Massive MU-MIMO Downlink TDD Systems with Linear Precoding and Downlink Pilots

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Motivation

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- Massive number of devices
- Higher Data Rates
- High Capacity
- Realiability
- Mobility

Contents

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Massive MU-MIMO Systems

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- Number of transmit antennas approaches to infinity.
- Payload transmission is based on linear processing at BS.
- Linear receive combining is applied to discriminate transmitted signal from interfering signals.
- Linear Precoding Schemes to focus each signal to a desired transmitter
  - Zero-Forcing (ZF)
  - MRT
- CSI is required at the Base Station.

FDD vs TDD

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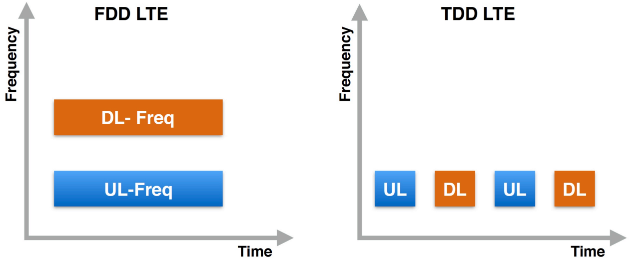


DL - Downlink
UL - Uplink

Channel estimation in FDD

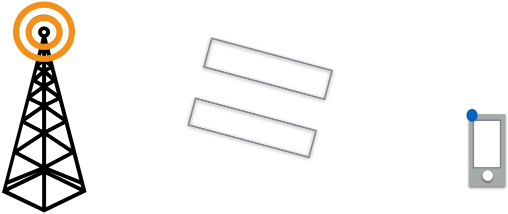


Figure: FDD channel estimation

Channel estimation in TDD

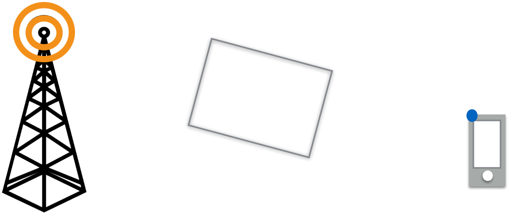
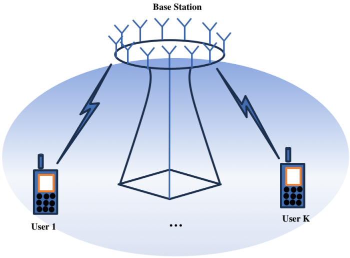


Figure: TDD channel estimation

System Model



- $M \gg K$,
- Time Division Duplexing Scheme,
- ZF and MRT for precoding,
- $\mathbf{H}$ the channel matrix has i.i.d Gaussian distributed elements with zero mean and unit variance.

Uplink Training

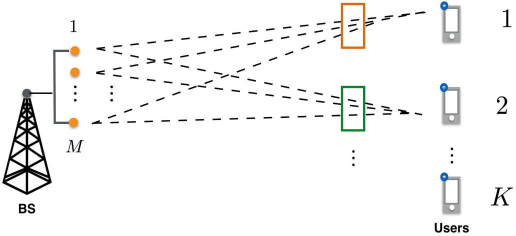
MMSE Channel estimate at the BS [3]

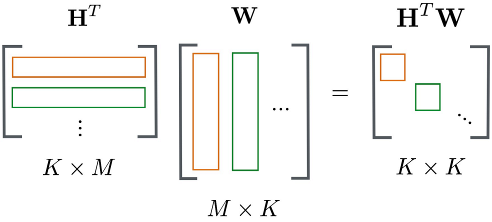
$$\hat{\mathbf{H}} = \frac{\rho_u \tau_u}{\rho_u \tau_u + 1}\mathbf{H} + \frac{\sqrt{\rho_u \tau_u}}{\rho_u \tau_u + 1}\mathbf{N}_u,$$

Where

- $\mathbf{H}$: $M \times K$ Dimensional i.i.d Gaussian matrix with zero mean unit variance,
- $\mathbf{N}_u$: Gaussian noise with i.i.d $C\mathcal{N}(0, 1)$,
- ρ_u: average transmit power of each uplink pilot symbol,
- τ_u: length of pilot symbols per coherence interval.

Precoding


1
M
BS
1
2
K
Users


$\mathbf{H}^T$
$\mathbf{W}$
$\mathbf{H}^T\mathbf{W}$
=
$K \times M$
$M \times K$
$K \times K$

Downlink Transmission

The precoded signal to be transmitted is given as:

$$x = \sqrt{\rho_d}\mathbf{W}\mathbf{s}$$

The collective vector of samples received at the K users is given by

$$\mathbf{y} = \sqrt{\rho_d}\mathbf{H}^T\mathbf{W}\mathbf{s} + \mathbf{n},$$

The signal received at the *kth* user is given by

$$y_k = a_{kk}s_k\sqrt{\rho_d} + \sqrt{\rho_d}\sum_{i \neq k}^{K} a_{ki}s_i + n_k,$$

Where

- s_k : symbol transmitted to *kth* user with ρ_d average transmit power at BS,
- $a_{ki} \triangleq h_k^T w_i$,
- $\mathbf{W}$: $M \times K$ precoding matrix dependent on $\hat{\mathbf{H}}$.

Beamforming Training Scheme

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The pilot matrix is given by

$$\mathbf{S}_p = \sqrt{\rho_d \tau_d} \phi$$

The received pilot matrix at the K users is given by

$$\mathbf{Y}_p^T = \sqrt{\rho_d \tau_d} \mathbf{H}^T \mathbf{W} \phi + \mathbf{N}_p^T,$$

The received processed signal at the K users is given by

$$\tilde{\mathbf{Y}}_p^T = \mathbf{Y}_p^T \phi^H$$

- ϕ : Pairwise orthonormal pilot symbols.

Achievable Downlink Rate

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- **Maximum-Ratio Transmission**
  Channel estimate $\hat{a}_{ki}$ and the lower bound on achievable rate R_k is given as proposed in[2]

$$\hat{a}_{ki} = \frac{\sqrt{\rho_d \tau_d}}{\rho_d \tau_d + K} \tilde{\mathbf{y}}_{\mathbf{p},\mathbf{ki}} + \frac{K}{\rho_d \tau_d + K} \sqrt{\frac{\rho_u \tau_u M}{K(\rho_u \tau_u + 1)}} \delta_{ki},$$

$$R_k = \mathbb{E}\left\{ \log_2 \left(1 + \frac{\rho_d |\hat{a}_{kk}|^2}{\rho_d \sum_{i \neq k}^{K} |\hat{a}_{ki}|^2 + \frac{K \rho_d}{\rho_d \tau_d + K} + 1} \right) \right\}, \tag{1}$$

Achievable Downlink Rate

- **Zero-Forcing**
  Channel estimate $\hat{a}_{ki}$ and the lower bound on achievable rate R_k is given as proposed in [2]

$$
\begin{aligned}
\hat{a}_{ki} &= \frac{\sqrt{\rho_d \tau_d}}{\rho_d \tau_d + K(\rho_u \tau_u + 1)} \tilde{\mathbf{y}}_{\mathbf{p},\mathbf{ki}} + \frac{\sqrt{K(M-K)\rho_u \tau_u (\rho_u \tau_u + 1)}}{\rho_d \tau_d + K(\rho_u \tau_u + 1)} \delta_{ki}, \\
R_k &= \mathbb{E}\left\{ \log_2 \left(1 + \frac{\rho_d |\hat{a}_{kk}|^2}{\rho_d \sum_{i \neq k}^{K} |\hat{a}_{ki}|^2 + \frac{K \rho_d}{\rho_d \tau_d + K(\rho_u \tau_u + 1)} + 1} \right) \right\},
\end{aligned} \tag{2}
$$

Spectral Efficiency

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Beamforming spectral efficiency defined as sum-rate(in bits) per channel use :

$$S_{TB} = \frac{T - \tau_u - \tau_d}{T} \sum_{i \neq k}^{K} R_k, \tag{3}$$

- T : Length of coherence interval,
- τ_u: Length of uplink pilot symbols,
- τ_d: Length of downlink pilot symbols.

Simulation Results (1)

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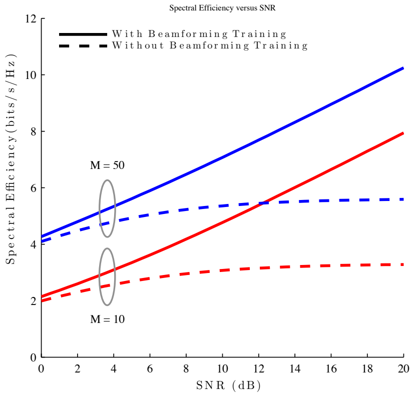

Figure: Spectral efficiency versus SNR for single-user setup (K=1, ρ_u = 0 dB and T=200)

Simulation Results (2)

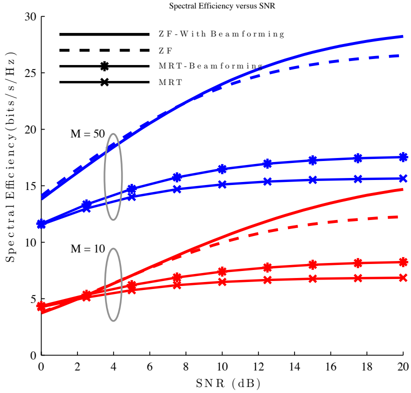

Figure: Spectral efficiency versus SNR for single-user setup (K=1, ρ_u = 0 dB and T=200)

Simulation Results (3)

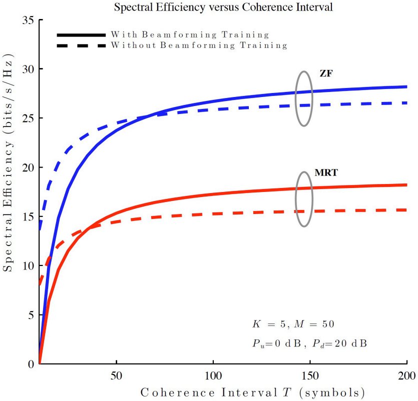

Figure: Spectral efficiency vs T for MRT, ZF precoding (K=5, M=50, ρ_u = 0 dB, ρ_d = 20 dB)

Simulation Results(4)

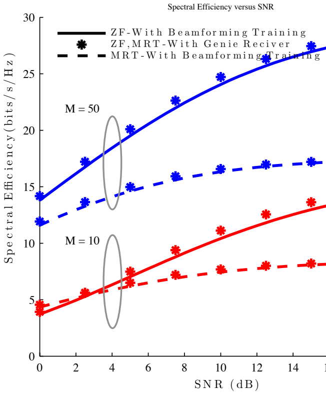

Figure: Spectral efficiency versus SNR with a genie receiver (K=5, ρ_u = 0 dB ,T=200)

Conclusion

- Beamforming training scheme is proposed and analyzed.
- Overhead of channel estimation is proportional to the number of users instead of number of BS antennas in a massive MIMO system.
- Spectral efficiency loss associated with transmission of downlink pilot symbols.

Reference

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Thank You