DISSERTATION

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Robust Methods in Small Area Estimation: Numerical Solutions and Their Implementations

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CONTENTS

Part I

THEORY

This is the chapter where I want to present the theoretical concepts underpinning the development of software and application. Most notably is the robust version of a Fay-Herriot Type model with different variance-covariance structures.

1 REVIEW OF ROBUST METHODS IN SMALL AREA ESTIMATION

1.1 UNIT-LEVEL MODELS

1.2 AREA-LEVEL MODELS

1.3 MEAN SQUARED ERROR

1.3.1 *pseudo linear FH*

This is the representation of the pseudo linear representation of the FH model. As it is introduced in Chambers, J. Chandra, and Tzavidis (2011) and Chambers, H. Chandra, et al. (2014).

Presenting the FH in pseudo linear form means to present the area means as a weighted sum of the response vector y. The FH model is given by

$$\theta_i = \gamma_i y_i + (1 - \gamma_i) x_i^\top \beta \tag{1}$$

where $\gamma_i = \frac{\sigma_u^2}{\sigma_u^2 + \sigma_e^2}$, so it can be represented as

$$\theta_i = w_i^\top y$$

where

$$w_i^\top = \gamma_i \mathbf{I}_i^\top + (1 - \gamma_i) x_i^\top \mathbf{A}$$

and

$$\mathbf{A} = \left(\mathbf{X} \mathbf{V}^{-1} \mathbf{U}^{\frac{1}{2}} \mathbf{W} \mathbf{U}^{-\frac{1}{2}} \mathbf{X} \right)^{-1} \mathbf{X}^\top \mathbf{V}^{-1} \mathbf{U}^{\frac{1}{2}} \mathbf{W} \mathbf{U}^{-\frac{1}{2}}$$

with

$$\mathbf{W} = Diag(w_j), \text{ with } j = 1, \ldots, n$$

and

$$w_j = \frac{\psi \left(u_j^{-\frac{1}{2}} (y_j - x_j^\top \beta) \right)}{u_j^{-\frac{1}{2}} (y_j - x_j^\top \beta}$$

Note that if ψ is the identity or equally a huber influence function with a large smoothing constant, i. e.inf:

$$\mathbf{A} = \left(\mathbf{X} \mathbf{V}^{-1} \mathbf{X} \right)^{-1} \mathbf{X}^\top \mathbf{V}^{-1}$$

This whole thing can also be addapted to define the random effects. If we define the model in an alternative way:

$$\theta_i = x_i^\top \beta + u_i \tag{2}$$

we can restate it similarly to the above as:

$$\theta_i = w_{s,i}^\top y$$

with

$$w_{s,i}^\top = x_i^\top \mathbf{A} + \mathbf{B}$$

where $\mathbf{A}$ is defined as above and

$$\mathbf{B} = \left(\mathbf{V}_e^{-\frac{1}{2}} \mathbf{W}_2 \mathbf{V}_e^{-\frac{1}{2}} + \mathbf{V}_u^{-\frac{1}{2}} \mathbf{W}_3 \mathbf{V}_u^{-\frac{1}{2}} \right)^{-1} \mathbf{V}_e^{-\frac{1}{2}} \mathbf{W}_2 \mathbf{V}_e^{-\frac{1}{2}}$$

with $\mathbf{W}_2$ as diagonal matrix with ith component:

$$w_{2i} = \frac{\psi\{\sigma_{e,i}^{-1}(y_i - x_i^\top \beta - u_i)\}}{\sigma_{e,i}^{-1}(y_i - x_i^\top \beta - u_i)}$$

and with $\mathbf{W}_3$ as diagonal matrix with ith component:

$$w_{3i} = \frac{\psi\{\sigma_u^{-1} u_i\}}{\sigma_u^{-1} u_i}$$

1.4 NEW SECTION

- item
  - item 1.1
  - item 1.2
  - new item

$$x_i = y_i \tag{3}$$
$$x_i + y_i = y_i \tag{4}$$

cite me: Abberger (1997)
Reference math: (3)
Math without numbering:

$$x_i = y_i$$
$$x_i + y_i = y_i$$

```
x <- 1
<<<<<<< HEAD
plot(1:10)
=======
```

```
ggplot(data.frame(x = 1:10, y = 11:20)) +
  geom_point(aes(x, y)) +
  theme_thesis()
>>>>>>> 202c45f790ff3fb900468c7f4559f74b835f5fa8
```

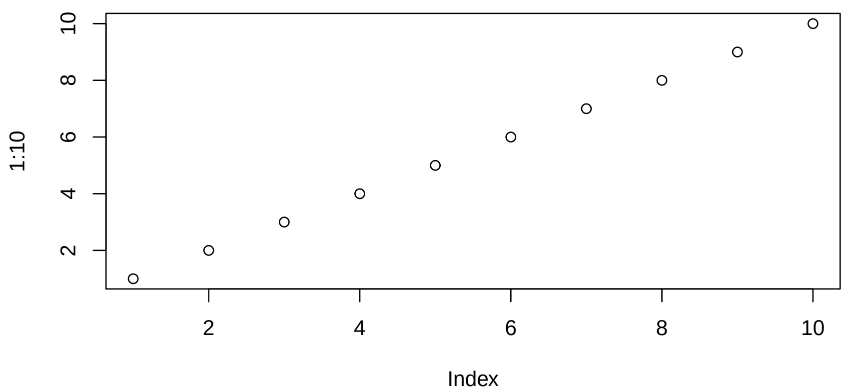

Figure 1: plot of chunk unnamed-chunk-2

2 ALGORITHMS

2.1 REVIEW OF NUMERICAL METHODS FOR NON-LINEAR OPTIMIZATIONS IN STATISTICS AND RELATED FIELDS

2.2 ALGORITHMS FOR ROBUST ESTIMATORS IN STATISTICS

2.3 PROPOSITIONS

Part II

IMPLEMENTATION

This is the part where I want to introduce the software where the theoretical concepts find implementation.

3 VERIFICATION OF RESULTS

4 PERFORMANCE OF ALGORITHMS, A STATISTICIANS PERSPECTIVE

5 VALIDATION OF POINT ESTIMATES, A USERS PERSPECTIVE

6 SOFTWARE

Part III

RESULTS

This is the part where I will present all results. Most certainly they will contain a lot of model- and design-based simulation studies for various settings. Maybe there will be more data available and I can present some applications.

7 NUMERICAL ACCURACY

8 STABILITY

9 SPEED OF CONVERGENCE

10 COMPUTATIONAL COMPLEXITY

Part IV

APPENDIX

BIBLIOGRAPHY

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