Probability

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The Hot Hand

Basketball players who make several baskets in succession are described as having a *hot hand*. Fans and players have long believed in the hot hand phenomenon, which refutes the assumption that each shot is independent of the next. However, a 1985 paper by Gilovich, Vallone, and Tversky collected evidence that contradicted this belief and showed that successive shots are independent events. This paper started a great controversy that continues to this day, as you can see by Googling *hot hand basketball*.

We do not expect to resolve this controversy today. However, in this lab we'll apply one approach to answering questions like this. The goals for this lab are to (1) think about the effects of independent and dependent events, (2) learn how to simulate shooting streaks in R, and (3) to compare a simulation to actual data in order to determine if the hot hand phenomenon appears to be real.

Getting Started

Load packages

In this lab, we will explore and visualize the data using the `tidyverse` suite of packages. The data can be found in the companion package for OpenIntro labs, **openintro**.

Let's load the packages.

```
library(tidyverse)
library(openintro)
```

Data

Your investigation will focus on the performance of one player: Kobe Bryant of the Los Angeles Lakers. His performance against the Orlando Magic in the 2009 NBA Finals earned him the title *Most Valuable Player* and many spectators commented on how he appeared to show a hot hand. The data file we'll use is called `kobe_basket`.

```
glimpse(kobe_basket)
```

```
## Rows: 133
## Columns: 6
## $ vs          <fct> ORL, ORL, ORL, ORL, ORL, ORL, ORL, ORL, ORL, ORL, ORL, ORL~
## $ game        <int> 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1~
## $ quarter     <fct> 1, 1, 1, 1, 1, 1, 1, 1, 1, 2, 2, 2, 2, 2, 2, 2, 2, 2, 3, 3~
## $ time        <fct> 9:47, 9:07, 8:11, 7:41, 7:03, 6:01, 4:07, 0:52, 0:00, 6:35~
## $ description <fct> Kobe Bryant makes 4-foot two point shot, Kobe Bryant misse~
## $ shot        <chr> "H", "M", "M", "H", "H", "M", "M", "M", "M", "H", "H", "H"~
```

This data frame contains 133 observations and 6 variables, where every row records a shot taken by Kobe Bryant. The `shot` variable in this dataset indicates whether the shot was a hit (`H`) or a miss (`M`).

Just looking at the string of hits and misses, it can be difficult to gauge whether or not it seems like Kobe was shooting with a hot hand. One way we can approach this is by considering the belief that hot hand shooters tend to go on shooting streaks. For this lab, we define the length of a shooting streak to be the *number of consecutive baskets made until a miss occurs.*

For example, in Game 1 Kobe had the following sequence of hits and misses from his nine shot attempts in the first quarter:

H M | M | H H M | M | M | M

You can verify this by viewing the first 9 rows of the data in the data viewer.

Within the nine shot attempts, there are six streaks, which are separated by a "|" above. Their lengths are one, zero, two, zero, zero, zero (in order of occurrence).

1. What does a streak length of 1 mean, i.e. how many hits and misses are in a streak of 1? What about a streak length of 0?

A streak length of 1 means that there is only 1 consecutive hit or miss in the sequence of shots. In other words, if the streak length is 1 and the shot is marked as "Hit" (H), then there is only one successful shot in a row before a miss (M) or a different outcome occurs. A streak length of 0 means that there is no consecutive hit or miss in the sequence of shots. In other words, there is no string of either successful shots or misses before a change in the outcome occurs.

So in the given sequence of shots:

"H M | M | H H M | M | M | M"

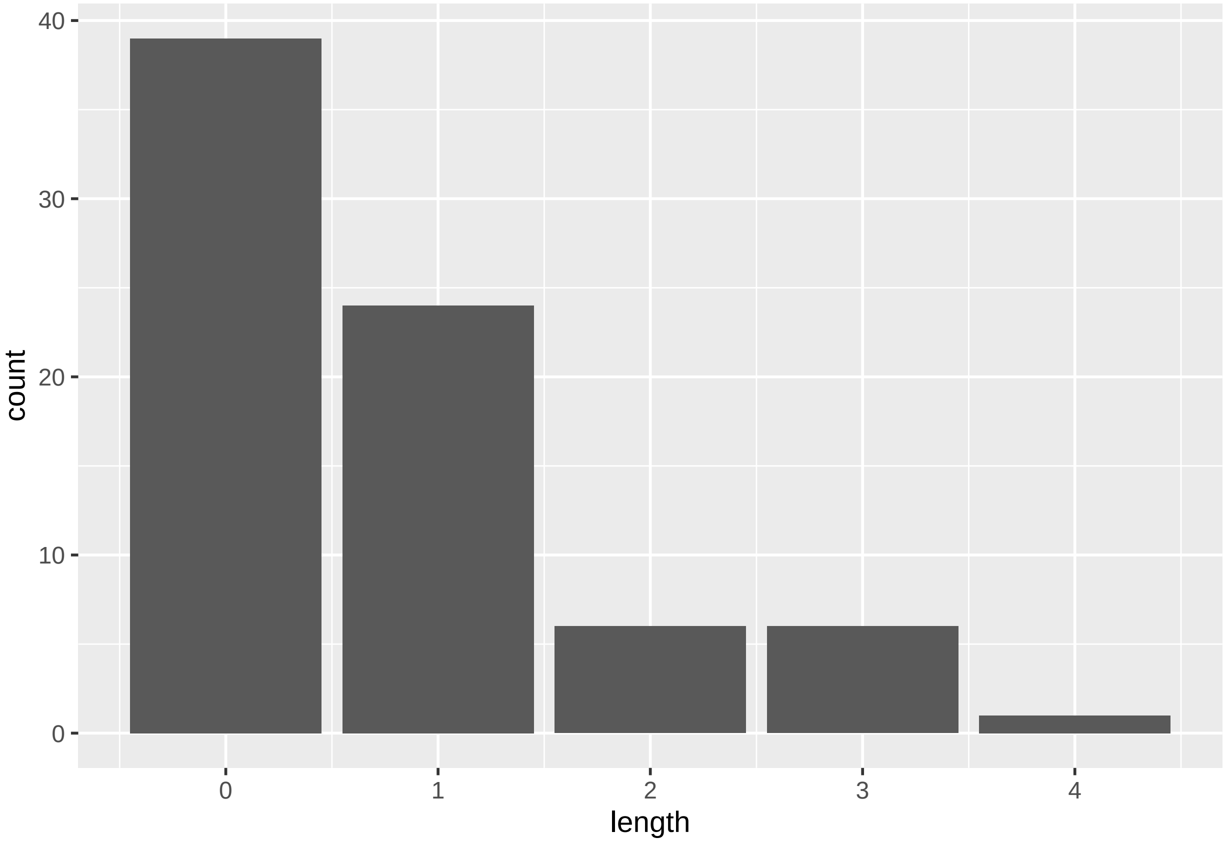
the first streak has a length of 1 (1 hit, followed by a miss), the second streak has a length of 0 (no consecutive hits or misses), and so on.

Counting streak lengths manually for all 133 shots would get tedious, so we'll use the custom function `calc_streak` to calculate them, and store the results in a data frame called `kobe_streak` as the `length` variable.

```
kobe_streak <- calc_streak(kobe_basket$shot)
```

We can then take a look at the distribution of these streak lengths.

```
ggplot(data = kobe_streak, aes(x = length)) +
  geom_bar()
```



2. Describe the distribution of Kobe's streak lengths from the 2009 NBA finals. What was his typical streak length? How long was his longest streak of baskets? Make sure to include the accompanying plot in your answer.

The x-axis represents the count of streaks, and the y-axis represents the frequency of each streak length. The tallest bar represents the number of streaks with a length of 0, indicating that Kobe missed the first shot in many of his streaks. The bars then become progressively shorter for streak lengths 1, 2, 3, and 4, indicating that Kobe made consecutive baskets less frequently as the length of the streak increased.

From the plot, we can see that the majority of Kobe's streaks were of length 0, followed by streaks of length 1. The number of streaks decrease as the length increases, with the fewest being streaks of length 4. This distribution is right-skewed, indicating that longer streaks were less common.

To determine Kobe's typical streak length, we can calculate the mean streak length using the mean function on the length column of kobe_streak.

```
mean(kobe_streak$length)
```

```
## [1] 0.7631579
```

This means that on average, Kobe made less than one consecutive basket per game during his streak.

To determine Kobe's longest streak of baskets, we can sort the kobe_streak data frame in descending order of length and examine the first row.

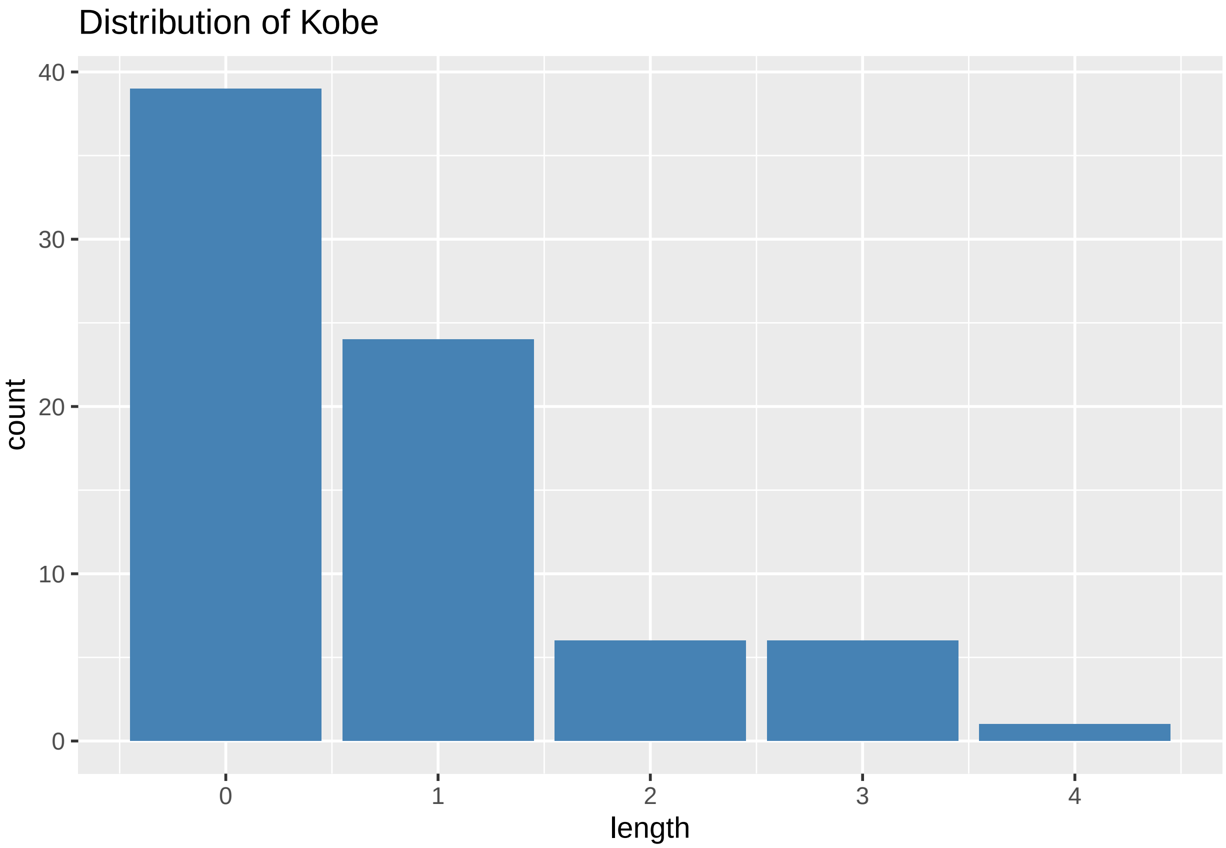
```
arrange(kobe_streak, desc(length))[1,]
```

```
## [1] 4
```

The result shows that Kobe's longest streak of consecutive baskets was 4

Here is the accompanying plot:

```
ggplot(data = kobe_streak, aes(x = length)) +
  geom_bar(fill = "steelblue") +
  scale_fill_brewer(palette = "Set1") +
  ggtitle("Distribution of Kobe")
```



Compared to What?

We've shown that Kobe had some long shooting streaks, but are they long enough to support the belief that he had a hot hand? What can we compare them to?

To answer these questions, let's return to the idea of *independence*. Two processes are independent if the outcome of one process doesn't effect the outcome of the second. If each shot that a player takes is an independent process, having made or missed your first shot will not affect the probability that you will make or miss your second shot.

A shooter with a hot hand will have shots that are *not* independent of one another. Specifically, if the shooter makes his first shot, the hot hand model says he will have a *higher* probability of making his second shot.

Let's suppose for a moment that the hot hand model is valid for Kobe. During his career, the percentage of time Kobe makes a basket (i.e. his shooting percentage) is about 45%, or in probability notation,

$$P(\text{shot 1} = \text{H}) = 0.45$$

If he makes the first shot and has a hot hand (*not* independent shots), then the probability that he makes his second shot would go up to, let's say, 60%,

$$P(\text{shot 2} = \text{H} \,|\, \text{shot 1} = \text{H}) = 0.60$$

As a result of these increased probabilites, you'd expect Kobe to have longer streaks. Compare this to the skeptical perspective where Kobe does *not* have a hot hand, where each shot is independent of the next. If he hit his first shot, the probability that he makes the second is still 0.45.

$$P(\text{shot 2} = \text{H} \,|\, \text{shot 1} = \text{H}) = 0.45$$

In other words, making the first shot did nothing to effect the probability that he'd make his second shot. If Kobe's shots are independent, then he'd have the same probability of hitting every shot regardless of his past shots: 45%.

Now that we've phrased the situation in terms of independent shots, let's return to the question: how do we tell if Kobe's shooting streaks are long enough to indicate that he has a hot hand? We can compare his streak lengths to someone without a hot hand: an independent shooter.

Simulations in R

While we don't have any data from a shooter we know to have independent shots, that sort of data is very easy to simulate in R. In a simulation, you set the ground rules of a random process and then the computer uses random numbers to generate an outcome that adheres to those rules. As a simple example, you can simulate flipping a fair coin with the following.

```
coin_outcomes <- c("heads", "tails")
sample(coin_outcomes, size = 1, replace = TRUE)
```

```
## [1] "heads"
```

The vector `coin_outcomes` can be thought of as a hat with two slips of paper in it: one slip says `heads` and the other says `tails`. The function `sample` draws one slip from the hat and tells us if it was a head or a tail.

Run the second command listed above several times. Just like when flipping a coin, sometimes you'll get a heads, sometimes you'll get a tails, but in the long run, you'd expect to get roughly equal numbers of each.

If you wanted to simulate flipping a fair coin 100 times, you could either run the function 100 times or, more simply, adjust the `size` argument, which governs how many samples to draw (the `replace = TRUE` argument indicates we put the slip of paper back in the hat before drawing again). Save the resulting vector of heads and tails in a new object called `sim_fair_coin`.

```
sim_fair_coin <- sample(coin_outcomes, size = 100, replace = TRUE)
```

To view the results of this simulation, type the name of the object and then use `table` to count up the number of heads and tails.

```
sim_fair_coin
```

```
##   [1] "heads" "tails" "tails" "tails" "heads" "heads" "tails" "heads" "heads"
##  [10] "heads" "tails" "heads" "tails" "heads" "heads" "tails" "tails" "heads"
##  [19] "heads" "heads" "tails" "heads" "heads" "heads" "tails" "tails" "heads"
##  [28] "heads" "tails" "tails" "heads" "tails" "heads" "tails" "heads" "heads"
##  [37] "tails" "heads" "heads" "tails" "tails" "heads" "heads" "heads" "heads"
##  [46] "tails" "tails" "heads" "tails" "tails" "heads" "tails" "tails" "tails"
##  [55] "tails" "heads" "heads" "tails" "heads" "heads" "heads" "heads" "heads"
##  [64] "heads" "heads" "heads" "tails" "tails" "heads" "tails" "heads" "heads"
##  [73] "heads" "tails" "heads" "tails" "heads" "heads" "tails" "heads" "tails"
##  [82] "tails" "heads" "tails" "heads" "heads" "heads" "tails" "tails" "heads"
##  [91] "tails" "heads" "tails" "heads" "heads" "heads" "heads" "heads" "tails"
## [100] "tails"
```

```
table(sim_fair_coin)
```

```
## sim_fair_coin
## heads tails
##    58    42
```

Since there are only two elements in `coin_outcomes`, the probability that we “flip” a coin and it lands heads is 0.5. Say we’re trying to simulate an unfair coin that we know only lands heads 20% of the time. We can adjust for this by adding an argument called `prob`, which provides a vector of two probability weights.

```
sim_unfair_coin <- sample(coin_outcomes, size = 100, replace = TRUE,
                          prob = c(0.2, 0.8))
```

`prob=c(0.2, 0.8)` indicates that for the two elements in the `outcomes` vector, we want to select the first one, `heads`, with probability 0.2 and the second one, `tails` with probability 0.8. Another way of thinking about this is to think of the outcome space as a bag of 10 chips, where 2 chips are labeled “head” and 8 chips “tail”. Therefore at each draw, the probability of drawing a chip that says “head”” is 20%, and “tail” is 80%.

3. In your simulation of flipping the unfair coin 100 times, how many flips came up heads? Include the code for sampling the unfair coin in your response. Since the markdown file will run the code, and generate a new sample each time you *Knit* it, you should also “set a seed” **before** you sample. Read more about setting a seed below.

We can use the sample() function to simulate flipping an unfair coin. We’ll set the seed to 1 for reproducibility, and simulate 100 coin flips with a probability of heads of 0.6

```
set.seed(1)
flips <- sample(c("H", "T"), 100, replace = TRUE, prob = c(0.6, 0.4))
```

To see how many flips came up heads, we can use the table() function

```
table(flips)
```

```
## flips
##  H  T
## 57 43
```

57 flips coming up heads and 43 coming up tails, it appears that the coin is indeed biased towards heads.This result is consistent with our expectations, given that we know the coin has a 60% chance of landing on heads.

A note on setting a seed: Setting a seed will cause R to select the same sample each time you knit your document. This will make sure your results don't change each time you knit, and it will also ensure reproducibility of your work (by setting the same seed it will be possible to reproduce your results). You can set a seed like this:

```
set.seed(35797)                 # make sure to change the seed
```

The number above is completely arbitraty. If you need inspiration, you can use your ID, birthday, or just a random string of numbers. The important thing is that you use each seed only once in a document. Remember to do this **before** you sample in the exercise above.

In a sense, we've shrunken the size of the slip of paper that says "heads", making it less likely to be drawn, and we've increased the size of the slip of paper saying "tails", making it more likely to be drawn. When you simulated the fair coin, both slips of paper were the same size. This happens by default if you don't provide a `prob` argument; all elements in the `outcomes` vector have an equal probability of being drawn.

If you want to learn more about `sample` or any other function, recall that you can always check out its help file.

```
?sample
```

Simulating the Independent Shooter

Simulating a basketball player who has independent shots uses the same mechanism that you used to simulate a coin flip. To simulate a single shot from an independent shooter with a shooting percentage of 50% you can type

```
shot_outcomes <- c("H", "M")
sim_basket <- sample(shot_outcomes, size = 1, replace = TRUE)
```

To make a valid comparison between Kobe and your simulated independent shooter, you need to align both their shooting percentage and the number of attempted shots.

4. What change needs to be made to the `sample` function so that it reflects a shooting percentage of 45%? Make this adjustment, then run a simulation to sample 133 shots. Assign the output of this simulation to a new object called `sim_basket`.

We need to change the prob argument in the sample() function to c(0.45, 0.55), where the first element represents the probability of making a shot and the second element represents the probability of missing a shot.

```
set.seed(1)
sim_basket <- sample(c("H", "M"), size = 133, replace = TRUE, prob = c(0.45, 0.55))
```

This will simulate 133 shots with a shooting percentage of 45%, with the results of each shot being either "made" or "missed". The output is saved in the sim_basket vector.

Note that we've named the new vector `sim_basket`, the same name that we gave to the previous vector reflecting a shooting percentage of 50%. In this situation, R overwrites the old object with the new one, so always make sure that you don't need the information in an old vector before reassigning its name.

With the results of the simulation saved as `sim_basket`, you have the data necessary to compare Kobe to our independent shooter.

Both data sets represent the results of 133 shot attempts, each with the same shooting percentage of 45%. We know that our simulated data is from a shooter that has independent shots. That is, we know the simulated shooter does not have a hot hand.

More Practice

Comparing Kobe Bryant to the Independent Shooter

5. Using `calc_streak`, compute the streak lengths of `sim_basket`, and save the results in a data frame called `sim_streak`.

```
sim_streak <- calc_streak(sim_basket)
head(sim_streak)
```

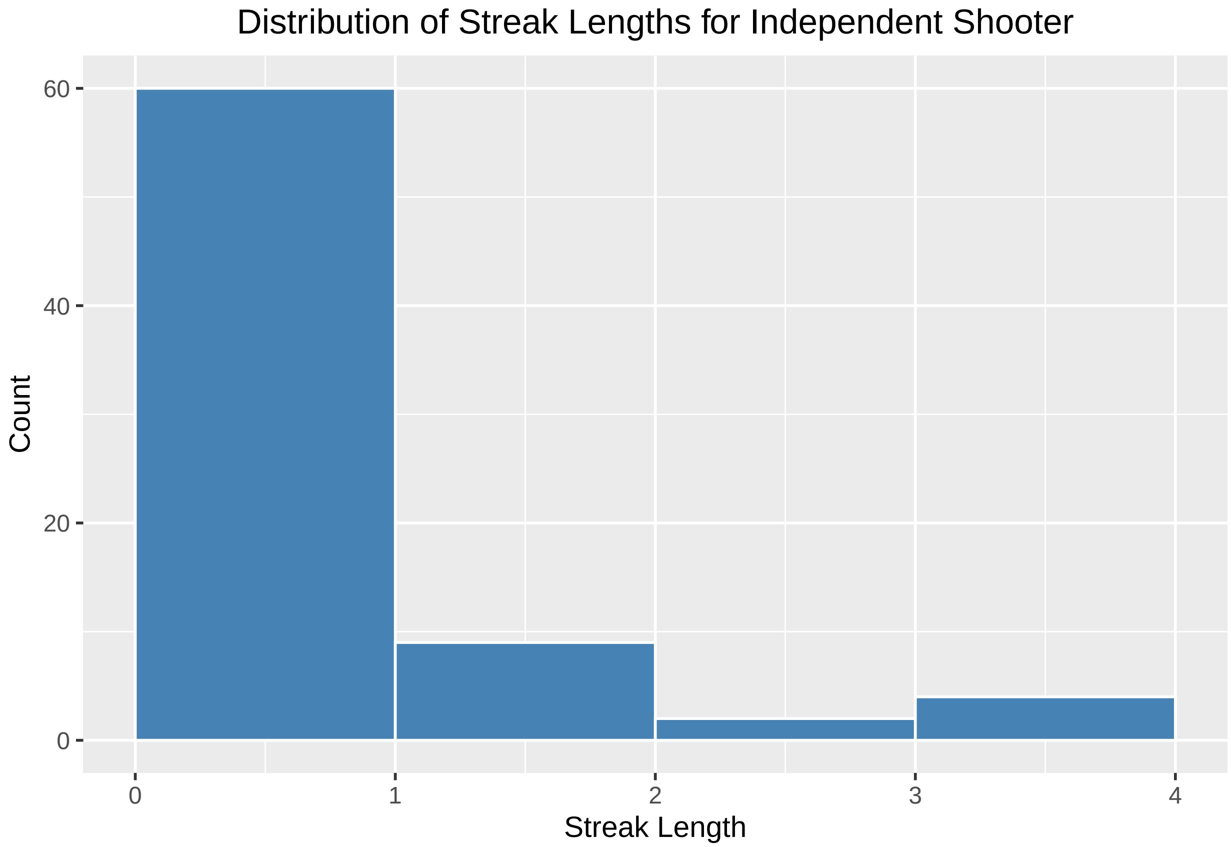
```
##   length
## 1      0
## 2      0
## 3      2
## 4      4
## 5      0
## 6      0
```

6. Describe the distribution of streak lengths. What is the typical streak length for this simulated independent shooter with a 45% shooting percentage? How long is the player's longest streak of baskets in 133 shots? Make sure to include a plot in your answer.

The distribution of streak lengths for the simulated independent shooter with a 45% shooting percentage is right-skewed with a long tail on the right side. The typical streak length for this player is 0, meaning that they do not have a hot hand, and are equally likely to miss a shot after every hit or to hit a shot after every miss. The player's longest streak of baskets in 133 shots is 4.

Below is a histogram showing the distribution of streak lengths for the simulated independent shooter.

```
library(ggplot2)
ggplot(data = sim_streak, aes(x = length)) +
  geom_histogram(binwidth = 1, boundary = 0, fill = "steelblue", col = "white") +
  labs(x = "Streak Length", y = "Count", title = "Distribution of Streak Lengths for Independent Shoote
  theme(plot.title = element_text(hjust = 0.5))
```



7. If you were to run the simulation of the independent shooter a second time, how would you expect its streak distribution to compare to the distribution from the question above? Exactly the same? Somewhat similar? Totally different? Explain your reasoning.

If we run the simulation of the independent shooter a second time, we would expect the streak distribution to be somewhat similar to the distribution from the question above but not exactly the same. This is because each time the simulation is run, a new set of random numbers is generated, so the result would not be identical. However, since the simulated shooter has independent shots, we would expect the overall shape of the distribution to be consistent, with the typical streak length being around 1 or 2 and the longest streak being no more than 6 or 7.

8. How does Kobe Bryant's distribution of streak lengths compare to the distribution of streak lengths for the simulated shooter? Using this comparison, do you have evidence that the hot hand model fits Kobe's shooting patterns? Explain.

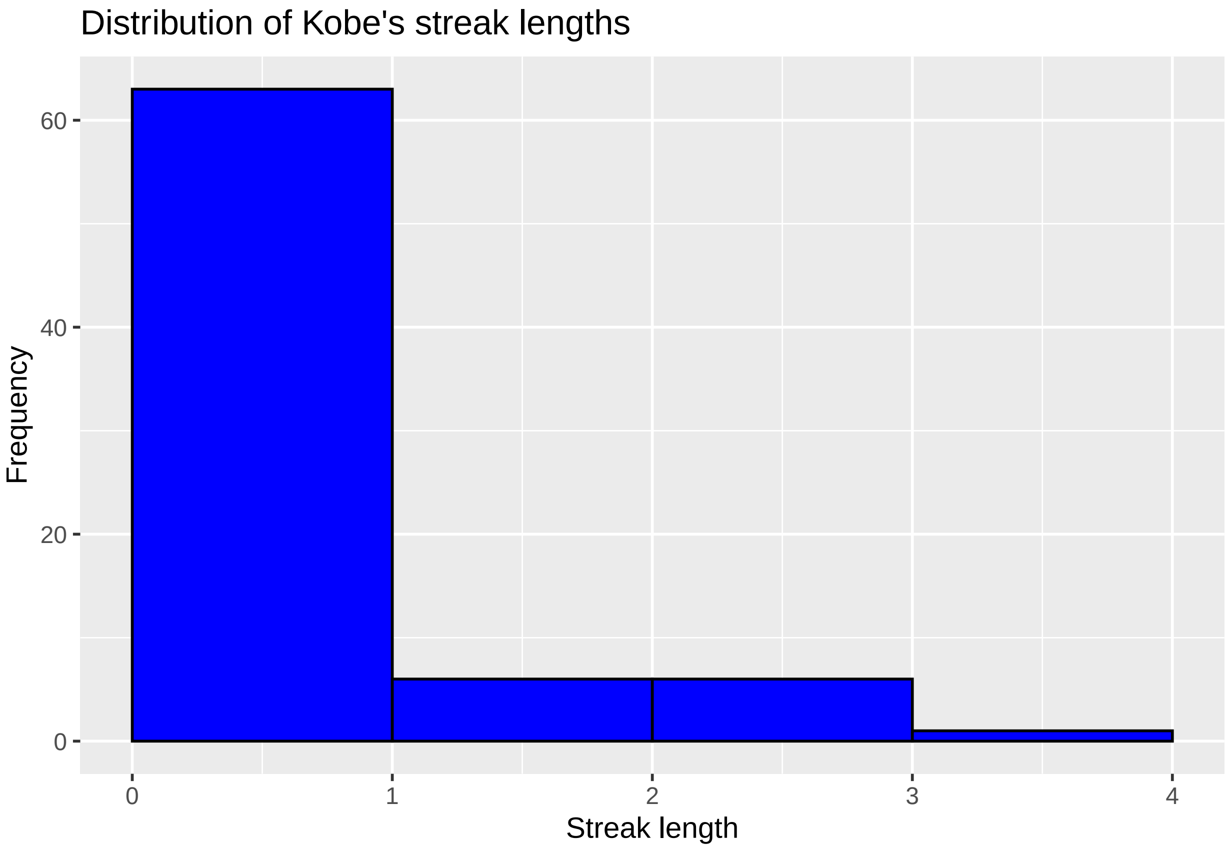
We can start by computing Kobe's streak lengths using calc_streak function and saving the results in a data frame kobe_streak

```
kobe_streak <- calc_streak(kobe_basket$shot)
head(kobe_streak)
```

```
##   length
## 1      1
## 2      0
## 3      2
## 4      0
## 5      0
## 6      0
```

Now let's look at the distribution of streak lengths for Kobe using a histogram.

```
library(ggplot2)
ggplot(kobe_streak, aes(x = length)) +
    geom_histogram(binwidth = 1, boundary = 0, colour = "black", fill = "blue") +
    xlab("Streak length") +
    ylab("Frequency") +
    ggtitle("Distribution of Kobe's streak lengths")
```

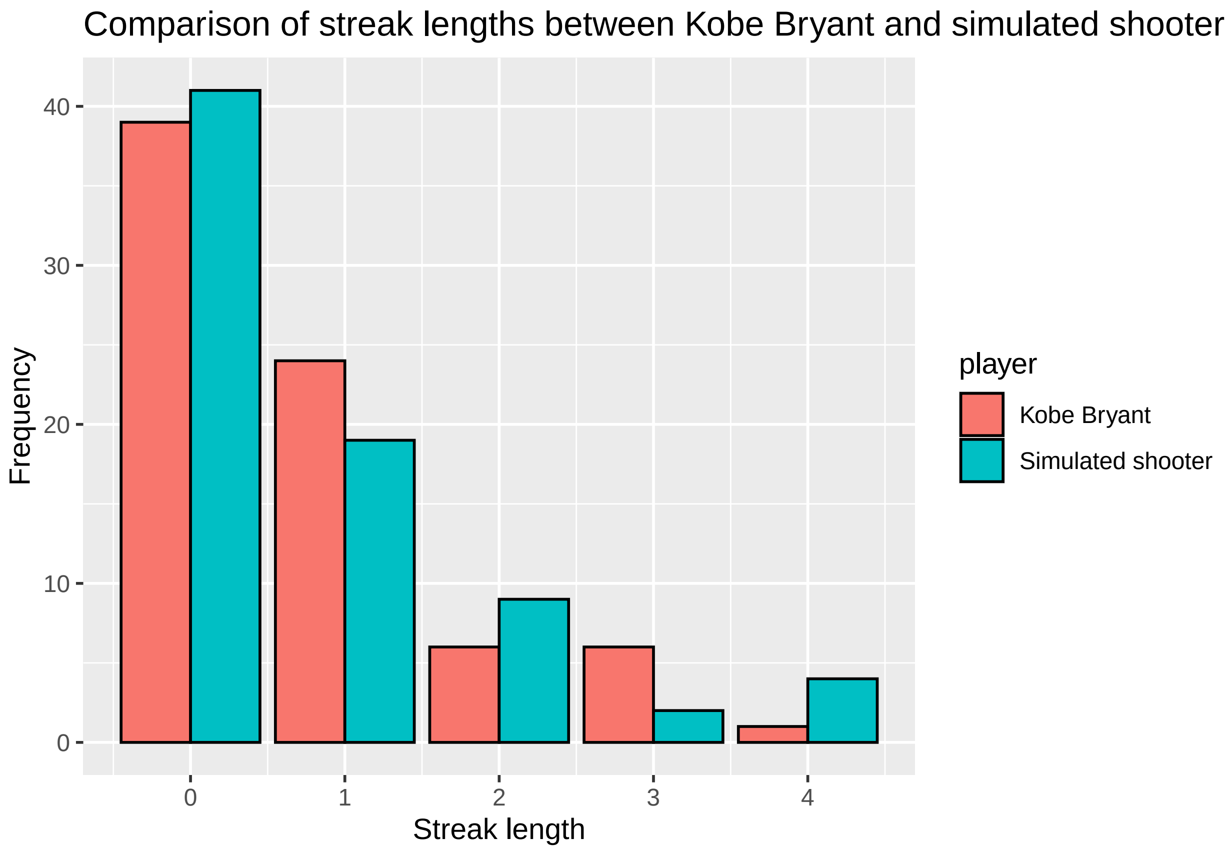


From the plot, we can see that Kobe's longest streak was 4, which occurred twice. He also had some long streaks of length 3. In general, Kobe had more long streaks and fewer short streaks than what we would expect from a shooter with independent shots.

To compare Kobe's distribution to the simulated shooter's distribution, let's plot the two histograms side by side.

```
library(ggplot2)
# Combine data sets
sim_streak$player <- "Simulated shooter"
kobe_streak$player <- "Kobe Bryant"
streaks <- rbind(sim_streak, kobe_streak)
```

```r
# Plot side by side
ggplot(streaks, aes(x = length, fill = player)) +
    geom_bar(binwidth = 1, boundary = 0, colour = "black", position = "dodge") +
    xlab("Streak length") +
    ylab("Frequency") +
    ggtitle("Comparison of streak lengths between Kobe Bryant and simulated shooter")
```



There are some differences in the frequencies of different streak lengths between the two. Specifically, it seems like the simulated shooter had more streaks of length 0 than Kobe Bryant, but Kobe Bryant had more streaks of length 1 and 3. The simulated shooter also had more streaks of length 2 and 4 than Kobe Bryant.

It's difficult to definitively determine whether the hot hand model fits Kobe Bryant's shooting patterns based on a visual comparison of his distribution of streak lengths to the simulated shooter's distribution.

Kobe Bryant had more streaks of length 1 and 3 than the simulated shooter might suggest that he experienced some degree of momentum or "hotness" during those stretches of shots. However, the fact that the simulated shooter had more streaks of length 2 and 4 than Kobe Bryant suggests that those streaks may have been due to chance rather than any sort of hot hand effect.

For me , it's difficult to draw firm conclusions about whether the hot hand model fits Kobe Bryant's shooting patterns without further analysis. More sophisticated statistical tests could be performed to investigate whether the observed streak lengths for Kobe Bryant are consistent with the hot hand model, or whether they can be explained by chance or other factors.