2}$

$\Rightarrow \frac{\cancel{4}-\cancel{5}+8}{\cancel{4}} / \frac{2-7+5}{2}$

$\Rightarrow 1/0$ ⇒ wrong

2

7) $\lim_{x \to a} \frac{x^m - a^m}{x^n - a^n} = \frac{m}{n} a^{m-n}$

8) $\lim_{x \to 2} \frac{x^6 - 64}{x^4 - 16} = \frac{3}{2} 2^{3-2} = 3$

9) $\lim_{x \to 3} \frac{x^2 - 9}{x - 3} = 6$

10) $\lim_{x \to 1} \frac{x^m - 1}{x - 1} = m$

11) $\lim_{x \to 0} \frac{e^x - 1}{x} = 1$

12) $\lim_{x \to 0} \frac{a^x - 1}{x} = \ln a$

12) $\lim_{x \to 0} \frac{e^{3x} - 1}{x} = 3$

13) $\lim_{x \to 0} \frac{25^x - 1}{2x} = \frac{1}{2} \ln 25 = \ln \sqrt{25} = \ln 5$

14) $\lim_{x \to 0} \frac{e^{mx} - e^{nx}}{x} = m - n$

15) $\lim_{x \to 0} \frac{e^{11x} - e^{3x}}{x} = 8$

$x / \cos x = 0$

16) $\lim_{x \to 0} \frac{5e^x - e^{-x} - 4}{x} = 6$

$x / \tan x = 1$

17) $\lim_{x \to 0} \frac{\sin x}{x} = 1$

18) $\lim_{x \to 0} \frac{\tan x}{x} = 1$

19) $\lim_{x \to 0} \frac{\cos x}{x} = 0$

20) $\lim_{x \to 0} \frac{\sin 7x}{x} = 7$

21) $\lim_{x \to 0} \frac{\sin 4x}{\sin 5x} = \frac{4}{5}$

22) $\lim_{x \to 0} \frac{\sin 5x}{\tan 8x} = \frac{5}{8} =$

23) $\lim_{x \to 0} \frac{\tan 5x}{\tan 8x} = \frac{5}{8}$

24) $\lim_{x \to 0} \frac{\sin^2 3x}{x^2} = 9$

25) $\lim_{x \to 0} \frac{\sin x^0}{x} = \frac{\pi}{180}$

⇒ square.

26) $\lim_{x \to 0} \frac{1 - \cos 3x}{1 - \cos 5x} = \frac{9}{25}$

27) $\lim_{x \to 0} \frac{1 - \cos 2X}{X^2} = 2$

For I.F
Lopital theorm
Differentiate
Numerator
and also
Dinomentor

28) $\lim_{x \to 2} \frac{x^6 - 64}{x^3 - 8} = \frac{6}{3} (2^{6-3}) = 2.2^3 = 2^4 = 16$

29) $\lim_{x \to 0} \frac{e^{4x} - 1}{x}$

30) $\lim_{x \to 0} \frac{5^x - 1}{2x} = \frac{1}{2} \ln 5 = \ln \sqrt{5}$

31) $\lim_{x \to 0} \frac{5^x - 3^x}{x} = \ln 5 - \ln 3 = \ln\left(\frac{5}{3}\right)$

32) $\lim_{x \to 0} \frac{2^x - 1}{3^x - 1} = \frac{\ln 2}{\ln 3}$

*32) $\lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^x = e$

*33) $\lim_{x \to 0} (1 + x)^{\frac{1}{x}} = e$

*34) $\lim_{x \to \infty} \left(1 + \frac{5}{x}\right)^x = e^5$

35) $\lim_{x \to 0} (1 + x)^{-3/x} = e^{-3} = 1/e^3$

Convergence : A sequence is a said to be convergence of $\lim_{n \to \infty} \{a_n\} = 0$

Divergence: A sequence is a said to be divergence of $\lim_{n \to \infty} \{a_n\} = \infty$

For a infinite Geometric Series if $r > 1$ (*Divergent*) and $r < 1$ (*Convergent*)

Eg: $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, , , , , , , , ,$ *ALSO* $\frac{1}{10}, \frac{1}{100}, \frac{1}{1000}, , , , , ,$ (*Convergent*)

Eg: $1, 3, 9, 27, 81, , , , , , , , , , ,$ *ALSO* $\frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, , , , ,$ (*Divergent*)

Limit of Sequence : 1) $an = \frac{n^2+4}{3n^2+2}$ =>> $\lim_{n \to \infty} \frac{n^2+4}{3n^2+2} = \frac{1}{3}$

2) $\lim_{n \to \infty} an = \frac{n+4}{n^3 - 5} = 0$

3) $\lim_{n \to \infty} an = \frac{n^3 + 4}{2n - 5} = \infty$

4) $\lim_{n \to \infty} an = 4^{\infty} = \infty$

Monotonic Increasing and Decreasing Sequence:

A sequence $\{a_n\}$ *is called* Monotonic Increasing *if* $a_n < a_{n+1} \; \forall n \in N$

A sequence $\{a_n\}$ *is called* Monotonic Decreasing *if* $a_n > a_{n+1} \; \forall n \in N$

Rajab-Ali

MATH XII STRAIGHT LINE & GENERAL EQUATION OF STRAIGHT LINE

A Straight line is the minimum distance b/w two points in co – ordinate geometry.

Distance Formula A (x_1, y_1), B $(x_2 y_2)$ $\Rightarrow$ $D = \overline{AB} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Mid point Formula: for a line segment A(x_1, y_1), B(x_2, y_2) $P(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.

Division or Ratio Formula: If a Point $P(x, y)$ divides a join A(x_1, y_1) and B(x_2, y_2) in the ratio $m_1 : m_2$

Internally. $P(x, y) = \left(\frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}, \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}\right)$

Externally. $P(x, y) = \left(\frac{m_1 x_2 - m_2 x_1}{m_1 - m_2}, \frac{m_1 y_2 - m_2 y_1}{m_1 - m_2}\right)$

Centeroid of Triangle with vertices A(x_1, y_1), B(x_2, y_2), C(x_3, y_3)

$P(x, y) = \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$

In center of Triangle $P(x, y) = \left(\frac{a x_1 + b x_2 + c x_3}{a + b + c}, \frac{a y_1 + b y_2 + c y_3}{a + b + c}\right)$ a, b, c are sides of Δ

Slope of Line: For a line with end points A(x_1, y_1), B(x_2, y_2)

Slope $= m = \frac{y_2 - y_1}{x_2 - x_1}$, also if Inclination θ *given* $slope = m = Tan\theta$

* *Slope of* x – *axis* $= 0$ *Slope of* y – *axis* $= 0$

* *Angle* $\frac{b}{w}$ *two lines from* l_1, l_2, $Tan\,\theta = m_2 - m_1 / 1 + m_1 m_2$

x – *intercept*: *If* a line cuts x – axis, for x – *intercept*: *put* $y = 0$

y – *intercept*: *If* a line cuts y – axis, for y – *intercept*: *put* $x = 0$

Equation of Lines

1) *Parallel Condition*: a) If line is parallel to x axis y = a ;

b) If line is parallel to y axis x = b ;

(a is distance upward or downward)

(b is distance rightward or leftward.)

2) Two Point Form: Points given (x_1, y_1), $(x_2 y_2)$

Equation of line $\Rightarrow$ $\frac{y - y_1}{y_2 - y_1} = \frac{y - x_1}{x_2 - x_1}$ or $\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0$

3) Slope Point Form: slope m and point (x_1, y_1) $\Rightarrow$ $y - y_1 = m(x - x_1)$

* For perpendicular m = - 1/m

4) Slope Intercept Form: $y = m\,x + c$ with slope m and c, y – intercept given

* y = m x line passing through origin.

5) Intercept Form: $\frac{x}{a} + \frac{y}{b} = 1$ where a = x - intercept b = y - intercept.

6) Normal Form / Perpendicular Form: $x\,Cos\,\alpha + y\,Sin\,\alpha = p$,

$\alpha = angle$ *and p is perpendicular or normal.*

7) Symmetric Form: $\frac{y - y_1}{Sin\,\theta} = \frac{x - x_1}{Cos\,\theta}$ *Point*(x_1, y_1) *and angle given.*

* Lines are *Parallel* if Slopes are Equal $m_1 = m_2$ Lines are *Perpendicular* if Product of Slopes are equals to -1. $m_1 \times m_2 = -1$ or $m_1 = \frac{-1}{m_2}$

General Equation of Line : $ax + by + c = 0$ Slope of Line $= m = \frac{-a}{b}$

$L_1 \rightarrow a_1 x + b_1 y + c_1 = 0$ $L_2 \rightarrow a_2 x + b_2 y + c_2 = 0$

Parallel if $a_1 b_2 - a_2 b_1 = 0$ *Perpendicular* if $a_1 a_2 + b_1 b_2 = 0$

Angle b/w two lines $Tan\theta = \frac{a_1 b_2 - a_2 b_1}{a_1 a_2 + b_1 b_2}$

Lines are concurrent $L_1 \rightarrow a_1 x + b_1 y + c_1 = 0$ $L_2 \rightarrow a_2 x + b_2 y + c_2 = 0$

$L_3 \rightarrow a_3 x + b_3 y + c_3 = 0$ $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$

Point of Intersection of two lines. $L_1 \rightarrow a_1 x + b_1 y + c_1 = 0$ $L_2 \rightarrow a_2 x + b_2 y + c_2 = 0$

$$P(x, y) = \left(\frac{\begin{vmatrix} -c_1 & b_1 \\ -c_2 & b_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}, \frac{\begin{vmatrix} a_1 & -c_1 \\ a_2 & -c_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}} \right).$$

Distance from Point (x_1, y_1) to line $ax + by + c = 0$ D = $\frac{|a_1 x + b_1 y + c_1|}{\sqrt{a^2 + b^2}}$

Distance b/w two lines (Parallel) : $L_1 \rightarrow a_1 x + b_1 y + c_1 = 0$ $L_2 \rightarrow a_2 x + b_2 y + c_2 = 0$

$$d = \frac{c_1}{\sqrt{a_1^2 + b_1^2}} - \frac{c_2}{\sqrt{a_2^2 + b_2^2}}$$

The point (x_1, y_1) *is* *above* the line $ax + by + c = 0$ if $a_1 x + b_1 y + c_1 > 0$ and below if $a_1 x + b_1 y + c_1 < 0$ (i.e $b > 0$)

Area of Triangle with vertices $(x_1, y_1), (x_2, y_2) (x_3, y_3)$ $A = \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

Area of Quadilateral with vertices

$(x_1, y_1)(x_2, y_2) (x_3, y_3) (x_4, y_4)$ $A = \frac{1}{2} \begin{vmatrix} x_1 - x_3 & y_1 - y_3 \\ x_2 - y_4 & y_2 - y_4 \end{vmatrix}$

If Three points $(x_1, y_1), (x_2, y_2) (x_3, y_3)$ *are collinear* $\begin{vmatrix} x_1 & y_1 & x_3 \\ x_2 & y_2 & y_3 \\ x_3 & y_3 & y_3 \end{vmatrix} = 0$.

Equation of Pair of lines, Combined Equaton , $ax^2 + 2hxy + by^2 = 0$ through origin and perpendicular to the given will be $bx^2 - 2hxy + ay^2 = 0$

Roots of Equation $m_1 = \frac{-h + \sqrt{h^2 - ab}}{b}$, $m_2 = \frac{-h - \sqrt{h^2 - ab}}{b}$,

Sum = $m_1 + m_2 = \frac{-2h}{b}$, Product = $m_1 m_2 = \frac{a}{b}$

(7)

Angle b/w Pair of lines $Tan\ \theta = \frac{2\sqrt{h^2-ab}}{a+b}$ $If\ a+b=0$ $lines\ are\ Perpendicular.\ \theta = \frac{\pi}{2}$

$Discriminant:$ $D = h^2 - ab$. 1) $If\ h^2 - ab = 0$ Lines are Coincident or Parallel .

2) $h^2 - ab > 0$ Lines are Real and Distinct. 3) $h^2 - ab < 0$ Lines are Imaginary.

Practice Questions.

1) The length of line segment joining the points (12 , - 3) and(14 , - 8) = $\sqrt{(14-12)^2 + (-8+3)^2}$ = $\sqrt{4+25}$ = $\sqrt{29}$ $units.$

2) If P(3,4) is the mid point of line segment A (7,8) and B .Find the co ordinates of B.

* Using Mid point formula Ans. (-1,0)

3) Find the point of division of line segment joining (1,-2)to(-3,4) in the Ratio 2:3 internally

* Using Ratio formula Ans. (- 3/5 , 2/5)

4) If a line is trisected means dividing in 3 equal parts with Ratio 1:2 and 2:1 .

5) If a point is 4/5 the way on $\overline{AB}$ line then $m_1 : m_2 = 4:1$ * $m_1 = 4\ and\ m_2 = 5 - 4 = 1$

6) The slope of line is ¾ . The slope of second line parallel to given = ¾ and perpendicular = $-4/3$

7) The slope of line joining the points (1,3)(-4,5) . Ans. m = -2/5 * Using $m = \frac{y_2 - y_1}{x_2 - x_1}$

8) The slope of with inclination 45^0 or $\frac{\pi}{4}$. Ans. m= 1 . Using $m = Tan\ \theta$

9) The x intercept of line 3x – 5 y – 1 5 = 0 . Ans. x = 5 * Put y = o =>> 3 x = 15 =>> , x = 5

10) The measure of angle from a line with slope 2/3 to the other line with slope - 3/2 . Ans. $\theta = \frac{\pi}{2}$

11) Find the equation of line Parallel to x – axis 4 units below . (Ans. y = - 4) . Parallel to y- axis 3 units right Ans. x = 3 Parallel to x- axis and passing through (2,-3) , y = -3 . Parallel to y- axis and passing through (2,-3) , x = 2 also Perpendicular to x – axis passing (2,-3) , x = 2

12) Equation of line through the points (7,-3) and (-4.1) Ans. 4x+11y+5 = 0 * Using two point form

13) Equation of line through the point (5,-2) with slope 4 Ans. 4x – y – 22 = 0 * Using Slope point form

14) The slope of a line is -3/4 .Find the Equation of line perpendicular the line (- 6 , 2) Ans. 4x – 3y +30 = 0

15) Equation of line with y –Intercept = 3 and slope = 2 Ans. 2x – y +3 = 0 * Using Slope intercept form

16) Equation of line with x – Intercept = 4 , y – Intercept = 3 Ans.3x +4 y – 12 = 0 * Using two intercept form

17) Equation of line with perpendicular from origin to the line P=3 and makes an angle $\theta = 60^0$

Ans. $x + \sqrt{3}y - 6 = 0$ Using Normal Form

18) Equation of line passing through (4, $\sqrt{3}$) and makes an angle 30^0 with x –axis . Ans. $x - \sqrt{3}y - 1 = 0$

19) The equation of line with point passing through (5,3) and perpendicular to to line 4x+5y=9.

Ans. 5x – 4 y – 13 = 0 * slope point formula.

20) The point (– 8. – 3) to the line 2x – 3 y + 4 = 0 Ans. (Above) * b > 0

21) The distance from point (6,-2) to the line 3 x – 4 y + 4 =0 Ans. 6 units . * Using D = $\frac{|a_1 x + b_1 y + c_1|}{\sqrt{a^2+b^2}}$

22) The distance b/w the parallel lines 5x-12y+10=0 and 5x-12y-16 = 0 Ans. 2 *d = $\frac{c_1}{\sqrt{a_1^2+b_1^2}} - \frac{c_2}{\sqrt{a_2^2+b_2^2}}$

23) The area of triangle whose vertices are (3 , 1) (- 2 , 5)(- 4,- 5) Ans.29 sq units * $A = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

24) The equations of pair of lines of x^2-5xy+6y^2 = 0 Ans. x – 3y = 0 and x – 2 y = 0 * factorize

25) The angle b/w lines represented by 3x^2+7xy+2y^2= 0 is Ans. $\theta = \frac{\pi}{4}$ * $Tan\ \theta = \frac{2\sqrt{h^2-ab}}{a+b}$

Rajdo - Ali - Kamlo

UNIT 6 CIRCLE

Circle : Points in a plane that are at the same distance(radius) from a fixed point (center).
R^2 - Space. A general **Quadratic equation** or **Second degree** with no term xy
coefficient of x^2 = **coefficient** of y^2

Radius : A line segment joining the center and any point on the circle.
Chord : A line segment joining the two points on the circle. **Area of Circle** = πr^2.
Diameter : A chord that passes through the center of the circle. $d = 2r$
Arc : The part of the circle containing two points.
Major Arc 360^0 - x Minor Arc < 180^0
Secant : Line that intersects a circle at than two points.

Tangent Line Line that touches the circle at one point.
Normal Line: Line that is **perpendicular** to the tangent line and lie towards the center of circle.

Standard Equation of Circle:
$$(x-a)^2+(y-b)^2=r^2$$ Center (a , b) : Radius : r
$$x^2+y^2=r^2$$ Center on origin (0 , 0)

General Equation of Circle:
$$x^2+y^2+2gx+2fy+c=0$$
where center : (- g , - f) **Radius :** $r=\sqrt{g^2+f^2-c}$

- If center on x – axis then f = 0 : $x^2+y^2+2gx+c=0$
- If center on y – axis then g = 0 : $x^2+y^2+2fy+c=0$
- If center on origin then f=0, g = 0 : $x^2+y^2+c=0$

- Equation of circle touching x axis : $x^2+y^2-2gx-2fy+g^2=0$
- Equation of circle touching y axis : $x^2+y^2-2gx-2fy+f^2=0$
- **Concentric Circles** means Circles with same center.
- Circle with no radius is called **Point Circle.**

Equation of circle with ends of Diameter are (x_1, y_1) (x_2, y_2) is
$$(x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0$$

- Equation of tangent to the circle $x^2+y^2=r^2$ at the point (x_1, y_1) is $xx_1+yy_1=r^2$
- Equation of Normal to the circle $x^2+y^2=r^2$ at the point (x_1, y_1) is $xy_1-yx_1=0$ $\Rightarrow \frac{x}{x_1}=\frac{y}{y_1}$

Length of the tangent line from a point P(x_1 , y_1) to the circle $x^2+y^2+2gx+2fy+c=0$ is
$$L=\sqrt{x_1^2+y_1^2+2gx_1+2fy_1+c}$$

- The point (x_1, y_1) lies the circle $x^2+y^2+2gx+2fy+c=0$ **inside / outside / on**
if $x_1^2+y_1^2+2gx_1+2fy_1+c$ < 0 or > 0 or = 0

- Parametric Equations of Circle: $x = r\ \text{Cos}\ \theta$, $y = r\ \text{Sin}\ \theta$
- Condition for line $y = mx + c$ to be tangent to the circle $x^2+y^2=a^2$ is
$c=\pm a\sqrt{1+m^2}$ OR $c^2=a^2(1+m^2)$
- Line intersect the circle at two points and its nature of points based as

Real and Distinct if: $a^2(1+m^2)-c^2>0$ **Real and Coincident if :** $a^2(1+m^2)-c^2=0$
Imaginary : $a^2(1+m^2)-c^2<0$
Director of circle $x^2+y^2=r^2$ is given by $x^2+y^2=2r^2$

Practice Questions

- The Equation of Circle with center (4 , - 2) and radius 8 units is Equation of Circle $x^2+y^2-8x+4y-44=0$
- The equation of circle with radius 5 and center on origin is $x^2+y^2=25$
- The length of tangent segment from a point (3 , 4) to the circle $x^2+y^2+2x-4y+7=0$ is $5\sqrt{2}$
- The Equation of tangent and Normal to the circle $x^2+y^2=25$ at (5 , 2) will be $5x+2y=25$ & $2x-5y=0$
- Circle through (0 , 0) (6 , 0) (0 , 4) will be $x^2+y^2-6x-4y=0$ its Circum - centre means center (3 , 2).

Kajal - Ali

UNIT 6 INTEGRATION / ANTI DERIVATIVE

The process of finding anti derivative of a function is called Integration.

$\int f(x)dx$ f(x) integrand * $\int dx$ opposite of d/dx

$\int x^n dx = \frac{x^{n+1}}{n+1} + c$ $\int dx = x + c$ * $\int 0\ dx = c$

linear

(1) $\int (ax + b)^n dx = \frac{(ax+b)^{n+1}}{a(n+1)} + c$ (2) $\int e^{ax} dx = \frac{e^{ax}}{a} + c$ (3) $\int a^{bx} dx = \frac{a^{bx}}{b\ln a} + c$

(4) $\int (f(x))^n\ f'(x)\,dx = \frac{(f(x))^{n+1}}{n+1} + c$ (5) $\int \frac{f'(x)}{f(x)}\,dx = \ln f(x) + c$

(6) $\int e^{f(x)}\ f'(x)\,dx = e^{f(x)} + c$ (7) $\int a^{f(x)}\ f'(x)\,dx = \frac{a^{f(x)}}{\ln a} + c$

(8) $\int e^{ax}\left(f(x) + f'(x)\right)dx = e^{ax} f(x) + c$

Trignometric Integerations

$\int \sin x\, dx = -\cos x + c$ $\int \cos x\, dx = \sin x + c$

$\int \tan\, dx = \ln(\sec x) + c$ $\int \cot x\, dx = \ln \sin x + c$

$\int \sec x\, dx = \ln(\sec x + \tan x) + c$ $\int \operatorname{cosec} x\, dx = \ln(\operatorname{cosec} x - \cot x) + c$

$\int \sec^2 x\, dx = \tan x) + c$ $\int \operatorname{cosec}^2 x\, dx = -\cot x) + c$

$\int \sec x \tan x\, dx = \sec x + c$ $\int \operatorname{cosec} x \cot x\, dx = -\operatorname{cosec} x + c$

Integeration by Formulae

$\int dx / x^2 - a^2 = \frac{1}{2a}\ln\ (x-a)/(x+a) + c$ $\int dx / a^2 - x^2 = \frac{1}{2a}\ln\ (a+x)/(a-x) + c$

$\int dx / \sqrt{a^2 - x^2} = \sin^{-1}(x/a) + c$ $\int dx / x^2 + a^2 = 1/a\ \tan^{-1}(x/a) + c$

$\int dx / x\sqrt{x^2 - a^2} = 1/a\ \sec^{-1}(x/a) + c$

$\int dx / \sqrt{x^2 + a^2} = \ln\ (x + \sqrt{x^2 + a^2}) + c$ $\int dx / \sqrt{x^2 - a^2} = \ln\ (x + \sqrt{x^2 - a^2}) + c$

$\int \sqrt{a^2 - x^2}\, dx = ½\ x \ln \sqrt{a^2 - x^2} + a^2 \sin^{-1} x/a + c$ Substitution $x = a \sin\theta$

$\int \sqrt{x^2 + a^2}\, dx = ½\ x \ln \sqrt{x^2 + a^2} + ½\ a^2 \ln\ (x + \sqrt{x^2 + a^2}) + c$ Substitution $x = a \tan\theta$

$\int \sqrt{x^2 - a^2}\, dx = ½\ x \ln \sqrt{x^2 - a^2} - ½\ a^2 \ln\ (x + \sqrt{x^2 - a^2}) + c$ Substitution $x = a \sec\theta$

* $\int e^{ax} dy = y\, e^{ax} + C$

Derivative = 0

54
680
880

-26

7

Integeration by Parts

$\int uv\,dx = u\int v\,dx - \int u' \int v\,dx$

$\int e^{ax}\sin bx\,dx = \frac{e^{ax}}{a^2+b^2}(a \sin x - b\cos x) + c$ $\int e^{ax}\cos bx\,dx = \frac{e^{ax}}{a^2+b^2}(a\cos x + b\sin x) + c$

Definite Integrals $\int_a^b f(x)dx = F(b) - F(a)$ $\int_a^a f(x)dx = 0$

Differential equations $\frac{dy}{dx} = x^2 + 9x + 3$

Find General solution $\int dy = \int (x^2 + 9x + 3)dx \Rightarrow y = x^3/3 + 9x^2/2 + 3x + c$

Area of Curve $A = \int_a^b y\,dx$

where y = curve given and a and b are the points of start and end of curve

PRACTICE QUESTIONS

* $\int (x^3 + 5x^{-2} + x^{2/3} - 4x - 3)dx = x^4/4 - 5/x + 3/5\ x^{5/3} - 2x^2 - 3x + c$

* $\int (3x-4)^3\,dx = (3x-4)^4/12 + c$ * $\int \sin 4x\,dx = -\cos 4x/4 + c$

* $\int \frac{1}{x}dx = \ln x + c$ $\int \frac{x-2}{x^2-2x+4}dx = \frac{1}{2}\ln(x^2 - 2x + 4) + c$

$\int dx/\tan^{-1}x\,(1+x^2) = \ln \tan^{-1} x + c$ $\int \sin^3 x \cos x\,dx = \frac{1}{4}\sin^4 x + c$

$\int \frac{(\ln)^4}{x} = \frac{(\ln)^5}{5} + c$ $\int (\cos^{-1}x)^3\,(-1/\sqrt{1-x^2})\,dx = \frac{1}{4}(\cos^{-1}x)^4 + c$

$\int e^{7x}dx = e^{7x}/7 + c$ $\int 4^{3x}dx = 4^{3x}/3\ln 4 + c$

$\int e^{\tan 4x}\sec^2 4x\,dx = \frac{1}{4}e^{\tan 4x} + c$ $\int x\,dx/x+2 = x - \ln(x+2) + c$

$\int e^x(\tan x + \sec^2 x)dx = e^x \tan x + c$ $\int dx/x^2+9 = 1/3\ \tan^{-1}(x/3) + c$

$dy/dx = x^2 + 4$, the solution of equation will be $y = \frac{1}{2}\tan^{-1}(x/2) + c$

BY PARTS $\int \ln x\,dx = x\ln x - x + c$ $\int x\sin x\,dx = -x\cos x + \sin x + c$

$\int \sin^2 x\,dx = (1 - \cos 2x/2) = \frac{1}{2}x - \frac{1}{4}\sin 2x + c$ $\int \cos^2 x\,dx = (1 + \cos 2x/2) = \frac{1}{2}x + \frac{1}{4}\sin 2x + c$

$\int \tan^2 x\,dx = (\sec^2 x - 1) = \tan x - x + c$ $\int e^{4x}\sin 3x\,dx = e^{4x}/25\ (4\sin 3x - 3\cos 3x) + c$

$\int e^{2x}\cos 4x\,dx = e^{2x}/20\ (2\cos 4x + 4\sin 4x) + c$ $\int e^x\,dx/e^x+3 = \ln(e^x + 3) + c$