

Foundations for statistical inference - Sampling distributions

Waheeb Algabri

In this lab, you will investigate the ways in which the statistics from a random sample of data can serve as point estimates for population parameters. We're interested in formulating a *sampling distribution* of our estimate in order to learn about the properties of the estimate, such as its distribution.

Setting a seed: We will take some random samples and build sampling distributions in this lab, which means you should set a seed at the start of your lab. If this concept is new to you, review the lab on probability.

Getting Started

Load packages

In this lab, we will explore and visualize the data using the **tidyverse** suite of packages. We will also use the **infer** package for resampling.

Let's load the packages.

```
library(tidyverse)
library(openintro)
library(infer)
```

The data

A 2019 Gallup report states the following:

The premise that scientific progress benefits people has been embodied in discoveries throughout the ages – from the development of vaccinations to the explosion of technology in the past few decades, resulting in billions of supercomputers now resting in the hands and pockets of people worldwide. Still, not everyone around the world feels science benefits them personally.

Source: World Science Day: Is Knowledge Power?

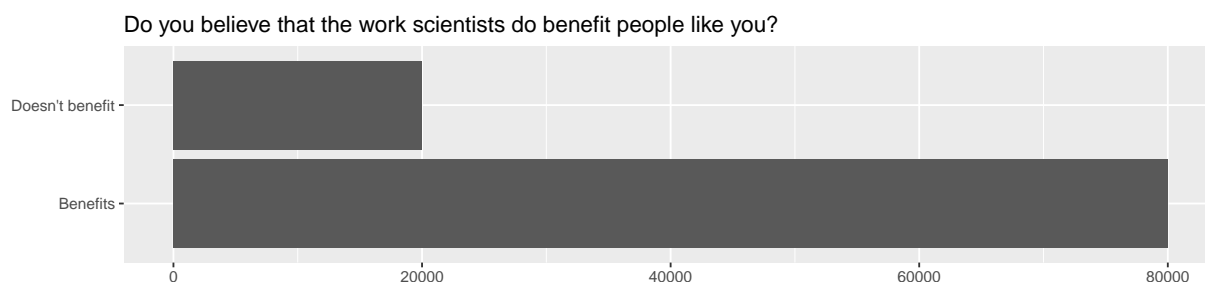
The Wellcome Global Monitor finds that 20% of people globally do not believe that the work scientists do benefits people like them. In this lab, you will assume this 20% is a true population proportion and learn about how sample proportions can vary from sample to sample by taking smaller samples from the population. We will first create our population assuming a population size of 100,000. This means 20,000 (20%) of the population think the work scientists do does not benefit them personally and the remaining 80,000 think it does.

```
global_monitor <- tibble(
  scientist_work = c(rep("Benefits", 80000), rep("Doesn't benefit", 20000))
)
```

The name of the data frame is `global_monitor` and the name of the variable that contains responses to the question “Do you believe that the work scientists do benefit people like you?” is `scientist_work`.

We can quickly visualize the distribution of these responses using a bar plot.

```
ggplot(global_monitor, aes(x = scientist_work)) +  
  geom_bar() +  
  labs(  
    x = "", y = "",  
    title = "Do you believe that the work scientists do benefit people like you?"  
  ) +  
  coord_flip()
```



We can also obtain summary statistics to confirm we constructed the data frame correctly.

```
global_monitor %>%  
  count(scientist_work) %>%  
  mutate(p = n / sum(n))  
  
## # A tibble: 2 x 3  
##   scientist_work      n      p  
##   <chr>          <int> <dbl>  
## 1 Benefits      80000  0.8  
## 2 Doesn't benefit 20000  0.2
```

The unknown sampling distribution

In this lab, you have access to the entire population, but this is rarely the case in real life. Gathering information on an entire population is often extremely costly or impossible. Because of this, we often take a sample of the population and use that to understand the properties of the population.

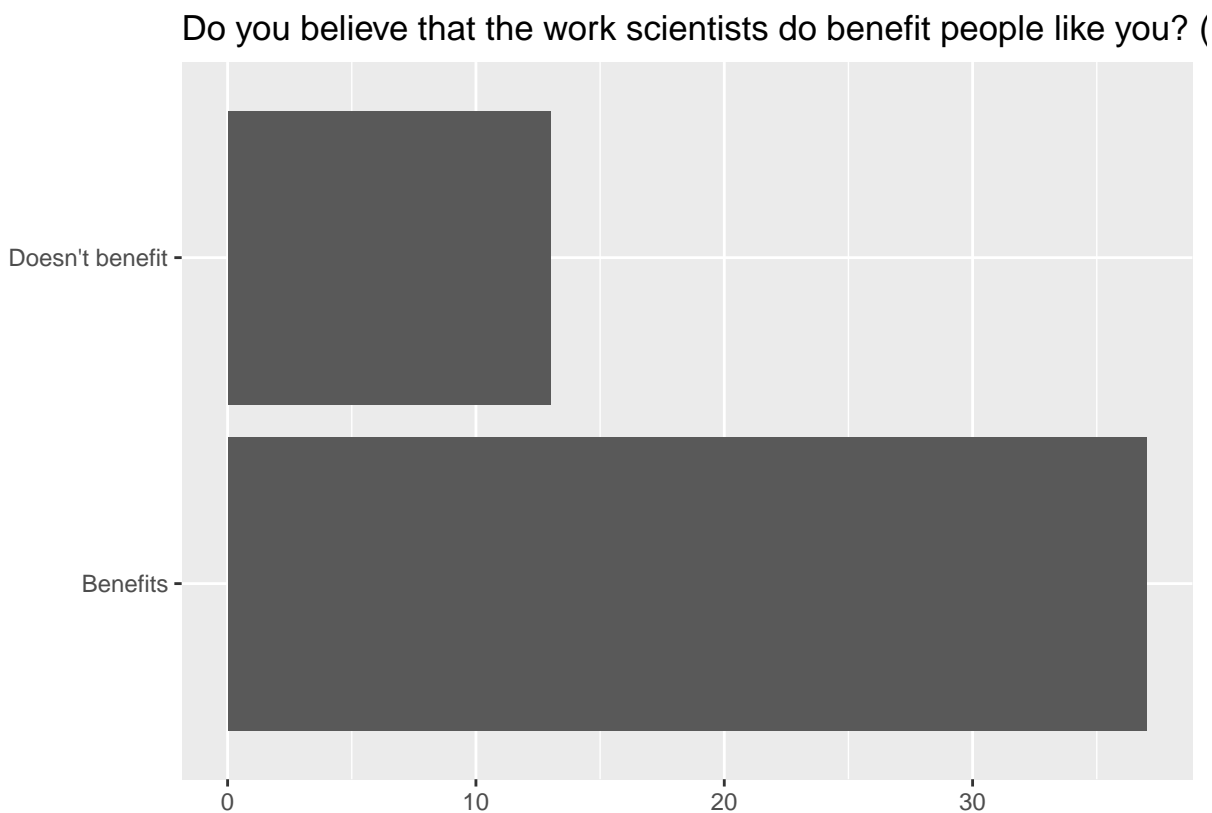
If you are interested in estimating the proportion of people who don't think the work scientists do benefits them, you can use the `sample_n` command to survey the population.

```
samp1 <- global_monitor %>%  
  sample_n(50)
```

This command collects a simple random sample of size 50 from the `global_monitor` dataset, and assigns the result to `samp1`. This is similar to randomly drawing names from a hat that contains the names of all in the population. Working with these 50 names is considerably simpler than working with all 100,000 people in the population.

1. Describe the distribution of responses in this sample. How does it compare to the distribution of responses in the population. **Hint:** Although the `sample_n` function takes a random sample of observations (i.e. rows) from the dataset, you can still refer to the variables in the dataset with the same names. Code you presented earlier for visualizing and summarizing the population data will still be useful for the sample, however be careful to not label your proportion `p` since you're now calculating a sample statistic, not a population parameters. You can customize the label of the statistics to indicate that it comes from the sample.

```
ggplot(samp1, aes(x = scientist_work)) +  
  geom_bar() +  
  labs(  
    x = "", y = "",  
    title = "Do you believe that the work scientists do benefit people like you? (Sample 1)"  
  ) +  
  coord_flip()
```



```
samp1 %>%  
  count(scientist_work) %>%  
  mutate(sample_proportion = n / sum(n))
```

```
## # A tibble: 2 x 3  
##   scientist_work      n sample_proportion  
##   <chr>          <int>          <dbl>  
## 1 Benefits         37            0.74  
## 2 Doesn't benefit  13            0.26
```

The sample has 37 individuals who believe that the work scientists do benefits people like them (or 74% of the sample) and 13 individuals who believe it does not benefit them (or 26% of the sample).

Compared to the population distribution, the sample distribution is more variable. This is because the sample size is smaller and there is greater potential for sampling variability when the sample size is smaller.

If you're interested in estimating the proportion of all people who do not believe that the work scientists do benefits them, but you do not have access to the population data, your best single guess is the sample mean.

```
samp1 %>%
  count(scientist_work) %>%
  mutate(p_hat = n / sum(n))

## # A tibble: 2 x 3
##   scientist_work      n p_hat
##   <chr>          <int> <dbl>
## 1 Benefits        37  0.74
## 2 Doesn't benefit  13  0.26
```

Depending on which 50 people you selected, your estimate could be a bit above or a bit below the true population proportion of 0.26. In general, though, the sample proportion turns out to be a pretty good estimate of the true population proportion, and you were able to get it by sampling less than 1% of the population.

2. Would you expect the sample proportion to match the sample proportion of another student's sample? Why, or why not? If the answer is no, would you expect the proportions to be somewhat different or very different? Ask a student team to confirm your answer.

No, I would not expect the sample proportion of another student's sample to match the sample proportion of my sample exactly. This is because each sample is a random subset of the population, and different samples may yield different proportions due to chance. However, I would expect the proportions to be somewhat similar, especially if the sample sizes are similar. If the sample sizes are small, there may be more variability in the sample proportions. If the sample sizes are large, the sample proportions should be more consistent.

3. Take a second sample, also of size 50, and call it `samp2`. How does the sample proportion of `samp2` compare with that of `samp1`? Suppose we took two more samples, one of size 100 and one of size 1000. Which would you think would provide a more accurate estimate of the population proportion?

To take a second sample of size 50, we can use the `sample_n()`

```
set.seed(123)
samp2 <- global_monitor %>%
  sample_n(size = 50)
```

We can calculate the sample proportion for `samp2` as we did for `samp1`:

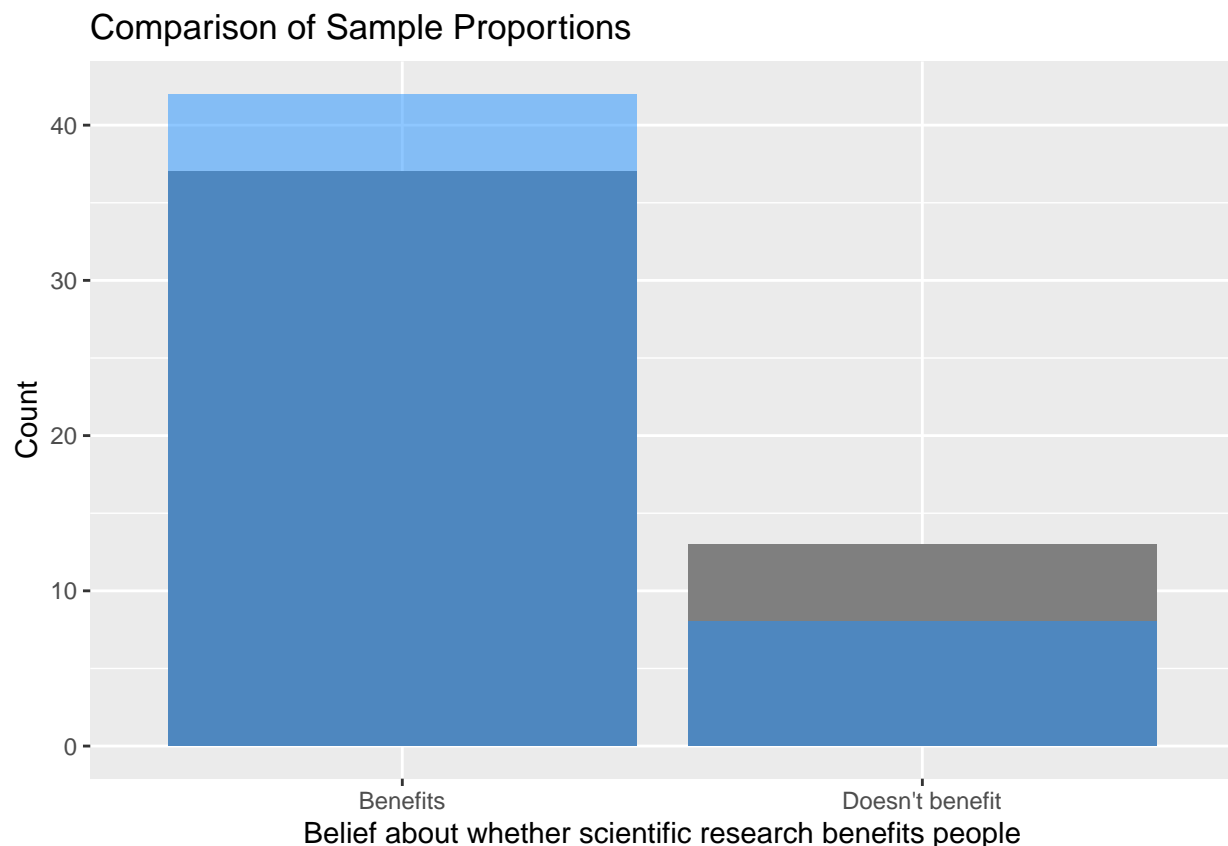
```
samp2 %>%
  count(scientist_work) %>%
  mutate(sample_proportion = n / sum(n))

## # A tibble: 2 x 3
##   scientist_work      n sample_proportion
##   <chr>          <int>          <dbl>
## 1 Benefits        42           0.84
## 2 Doesn't benefit   8           0.16
```

The sample proportion of samp2 is quite different from that of samp1. In samp2, only 16% of respondents said that the work scientists do doesn't benefit them, compared to 26% in samp1. This difference suggests that the sample proportion can vary quite a bit from sample to sample, even when the sample size is relatively large.

To compare the sample proportions of samp1 and samp2

```
ggplot() +
  geom_bar(data = samp1, aes(x = scientist_work), fill = "gray50") +
  geom_bar(data = samp2, aes(x = scientist_work), fill = "dodgerblue", alpha = 0.5) +
  labs(x = "Belief about whether scientific research benefits people",
       y = "Count",
       title = "Comparison of Sample Proportions")
```



Not surprisingly, every time you take another random sample, you might get a different sample proportion. It's useful to get a sense of just how much variability you should expect when estimating the population mean this way. The distribution of sample proportions, called the *sampling distribution (of the proportion)*, can help you understand this variability. In this lab, because you have access to the population, you can build up the sampling distribution for the sample proportion by repeating the above steps many times. Here, we use R to take 15,000 different samples of size 50 from the population, calculate the proportion of responses in each sample, filter for only the *Doesn't benefit* responses, and store each result in a vector called `sample_props50`. Note that we specify that `replace = TRUE` since sampling distributions are constructed by sampling with replacement.

```
sample_props50 <- global_monitor %>%
  rep_sample_n(size = 50, reps = 15000, replace = TRUE) %>%
```

```
count(scientist_work) %>%
mutate(p_hat = n / sum(n)) %>%
filter(scientist_work == "Doesn't benefit")
```

And we can visualize the distribution of these proportions with a histogram.

```
ggplot(data = sample_props50, aes(x = p_hat)) +
  geom_histogram(binwidth = 0.02) +
  labs(
    x = "p_hat (Doesn't benefit)",
    title = "Sampling distribution of p_hat",
    subtitle = "Sample size = 50, Number of samples = 15000"
  )
```

Next, you will review how this set of code works.

4. How many elements are there in `sample_props50`? Describe the sampling distribution, and be sure to specifically note its center. Make sure to include a plot of the distribution in your answer.

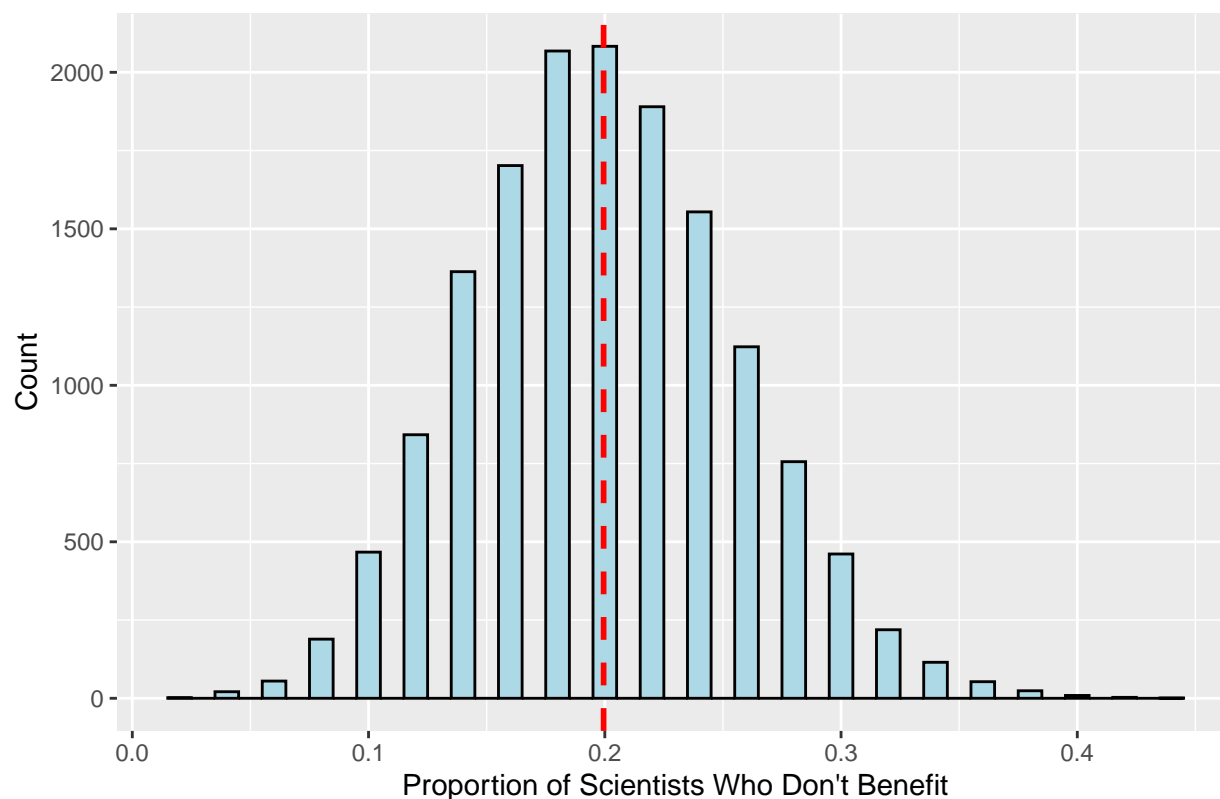
There are 15,000 elements in `sample_props50`, which is the result of taking 15,000 different samples of size 50 from the population and calculating the proportion of “Doesn’t benefit” responses in each sample.

The sampling distribution is centered around the true population proportion of people who do not believe that the work scientists do benefits them, which is 0.24. The histogram shows that the distribution is approximately normal, with a slight skewness to the right. This shape is expected, as the sample size is relatively small ($n=50$). As the sample size increases, the sampling distribution tends to become more symmetric and bell-shaped.

```
center <- mean(sample_props50$p_hat)
```

```
ggplot(sample_props50, aes(x = p_hat)) +
  geom_histogram(binwidth = 0.01, fill = "lightblue", color = "black") +
  geom_vline(xintercept = center, linetype = "dashed", color = "red", size = 1) +
  labs(title = "Distribution of Proportions",
    x = "Proportion of Scientists Who Don't Benefit",
    y = "Count")
```

Distribution of Proportions



Interlude: Sampling distributions

The idea behind the `rep_sample_n` function is *repetition*. Earlier, you took a single sample of size `n` (50) from the population of all people in the population. With this new function, you can repeat this sampling procedure `rep` times in order to build a distribution of a series of sample statistics, which is called the **sampling distribution**.

Note that in practice one rarely gets to build true sampling distributions, because one rarely has access to data from the entire population.

Without the `rep_sample_n` function, this would be painful. We would have to manually run the following code 15,000 times

```
global_monitor %>%
  sample_n(size = 50, replace = TRUE) %>%
  count(scientist_work) %>%
  mutate(p_hat = n / sum(n)) %>%
  filter(scientist_work == "Doesn't benefit")
```

```
## # A tibble: 1 x 3
##   scientist_work      n p_hat
##   <chr>          <int> <dbl>
## 1 Doesn't benefit      7  0.14
```

as well as store the resulting sample proportions each time in a separate vector.

Note that for each of the 15,000 times we computed a proportion, we did so from a **different** sample!

5. To make sure you understand how sampling distributions are built, and exactly what the `rep_sample_n` function does, try modifying the code to create a sampling distribution of **25 sample proportions** from **samples of size 10**, and put them in a data frame named `sample_props_small`. Print the output. How many observations are there in this object called `sample_props_small`? What does each observation represent?

```
sample_props_small <- global_monitor %>%
  rep_sample_n(size = 10, reps = 25, replace = TRUE) %>%
  count(scientist_work) %>%
  mutate(p_hat = n / sum(n)) %>%
  filter(scientist_work == "Doesn't benefit")

print(sample_props_small)
```

```
## # A tibble: 21 x 4
## # Groups:   replicate [21]
##   replicate scientist_work      n p_hat
##   <int> <chr>          <int> <dbl>
## 1         1 Doesn't benefit      1  0.1
## 2         3 Doesn't benefit      2  0.2
## 3         4 Doesn't benefit      1  0.1
## 4         5 Doesn't benefit      3  0.3
## 5         6 Doesn't benefit      3  0.3
## 6         7 Doesn't benefit      1  0.1
## 7         8 Doesn't benefit      4  0.4
## 8         9 Doesn't benefit      3  0.3
## 9        10 Doesn't benefit      1  0.1
## 10       11 Doesn't benefit      1  0.1
## # ... with 11 more rows
```

There should be 25 observations in the `sample_props_small` object, with each observation representing the proportion of people who responded “Doesn’t benefit” in each of the 25 samples of size 10.

Sample size and the sampling distribution

Mechanics aside, let’s return to the reason we used the `rep_sample_n` function: to compute a sampling distribution, specifically, the sampling distribution of the proportions from samples of 50 people.

```
ggplot(data = sample_props50, aes(x = p_hat)) +
  geom_histogram(binwidth = 0.02)
```

The sampling distribution that you computed tells you much about estimating the true proportion of people who think that the work scientists do doesn’t benefit them. Because the sample proportion is an unbiased estimator, the sampling distribution is centered at the true population proportion, and the spread of the distribution indicates how much variability is incurred by sampling only 50 people at a time from the population.

In the remainder of this section, you will work on getting a sense of the effect that sample size has on your sampling distribution.

6. Use the app below to create sampling distributions of proportions of *Doesn’t benefit* from samples of size 10, 50, and 100. Use 5,000 simulations. What does each observation in the sampling distribution

represent? How does the mean, standard error, and shape of the sampling distribution change as the sample size increases? How (if at all) do these values change if you increase the number of simulations? (You do not need to include plots in your answer.)

Each observation in the sampling distribution represents the proportion of people in that particular sample who believe that the work scientists do doesn't benefit them. The mean of the sampling distribution represents the expected proportion, and the standard error represents the variability of the sample proportion.

As the sample size increases, the standard error of the sampling distribution decreases, indicating less variability in the sample proportions. The shape of the distribution becomes more bell-shaped and symmetrical, and the mean of the distribution approaches the true population proportion.

More Practice

So far, you have only focused on estimating the proportion of those you think the work scientists doesn't benefit them. Now, you'll try to estimate the proportion of those who think it does.

Note that while you might be able to answer some of these questions using the app, you are expected to write the required code and produce the necessary plots and summary statistics. You are welcome to use the app for exploration.

7. Take a sample of size 15 from the population and calculate the proportion of people in this sample who think the work scientists do enhances their lives. Using this sample, what is your best point estimate of the population proportion of people who think the work scientists do enhances their lives?

```
set.seed(123) # for reproducibility
samp15 <- global_monitor %>%
  sample_n(15)

prop <- mean(samp15$scientist_work == "Benefits")
prop
```

```
## [1] 0.8666667
```

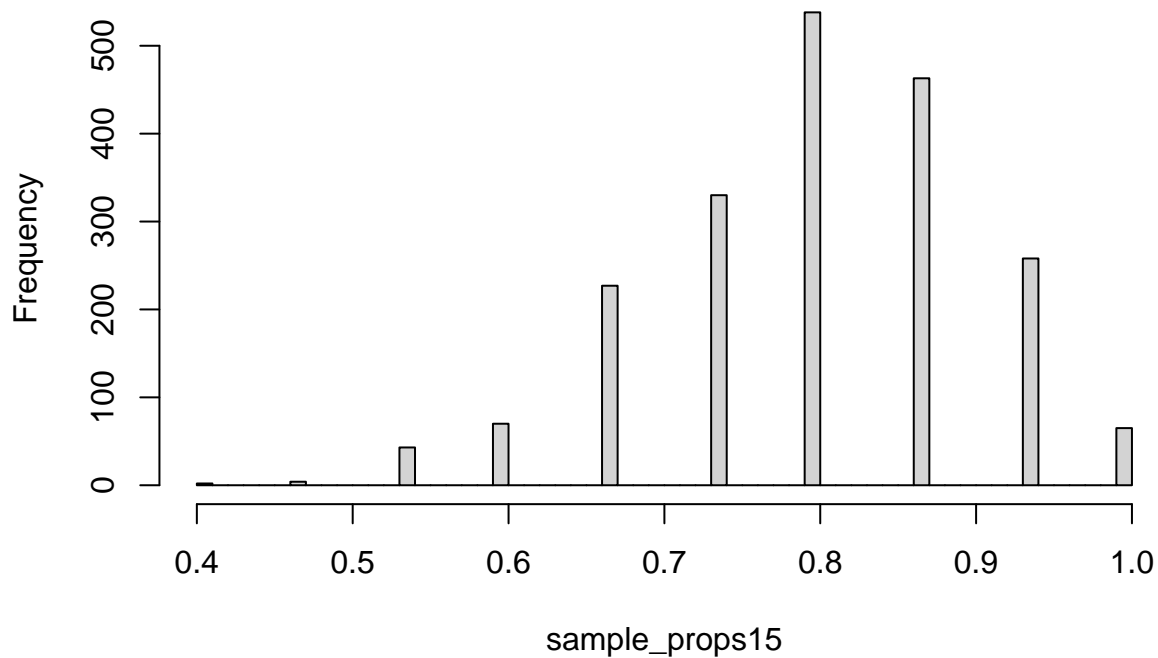
This means that approximately 87% of the 15 individuals in the sample believe that the work scientists do enhances their lives.

8. Since you have access to the population, simulate the sampling distribution of proportion of those who think the work scientists do enhances their lives for samples of size 15 by taking 2000 samples from the population of size 15 and computing 2000 sample proportions. Store these proportions in as `sample_props15`. Plot the data, then describe the shape of this sampling distribution. Based on this sampling distribution, what would you guess the true proportion of those who think the work scientists do enhances their lives to be? Finally, calculate and report the population proportion.

```
set.seed(123) # for reproducibility
n_sims <- 2000
sample_props15 <- replicate(n_sims, mean(sample(global_monitor$scientist_work, 15, replace = TRUE) == "Benefits"))

hist(sample_props15, breaks = sqrt(length(sample_props15)),
     main = "Sampling Distribution of Proportion of Those Who Think Scientists Enhance Lives (n=15)")
```

Sampling Distribution of Proportion of Those Who Think Scientists Enhance Lives



To calculate and report the population proportion, we can use the table function to summary of the scientist_work variable

```
table(global_monitor$scientist_work)
```

```
##
##      Benefits Doesn't benefit
##      80000      20000
```

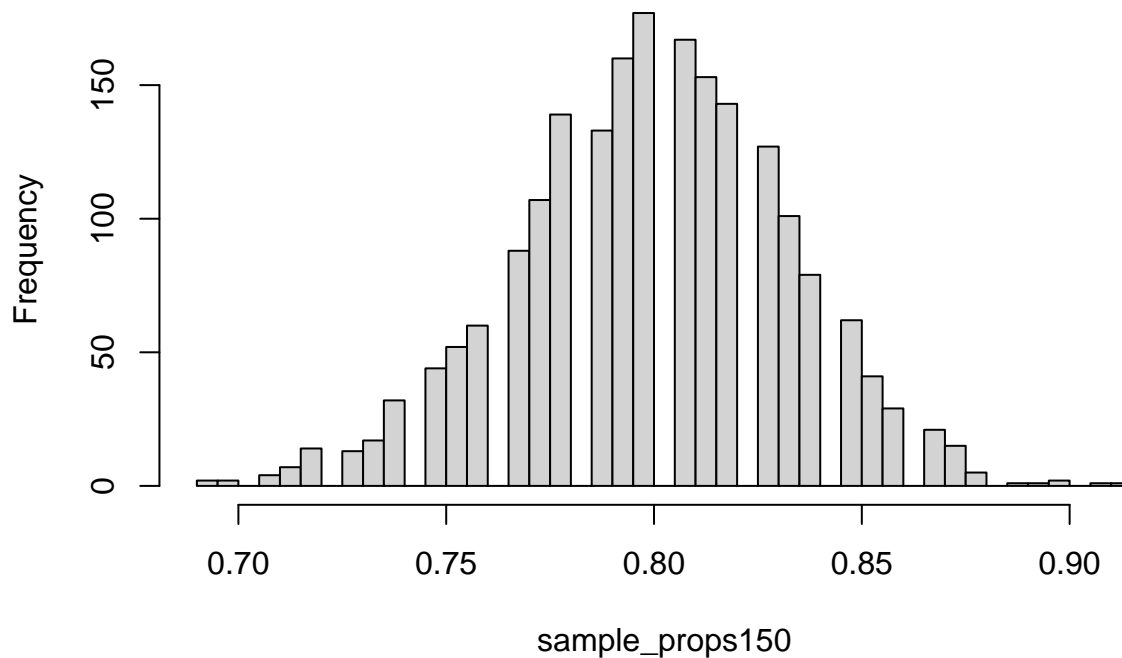
9. Change your sample size from 15 to 150, then compute the sampling distribution using the same method as above, and store these proportions in a new object called `sample_props150`. Describe the shape of this sampling distribution and compare it to the sampling distribution for a sample size of 15. Based on this sampling distribution, what would you guess to be the true proportion of those who think the work scientists do enhances their lives?

```
set.seed(123) # for reproducibility
sample_props150 <- replicate(2000, mean(sample(global_monitor$scientist_work, 150) == "Benefits"))
summary(sample_props150)
```

```
##      Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
## 0.6933  0.7800   0.8000  0.8006  0.8200  0.9133
```

```
hist(sample_props150, breaks = sqrt(length(sample_props150)),
     main = "Sampling Distribution of Proportion of Those Who Think Scientists Enhance Lives (n=150)")
```

ig Distribution of Proportion of Those Who Think Scientists Enhance L



Based on this sampling distribution, I would guess that the true proportion of those who think the work scientists do enhances their lives is around 0.8, which is consistent with the population proportion we calculated earlier.

10. Of the sampling distributions from 2 and 3, which has a smaller spread? If you're concerned with making estimates that are more often close to the true value, would you prefer a sampling distribution with a large or small spread?

The sampling distribution from part 3 with a sample size of 150 has a smaller spread compared to the sampling distribution from part 2 with a sample size of 15. If we're concerned with making estimates that are more often close to the true value, we would prefer a sampling distribution with a smaller spread. This is because a smaller spread indicates that the sample proportions are more tightly clustered around the true population proportion, which increases the chances of obtaining an estimate that is close to the true value.