Massive MU-MIMO Downlink TDD Systems with Linear Precoding and Downlink Pilots

Wahaj Aslam
Mariam Sohail

TECHNISCHE UNIVERSITÄT DARMSTADT

TECHNISCHE UNIVERSITÄT DARMSTADT

Contents

1. Introduction

Massive MIMO also known as Large Scale Antenna System makes use of larger number of antennas to focus the transmission and reception of signal energy for smaller regions of space [4]. This brings huge growth in throughput and energy efficiency. Large number of paths between transmitting and receiving end provides a very high data rate and reliability. It makes use of very simple precoding schemes such as Zero-Forcing (ZF) and Maximum ratio transmission (MRT) for uplink and downlink transmissions. In order to fully acknowledge the advantages of such large number of antennas, channel state information (CSI) is required at the base station (BS) and/or the users.

In case where the number of antennas is relatively small, each user can estimate the CSI based on the downlink training symbols and then feed it back to the BS using frequency-division duplex (FDD) in uplink. However due to large number of antennas in a massive MIMO system this is a difficult task, as the downlink resources for pilots, required for the feedback is relative to the number of the antennas at the BS. The time required to transmit the downlink pilot symbols is proportional to the number of antennas at the BS. As the number of BS antennas grows large, the traditional downlink channel estimation strategy for FDD systems becomes infeasible.Therefore, Massive MIMO system makes use of Time-Domain Duplexing (TDD) to take the advantage of channel reciprocity [4]. With the use of TDD, Massive MIMO can achieve the CSI through an open loop with the uplink transmission. The signal transmitted from the user can be decoded using the channel information, and also for downlink transmission BS can precode the signals to be transmitted. However, the user does not have the channel information to reliably decode the transmitted signals in the downlink. A simple pilot scheme could solve the issue and the user can decode the data using CSI obtained from the pilot symbols. However, the pilot overhead will again be proportional to the large number of BS antennas in downlink.

A beamforming scheme is proposed to measure the CSI at each user. The pilot length (symbols) proportional to the number of users instead of number of BS antennas in the system. BS applies beamforming at the downlink transmission so that each user can estimate the channel.

2. System Model

As illustrated in figure 2.1, uplink and downlink transmissions are considered in a MU-MIMO system with M antennas at BS and K users with the following assumptions:

- The number of BS antennas are much larger than number of users i.e. M»K
- TDD operation is considered so that the channel response is assumed to be equal for both uplink and downlink transmission.
- Linear precoding scheme at BS is applied to process the signal before transmitting it to the user. This requires the CSI.
- **H** is the channel matrix between the BS and the users and each element of the channel matrix **H** is i.i.d Gaussian distributed with zero mean and unit variance.

The performance of the proposed beamforming technique is evaluated with the derivation of the lower bound on the capacity measure of two linear precoding schemes, i.e. , MRT and ZF.

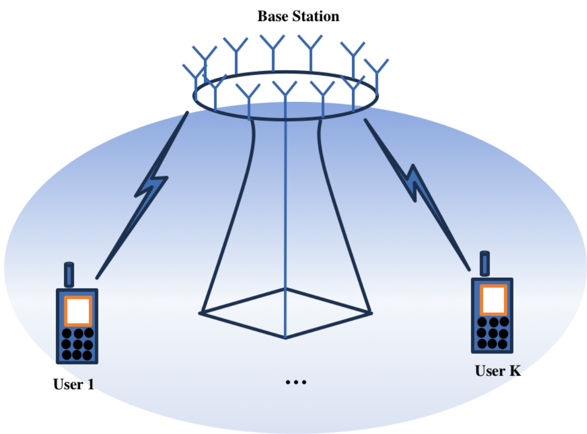

Figure 1: Massive MU-MIMO downlink system model

CSI is obtained through uplink training symbols for estimating precoding matrix at the BS. All users simultaneously transmit pilot symbols of length τ_u, which are pairwise orthogonal. MMSE estimate of the channel at the BS through uplink is given by [3]

$$\hat{\mathbf{H}} = \frac{\rho_u \tau_u}{\rho_u \tau_u + 1}\mathbf{H} + \frac{\sqrt{\rho_u \tau_u}}{\rho_u \tau_u + 1}\mathbf{N}_u, \tag{1}$$

Where $\mathbf{H}$ is of size $M \times K$, $\mathbf{N}_u$ is the Gaussian noise with i.i.d entries with zero mean and unit variance and ρ_u is the average transmit power of every uplink pilot symbol.

2.1 Downlink Transmission

Let s_k be the symbol to be transmitted to the $\mathbf{k}th$ user, $\mathbf{W}$ be the $M \times K$ precoding matrix that depends on $\hat{\mathbf{H}}$ and estimated as proposed in [1]. The signal received at the kth user is given by

$$y_k = a_{kk} s_k \sqrt{\rho_d} + \sqrt{\rho_d} \sum_{i \neq k}^{K} a_{ki} s_i + n_k, \tag{2}$$

where n_k is the kth element of the additive noise vector $\mathbf{n}$. Define $a_{ki} \triangleq h_k^T w_i$, where h_i and w_i are the ith columns of $\mathbf{H}$ and $\mathbf{W}$, respectively. CSI is required at each user for coherent detection of the sent symbols, but due to large M it is inefficient to simply use downlink pilot scheme. From (2) it can be seen that complete knowledge of $\mathbf{H}$ is not required, instead only a_{kk} is required to reliably decode the signal. Hence, small amount of coherence interval on downlink training is needed to acquire a_{kk} at each user. The beamforming training scheme to estimate a_{kk} provided in this paper is explained in next section. The advantage of this scheme is that the training overhead is proportional to number of users K.

2.2 Beamforming Training Scheme

The BS beamforms the pilot symbols and a_{ki} is estimated at receiver using received pilot symbols. The received signal at kth user is given by

$$\mathbf{Y}_p^T = \sqrt{\rho_d \tau_d} \mathbf{H}^T \mathbf{W} \boldsymbol{\phi} + \mathbf{N}_p^T, \tag{3}$$

where τ_d is the duration of downlink training, ϕ is a part of pilot matrix with pairwise orthonormal rows. For channel estimation it is sufficient to use:

$$\hat{\mathbf{Y}_p^T} \triangleq \mathbf{Y}_p^T \boldsymbol{\phi}^H \tag{4}$$

The channel is estimated according to the proposed beamforming training scheme and is used it to find the lower bound on achievable downlink rate for ZF and MRT precoding techniques.

Maximum-Ratio Transmission

For MRT the channel estimate $\hat{a}_{ki}$ and the lower bound on achievable rate R_k is given as proposed in [2]

$$\hat{a}_{ki} = \frac{\sqrt{\rho_d \tau_d}}{\rho_d \tau_d + K} \tilde{\mathbf{y}}_{\mathbf{p,ki}} + \frac{K}{\rho_d \tau_d + K} \sqrt{\frac{\rho_u \tau_u M}{K(\rho_u \tau_u + 1)}} \delta_{ki}, \tag{5}$$

$$R_k = \mathbb{E}\left\{ \log_2 \left(1 + \frac{\rho_d |\hat{a}_{kk}|^2}{\rho_d \sum_{i \neq k}^{K} |\hat{a}_{ki}|^2 + \frac{K\rho_d}{\rho_d \tau_d + K} + 1} \right) \right\}, \tag{6}$$

Zero-Forcing

For ZF the channel estimate $\hat{a_{ki}}$ and the lower bound on achievable rate R_k is given as proposed in [2]

$$\hat{a}_{ki} = \frac{\sqrt{\rho_d \tau_d}}{\rho_d \tau_d + K(\rho_u \tau_u + 1)} \tilde{\mathbf{y}}_{\mathbf{p,ki}} + \frac{\sqrt{K(M-K)\rho_u \tau_u (\rho_u \tau_u + 1)}}{\rho_d \tau_d + K(\rho_u \tau_u + 1)} \delta_{ki}, \tag{7}$$

$$R_k = \mathbb{E}\left\{ \log_2 \left(1 + \frac{\rho_d |\hat{a}_{kk}|^2}{\rho_d \sum_{i \neq k}^{K} |\hat{a}_{ki}|^2 + \frac{K\rho_d}{\rho_d \tau_d + K(\rho_u \tau_u + 1)} + 1} \right) \right\}, \tag{8}$$

3. Results and Simulations

In this section the performance of the proposed scheme is studied with the help of spectral efficiency which is defined as sum-rate(in bits) per channel use. If T is the length of coherence interval then the beamforming spectral efficiency is given as

$$S_{TB} = \frac{T - \tau_u - \tau_d}{T} \sum_{i \neq k}^{K} R_k, \tag{9}$$

In the following simulations $\tau_u = \tau_d = K$, $\rho_u = 0dB$ and SNR=ρ_d.

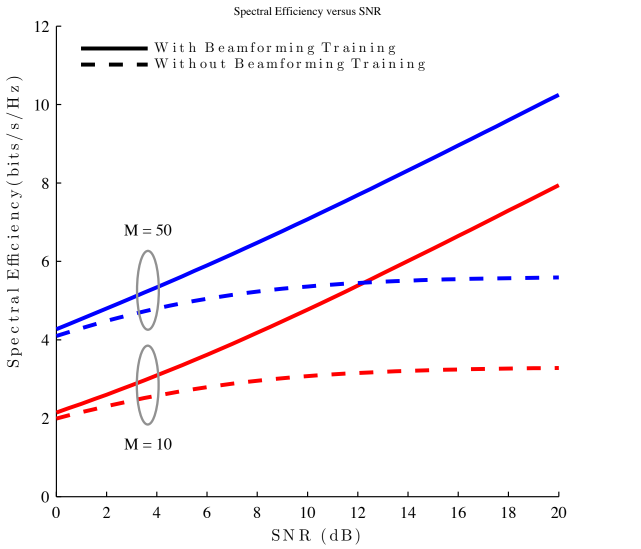

Figure 2: Spectral efficiency versus SNR for single-user setup (K=1, $\rho_u = 0$ dB and T=200)

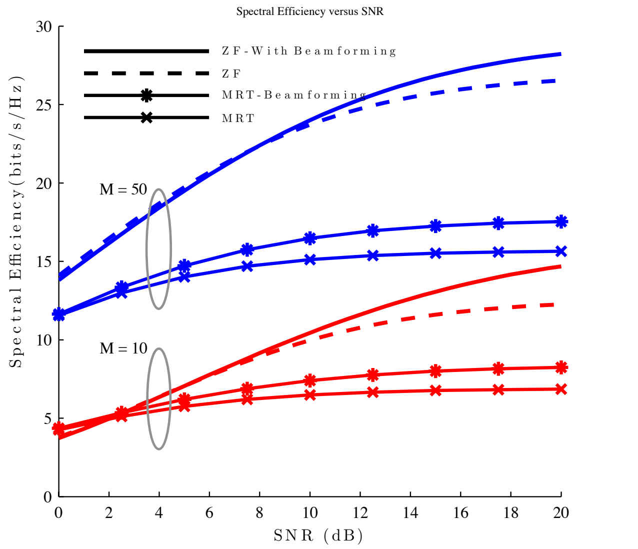

Figure 3: Spectral efficiency versus SNR for multi-user setup (K=5, $\rho_u = 0$ dB and T=200)

In Figure 2, single user setup (K=1) is considered. In this case the performance for both precoding schemes is approximately the same. With different number of BS antennas (M=10, 50) it is observed that the spectral efficiency of beamforming scheme outperformes the efficiency of the system without beamforming. As the SNR increases the performance gap increases, since increase in SNR leads to more accurate channel estimate at each user.

In Figure 3 multiuser setup is considered, i.e., K=5. The observation is made for M=10 and M=50, and coherence interval T=200. Again the same trend is observed, the beamforming scheme performs better with higher spectral efficiency as the SNR increases. It is also observed that ZF performs better than MRT, which is due to the fact that the randomness of the channel gain is smaller for ZF than with MRT.

Furthermore, the effect of coherence interval on the system performance is observed. In Figure 4 spectral efficiency versus length of coherence interval T at M=50, K=5 and ρ_d = 20 dB is investigated. In a high mobility environment i.e. for short coherence interval the training duration is relatively large, so the beamforming scheme should not be used to esitmate CSI at each user. At moderate and large coherence interval the training duration is relatively small compared to T, so in such a case beamforming scheme is suitable.

Finally the genie receiver is considered which assumes that the k_{th} user has perfect channel estimate a_{ki}. Spectral effieciency versus SNR with genie receiver for the proposed scheme is studied in Figure 5. Here, K=5 and T=200. The comparison shows that the performance gap between the two cases is very small. This proves that the estimate of the elements of a_{ki} is quiet reasonable.

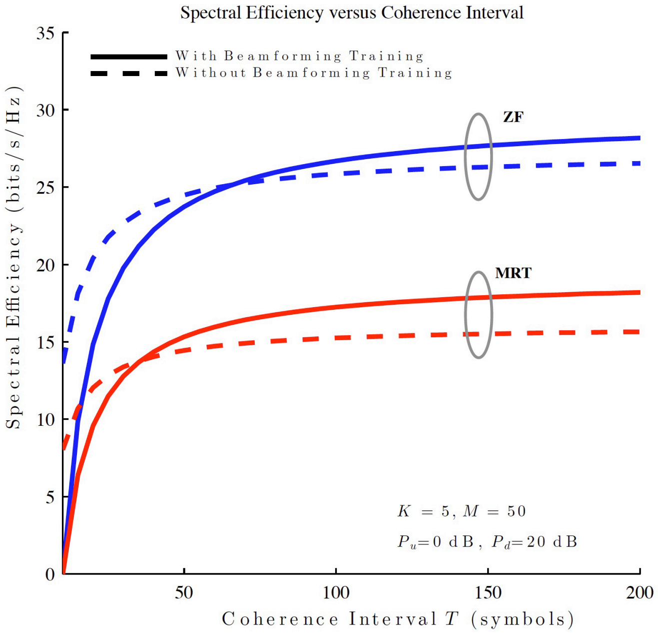

Figure 4: Spectral efficiency versus coherence interval for MRT, ZF precoding (K=5, M=50, $\rho_u = 0$ dB, $\rho_d = 20$ dB)

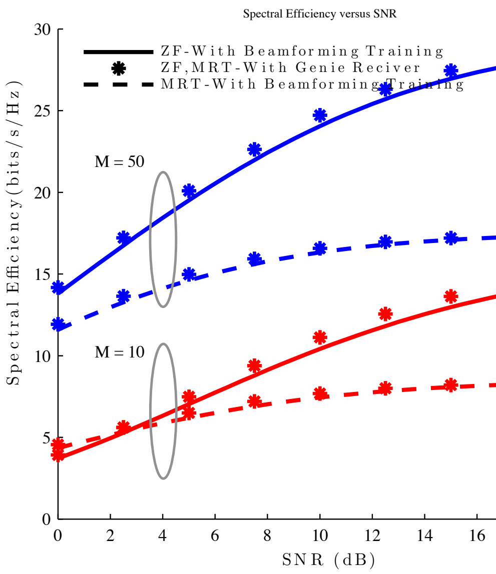

Figure 5: Spectral efficiency versus SNR with a genie receiver (K=5, $\rho_u = 0$ dB ,T=200)

4. Conclusion

In this paper [1], beamforming training scheme is proposed and analyzed. In this scheme CSI is acquired at each user in the downlink of a massive MU-MIMO system to coherently detect the transmitted signal. BS processes the training sequence using precoding techniques and sends it to the user for downlink channel estimation. The overhead of channel estimation of the proposed scheme is proportional to the number of users instead of number of BS antennas in a massive MIMO system.

Bibliography

[1] Ngo, Hien Quoc, Erik G. Larsson, and Thomas L. Marzetta. *Massive MU-MIMO downlink TDD systems with linear precoding and downlink pilots*. Communication, Control, and Computing (Allerton), 2013 51st Annual Allerton Conference on. IEEE, 2013.

[2] Ngo, Hien Quoc, Erik G. Larsson, and Thomas L. Marzetta. *The multicell multiuser MIMO uplink with very large antenna arrays and a finite-dimensional channel.* IEEE Transactions on Communications 61.6 (2013): 2350-2361.

[3] Sengijpta, Sailes K. *Fundamentals of statistical signal processing: Estimation theory.* Technometrics 37.4 (1995): 465-466.

[4] Larsson, Erik G., et al. *Massive MIMO for next generation wireless systems.* IEEE Communications Magazine 52.2 (2014): 186-195