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Example 8.1

Leakage Flow Past a Piston:-

Given:-

Gage Pressure = 20 MPa at 55°C

fluid is SAE 10W oil

Diameter = 25 mm

low Pressure = 1.0 MPa

mean radial clearance = 0.005 mm

To Find:-

Leakage flow rate:?

Solution:-

Governing eq = $\frac{Q}{l} = \frac{a^3 \Delta P}{12 \mu L}$

Assumption:-

(1) Laminar flow

(2) steady flow

(3) Incompressible flow

(4) Fully developed flow

So plate width is $l = \pi D$

eq be come

For SAE oil at 55° $\mu = 0.018$ kg/m.s

$$Q = \frac{1}{12} \times 25\ mm \times (0.0005)^3\ mm^3 \times (20-1) \times 10^6 \frac{N}{m^2}$$
$$\times \frac{m \cdot s}{0.018\,kg} \times \frac{1}{15\,mm} \times \frac{kg \cdot m}{N \cdot s^2}$$

$$Q = 57.6\ mm^3/s$$

Example 8.2

Torque and Power in a Journal Bearing:-

Given:-

SAE 30 oil at 210°F

diameter is = 3 in

diametral clearance is 0.0025 in.

Shaft rotates at 3600 rpm.

shaft lenght is 1.25 in.

Find:-

(a) Torque T.

(b) Power dissipated.

Solution:-

Governing eq = $\tau_{yx} = \mu \frac{U}{a} + a\left(\frac{\partial p}{\partial x}\right)\left[\frac{y}{a} - \frac{1}{2}\right]$

$\frac{\partial p}{\partial x} = 0$ due to flow is Symmetric

Assumption:-

1. Laminar flow
2. steady flow
3. Incompressible flow
4. fully developed flow
5. infinite width ($L/a = 1.25/0.00125 = 1000$
6. $\partial P/\partial x = 0$ (flow is symmetric).

So

$$\tau_{yx} = \mu \frac{U}{a} = \mu \frac{\omega R}{a} = \frac{\mu \omega D}{2a}$$

For SAE 30 at 210°F $\mu = 9.6 \times 10^{-3}$ N·s/m²

2.01×10^{-4}

$$\tau_{yx} = 2.01 \times 10^{-4} \frac{lbf.s}{ft^2} \times 3600 \frac{rev}{min} \times 2\pi \frac{rad}{rev} \times \frac{min}{60s} \times 3in \times \frac{1}{2} \times \frac{1}{0.00125\,in}$$

$\tau_{yx} = 90.9 \; lbf/ft^2$

$$T = FR$$

$$= \tau_{yx} \pi D L R = \frac{\pi}{2} \tau_{yx} D^2 L$$

$$= \frac{\pi}{2} \times 90.9 \frac{lbf}{ft^2} \times (3)^2 in^2 \times \frac{ft^2}{144\,in^2} \times 1.25\,in$$

$T = 11.2$ in. lbf

Power dissipated

$$W = FU = FR\omega = T\omega$$

$$= 11.2 \; in \cdot lbf \times 3600 \frac{rev}{min} \times \frac{min}{60s} \times 2\pi \frac{rad}{rev}$$

$$\times \frac{ft}{12\,in} \times \frac{hp \cdot s}{550\,ft \cdot lbf}$$

$W = 0.640$ hp

Example 8.3

Laminar Film on a vertical wall

Given:-

Fully developed lamina incompressible flow. Newtonian fluid down a vertical wall, thickness and film is constant.

$$\frac{\partial P}{\partial x} = 0$$

Find:- Expression for the velocity distribution

Solution:-

$$F_{S_x} + F_{B_x} = \frac{\partial}{\partial t}\int_{cv} u \rho \, dV + \int_{cs} u \rho \vec{V} \cdot d\vec{A}$$

For steady flow $\frac{\partial}{\partial t}\int_{cv} u \rho \, dV = 0$

For fully developed flow $= \int_{cs} u \rho \vec{V} \cdot d\vec{A} = 0$

Thus the momentum eq can be reduce to

$$F_{S_x} + F_{B_x} = 0$$

$$F_{B_x} = \rho g \, dV = \rho g \, dx \, dy \, dz$$

Shear stress on L.H.S $= \tau_{yx_L} = \left(\tau_{yx} - \frac{d\tau_{yx}}{dy}\frac{dy}{2}\right)$

Shear stress on R.H.S $= \tau_{yx_R} = \left(\tau_{yx} + \frac{d\tau_{yx}}{dy}\frac{dy}{2}\right)$

$$-\tau_{yx_L} \, dx \, dz + \tau_{yx_R} \, dx \, dz + \rho g \, dx \, dy \, dz = 0$$

$$-\left(\tau_{yx}-\frac{d\tau_{yx}}{dy}\frac{dy}{2}\right)dx\,dz+\left(\tau_{yx}+\frac{d\tau_{yx}}{dy}\frac{dy}{2}\right)dx\,dz$$
$$+\rho g\,dx\,dy\,dz=0$$

$$\frac{d\tau_{yx}}{dy}\cancel{dx\,dy\,dz}=-\rho g\,\cancel{dx\,dy\,dz}$$

$$\frac{d\tau_{yx}}{dy}=-\rho g$$

since

$$\tau_{yx}=\mu\frac{du}{dy}$$

Put in above result

$$\cancel{\frac{d\tau_{yx}}{dy}=-}$$

$$\mu\frac{d^2u}{dy^2}=-\rho g \Rightarrow \frac{d^2u}{dy^2}=\frac{-\rho g}{\mu}$$

Intigrating w.r.to y.

$$\frac{du}{dy}=-\frac{\rho g}{\mu}y+C_1$$

again Intgrating w.r.t y.

$$u=-\frac{\rho g}{\mu}\frac{y^2}{2}+C_1y+C_2$$

applying bondary condition.

$y=0$, $u=0$

$y=\delta$ $du/dy=0$

$C_2=0$

$0=-\frac{\rho g}{\mu}\delta+C_1 \Rightarrow$

$C_1=\frac{\rho g}{\mu}\delta$

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So

$$u = -\frac{\rho g}{\mu}\frac{y^2}{2} + \frac{\rho g}{\mu}\delta y$$

$$u = \frac{\rho g}{\mu}\delta^2\left[\left(\frac{y}{\delta}\right) - \frac{1}{2}\left(\frac{y}{\delta}\right)^2\right]$$

Example 8.4

Capillary viscometer:-

Given:-

Flow in capillary viscometer.

The flow rate is $Q = 880 \, mm^3/s$

Find:-

The fluid viscosity.

solution:-

Governing eq

$$Q = \frac{\pi \Delta P D^4}{128 \mu L}$$

Assumptions:-

(1) Laminar flow

(2) Steady flow

(3) Incompressible flow

(4) fully developed flow.

(5) Horizontal tube.

Then

$$\mu = \frac{\pi \Delta p D^4}{128 L Q}$$

$$= \frac{\pi}{128} \times 1.0 \times 10^6 \frac{N}{m^2} \times (0.50)^4 mm^4 \times \frac{s}{880\, mm^3} \times \frac{1}{1m} \times \left(\frac{m}{10^3 mm}\right)$$

$$= 1.74 \times 10^{-3}\ N.s/m^2$$