Formal Language & Automata Theory

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Equivalence of NFA and DFA

If DFA $D = (Q_D, \Sigma, \delta_D, \{q_0\}, F_D)$ is constructed from NFA $N = (Q_N, \Sigma, \delta_N, q_0, F_N)$ using subset construction then $L(D) = L(N)$

Proof Outline:

- Show the following by induction on $|w|$

$$\hat{\delta}_D(\{q_0\}, w) = \hat{\delta}_N(q_0, w)$$

Equivalence of NFA and DFA

- **Basis**

  Consider $|w| = 0$, then $w = \epsilon$

  Both $\hat{\delta}_D(\{q_0\}, \epsilon)$ and $\hat{\delta}_N(q_0, \epsilon)$ are $\{q_0\}$

  Hence, $\hat{\delta}_D(\{q_0\}, w) = \hat{\delta}_N(q_0, w)$
- **Induction**

  Assume that the $\hat{\delta}_D(\{q_0\}, w) = \hat{\delta}_N(q_0, w)$ holds for $|w| = n$

  Consider, $|\mathrm{w}| = \mathrm{n} + 1$ and $w = xa$ where a is the final symbol of w

  Hence, $|x| = n$ thus by induction hypothesis $\hat{\delta}_D(\{q_0\}, x) = \hat{\delta}_N(q_0, x)$

Equivalence of NFA and DFA

Consider that $\hat{\delta}_D(\{q_0\}, x) = \hat{\delta}_N(q_0, x) = \{p_1, p_2, \dots p_k\}$

Further we get;

$$\hat{\delta}_N(q_0, w) = \hat{\delta}_N(q_0, xa) = \delta_N\big(\hat{\delta}_N(q_0, x), a\big) = \bigcup_{i=1}^{k} \delta_N(p_i, a)$$

Now, from subset construction we get;

$$\delta_D(\{p_1, p_2, \dots p_k\}, a) = \bigcup_{i=1}^{k} \delta_N(p_i, a)$$

Equivalence of NFA and DFA

$$\hat{\delta}_D(q_0, w) = \hat{\delta}_D(\{q_0\}, xa) = \delta_D\big(\hat{\delta}_D(\{q_0\}, x), a\big)$$

$$= \delta_D(\{p_1, p_2, \ldots p_k\}, a) = \bigcup_{i=1}^{k} \delta_N(p_i, a)$$

Both D and N accepts w iff $\hat{\delta}_D(\{q_0\}, w)$ and $\hat{\delta}_N(q_0, w)$ contains a state in F_N.

Hence, $L(D) = L(N)$

Operations on DFA

Complement of DFA:

If $D = (Q, \Sigma, \delta, q_0, F)$ then complement of D is denoted as follows:

$$\overline{D} = (Q, \Sigma, \delta, q_0, Q - F)$$

Non final states become final states and vice-versa.

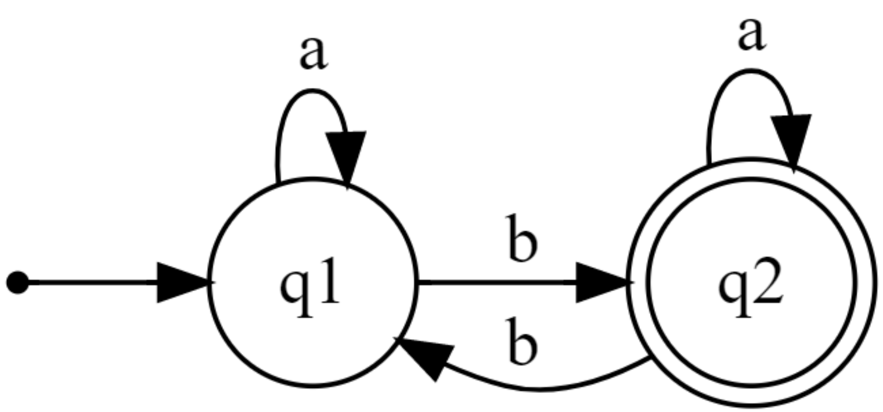
Every thing else remains same.

Operations on DFA

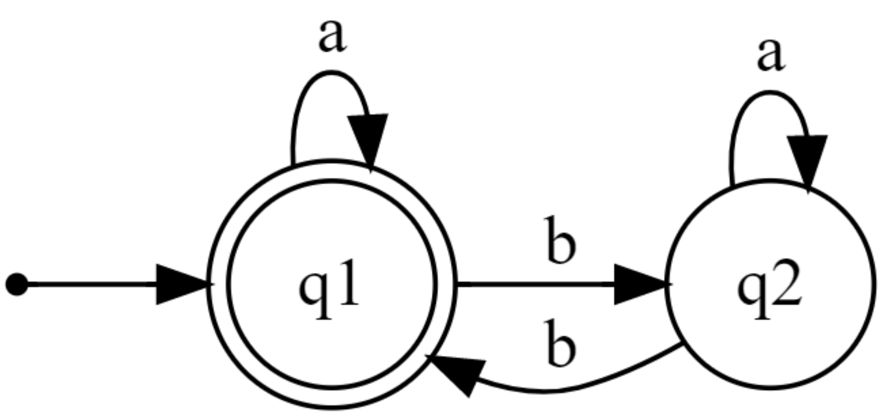
Complement of DFA (Example):

Design a DFA that accepts the following language and find its complement:

$L = \{w \mid w\ contains\ odd\ number\ of\ \boldsymbol{b}\}$ over $\Sigma = \{a, b\}$



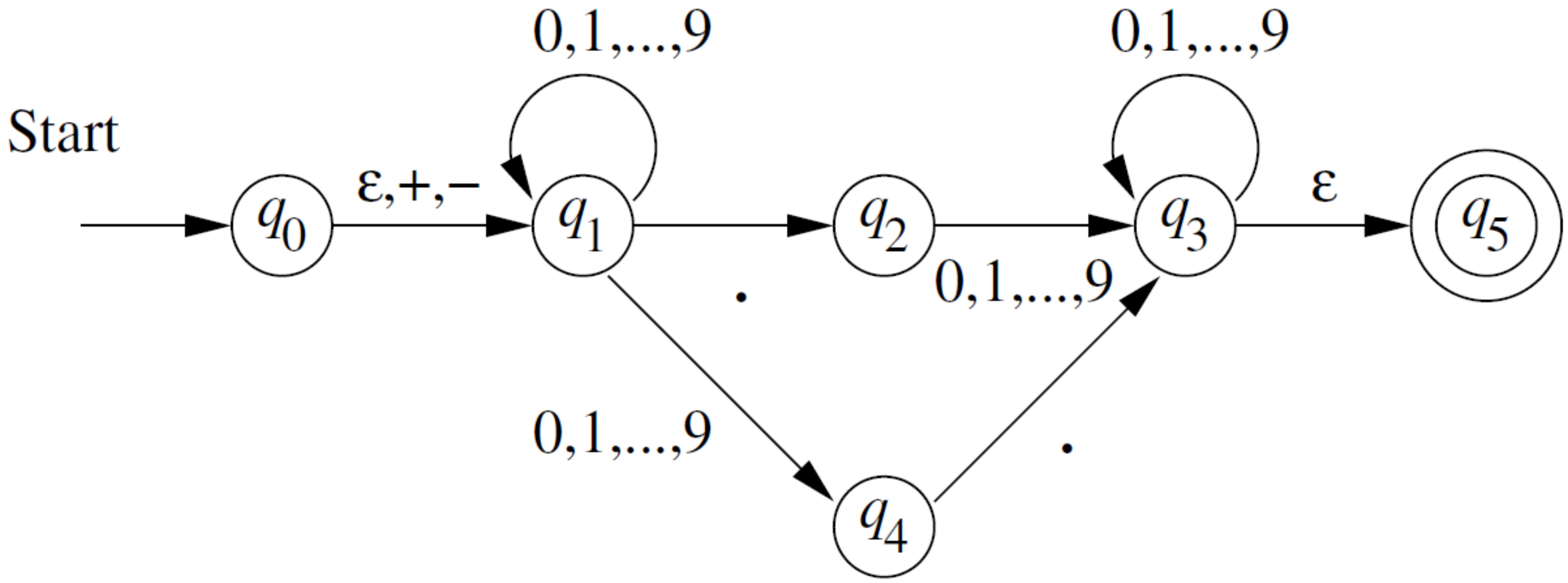
Original DFA accepting **odd** number of **b**



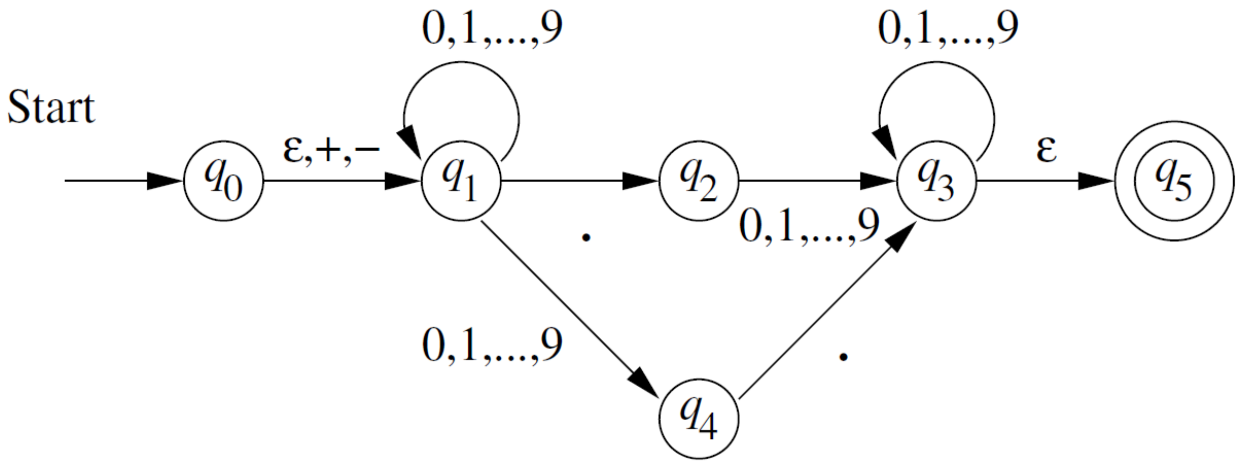
Complemented DFA accepting **even** number of **b**

NFA with ϵ Transition

- NFA where ϵ-transitions are allowed
- Only difference with NFA: $\delta: Q \times (\Sigma \cup \epsilon) \rightarrow P(Q)$
- Example: ϵ-NFA that accepts decimal numbers



NFA with ϵ Transition



Transition Diagram

	ϵ	$+, -$	$.$	$0, 1, \ldots, 9$
q_0	$\{q_1\}$	$\{q_1\}$	$\emptyset$	$\emptyset$
q_1	$\emptyset$	$\emptyset$	$\{q_2\}$	$\{q_1, q_4\}$
q_2	$\emptyset$	$\emptyset$	$\emptyset$	$\{q_3\}$
q_3	$\{q_5\}$	$\emptyset$	$\emptyset$	$\{q_3\}$
q_4	$\emptyset$	$\emptyset$	$\{q_3\}$	$\emptyset$
q_5	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$

Transition Table

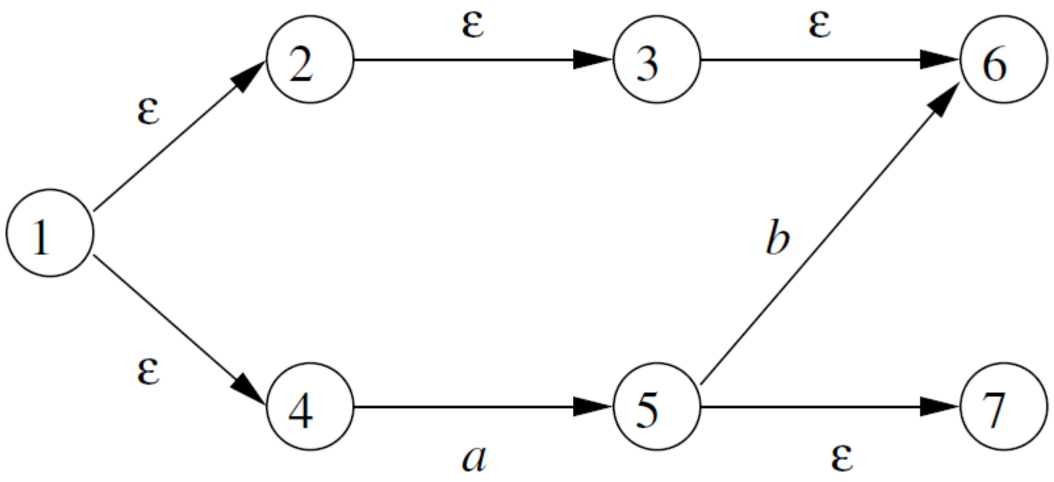
ϵ-closure

Find ϵ-close of state q by following all transitions out of q with label ϵ

$\underline{ECLOSE(q):}$
$E \ = \{q\}$

1. For all p in $\delta(q, \epsilon) - q$
   i. $E \leftarrow E \cup ECLOSE(p)$

2. Return E



ECLOSE(1) = {1,2,3,6,4}

Transition Function of ϵ-NFA

- Consider $w = xa$ where a is the last symbol and x is a rest of w.
- Assume $\hat{\delta}(q, x) = \{p_1, p_2, \dots p_k\}$

If

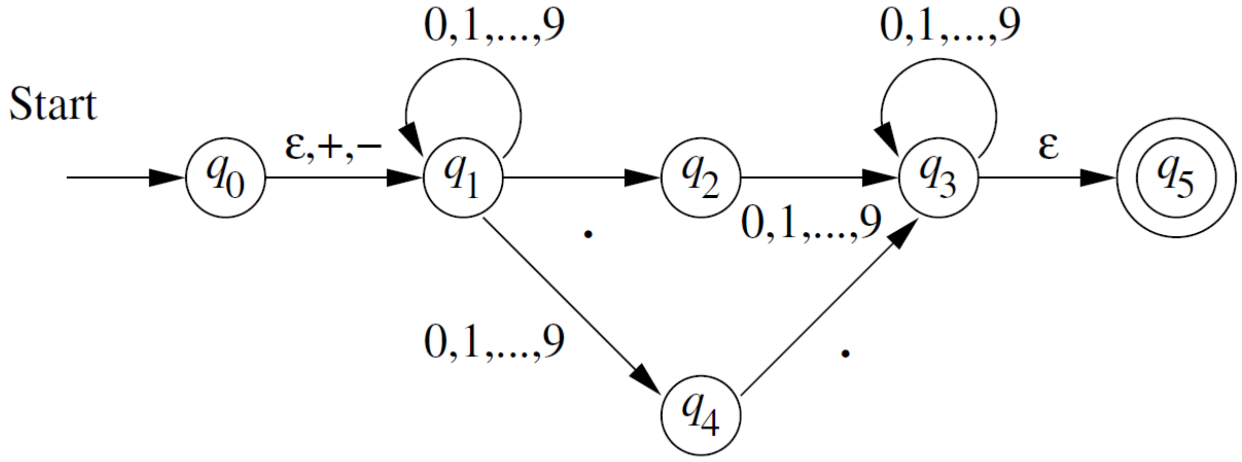
$$\bigcup_{i=1}^{k} \delta(p_i, a) = \{r_1, r_2, \dots r_m\}$$

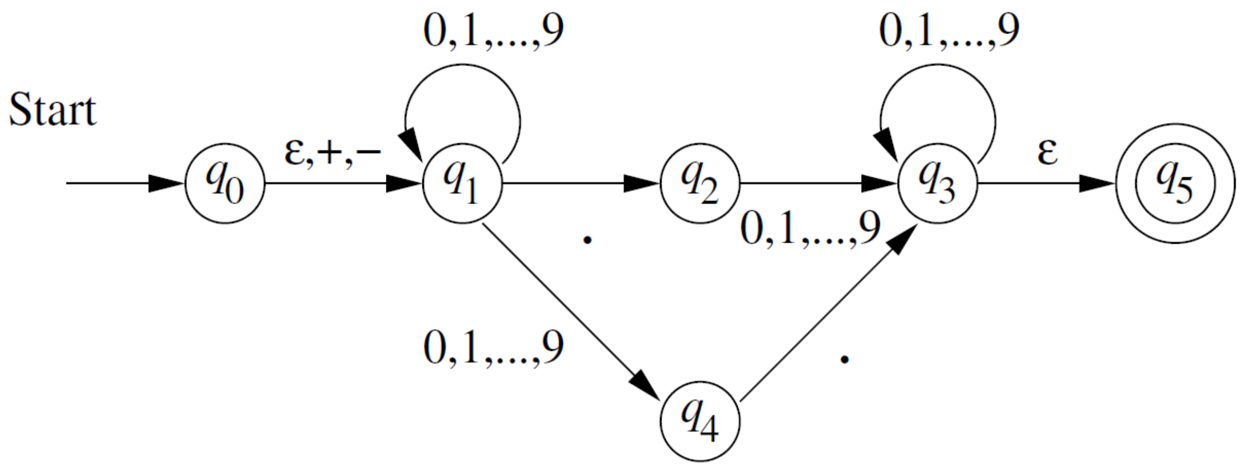
then

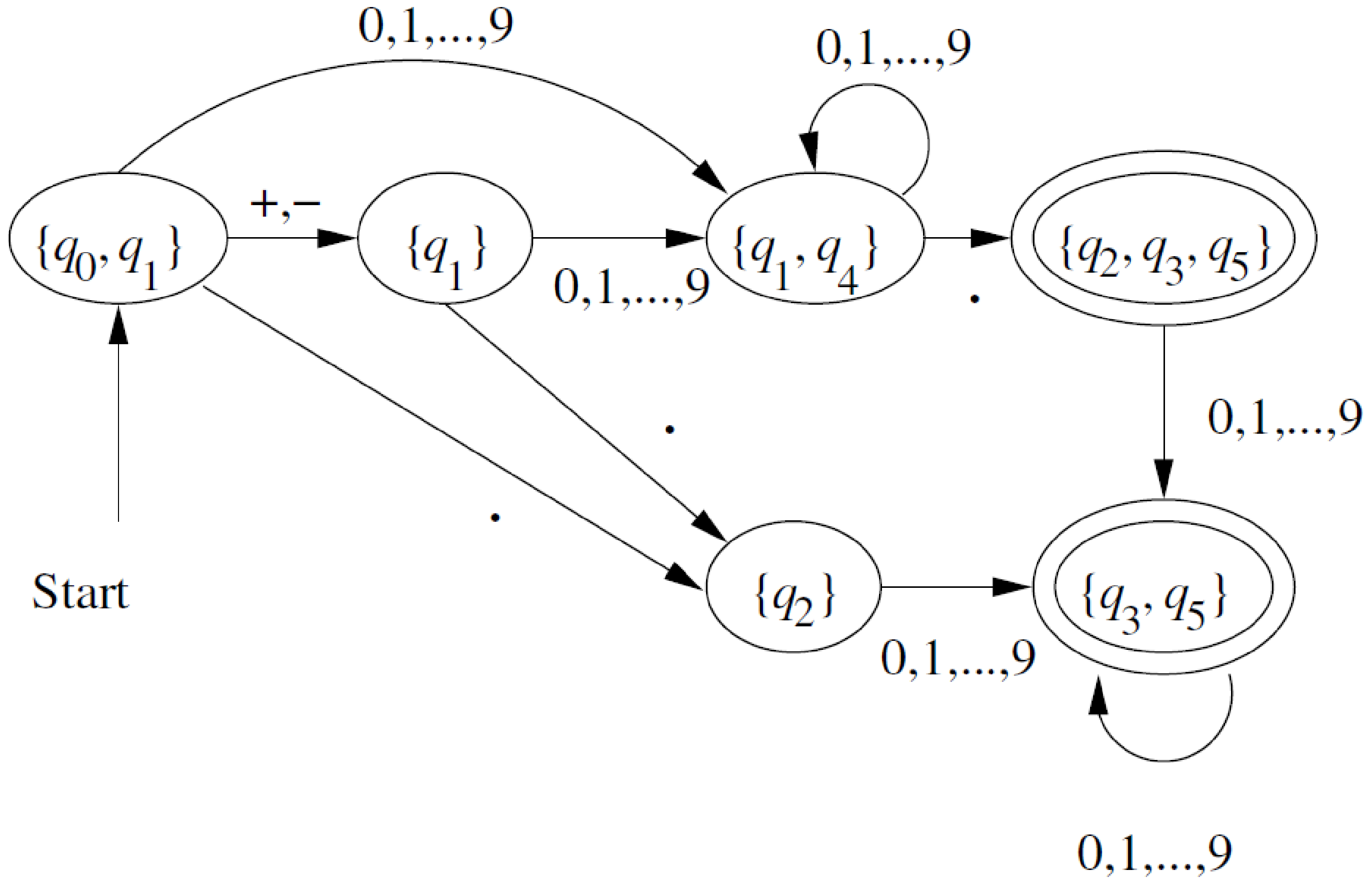
$$\hat{\delta}(q, w) = ECLOSE\{r_1, r_2, \dots r_m\}$$

Eliminating ϵ-Transition

- Consider ϵ-NFA $\boldsymbol{E} = (\boldsymbol{Q_E}, \boldsymbol{\Sigma}, \boldsymbol{\delta_E}, \boldsymbol{q_0}, \boldsymbol{F_E})$
- Consider DFA $\boldsymbol{D} = (\boldsymbol{Q_D}, \boldsymbol{\Sigma}, \boldsymbol{\delta_D}, \boldsymbol{q_D}, \boldsymbol{F_D})$
- Conversion:
  - $Q_D = \{S | S \subseteq Q_E and\ \epsilon - closed\}$
  - Σ remains same
  - $\delta_D(S, a)$ is computed as follows:
    - Assume $S = \{p_1, p_2, \ldots p_k\}$
    - $\bigcup_{i=1}^{k} \delta_E(p_i, a) = \{r_1, r_2, \ldots r_m\}$
    - $\delta_D(S, a) = ECLOSE(\{r_1, r_2, \ldots r_m\})$
  - $q_D = ECLOSE(q_0)$
  - $F_D = \{S | S \subseteq Q_D\ and\ S \cap F_E \neq \emptyset\}$




Start
ε,+,−
0,1,...,9
.
ε
q0
q1
q2
q3
q4
q5


Start
+,−
0,1,...,9
.
{q0, q1}
{q1}
{q1, q4}
{q2, q3, q5}
{q2}
{q3, q5}