COMPLEX NUMBER

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- Definition
- Different forms
- Modulus
- Argument

IMAGINARY UNIT

- You can't take the square root of a negative number. If you use imaginary units, you can!
- The imaginary unit is 'i'
- $i = \sqrt{-1}$
- It is used to write the square root of a negative number

Property of the square root of negative numbers

- If r is a positive real number, then

$$\sqrt{-r} = i\sqrt{r}$$

Examples:

$$\sqrt{-2} = i\sqrt{2} \qquad \sqrt{-9} = i\sqrt{9} = 3i$$

THE POWERS OF i

$i = \sqrt{-1}$, then:

$i^2 = -1$	$i^3 = -i$	$i^4 = 1$	$i^5 = i$
$i^6 = -1$	$i^7 = -i$	$i^8 = 1$	*etc.*

For i^n … divide n by 4 …

- If n is evenly divisible by 4 then $i^n = 1$
- If the remainder is 1, then $i^n = i$
- If the remainder is 2, then $i^n = -1$
- If the remainder is 3, then $i^n = -i$

DEFINITION

For real numbers x and y the number $z = x + iy$ is a complex number.

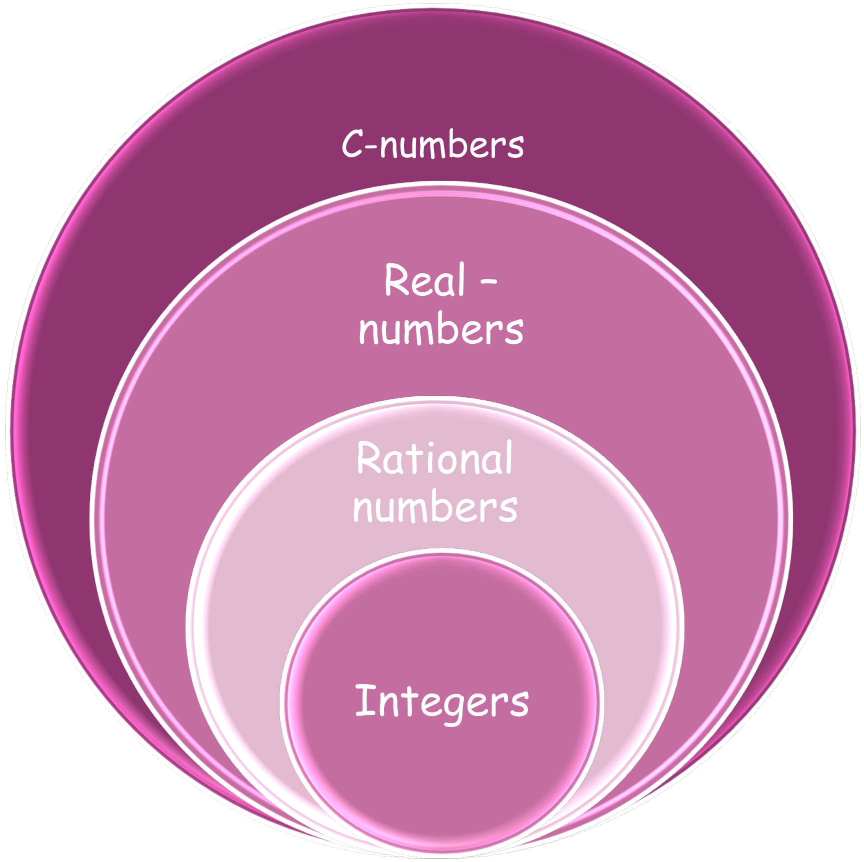
$$z = x + iy$$

Real part Imaginary part

- All numbers can be expressed as complex numbers.

$3 = 3 + i \cdot 0$ $-6i = 0 + i \cdot (-6)$

- The complex conjugate of a complex number, $z = x + iy$, denoted by $\bar{z}$, is given by $\bar{z} = x - iy$
- Two complex numbers $x + iy$ and $c + id$ are equal, if $x = c$ and $y = d$



ALGEBRAIC FORM OF z

$$\boxed{z = x + iy}$$

where: x and y real number

$$(\pm \boldsymbol{i})^2 = -\mathbf{1}$$

Examples :

- $z = 4 + 5\,i$
- $z = 4 - 15\,i = 4 + (-15)i$
- $z = -44 - 35\,i = -44 + (-35)i$
- $z = 4 = 4 + (0)i$
- $z = 6i = 0 + 6i$

2 IMPORTANT CONCEPTS

$$z = x + iy$$

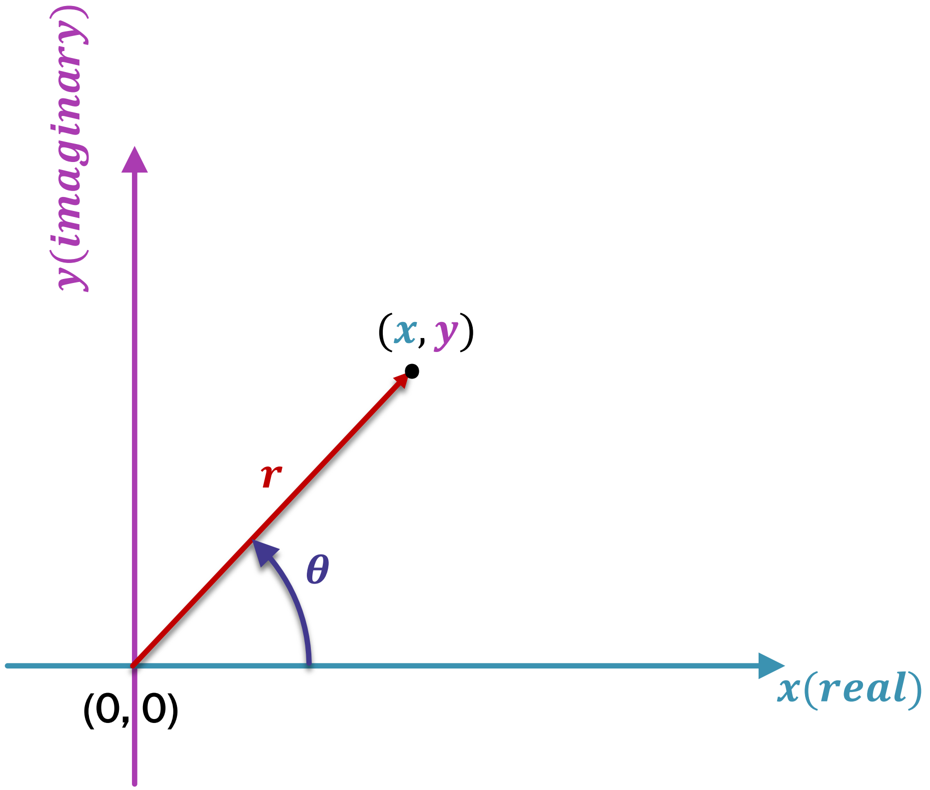
Modulus of z :

Notation : $|\, z \,| = r$

Rule : $|\, z \,| = \sqrt{x^2 + y^2}$

Argument of z :

Notation : $\arg(z)$ or θ



FINDING MODULUS OF Z

$$z = x + iy$$

Modulus of z :

- Notation : $|\, z \,|$
- Rule : $|\, z \,| = \sqrt{x^2 + y^2}$

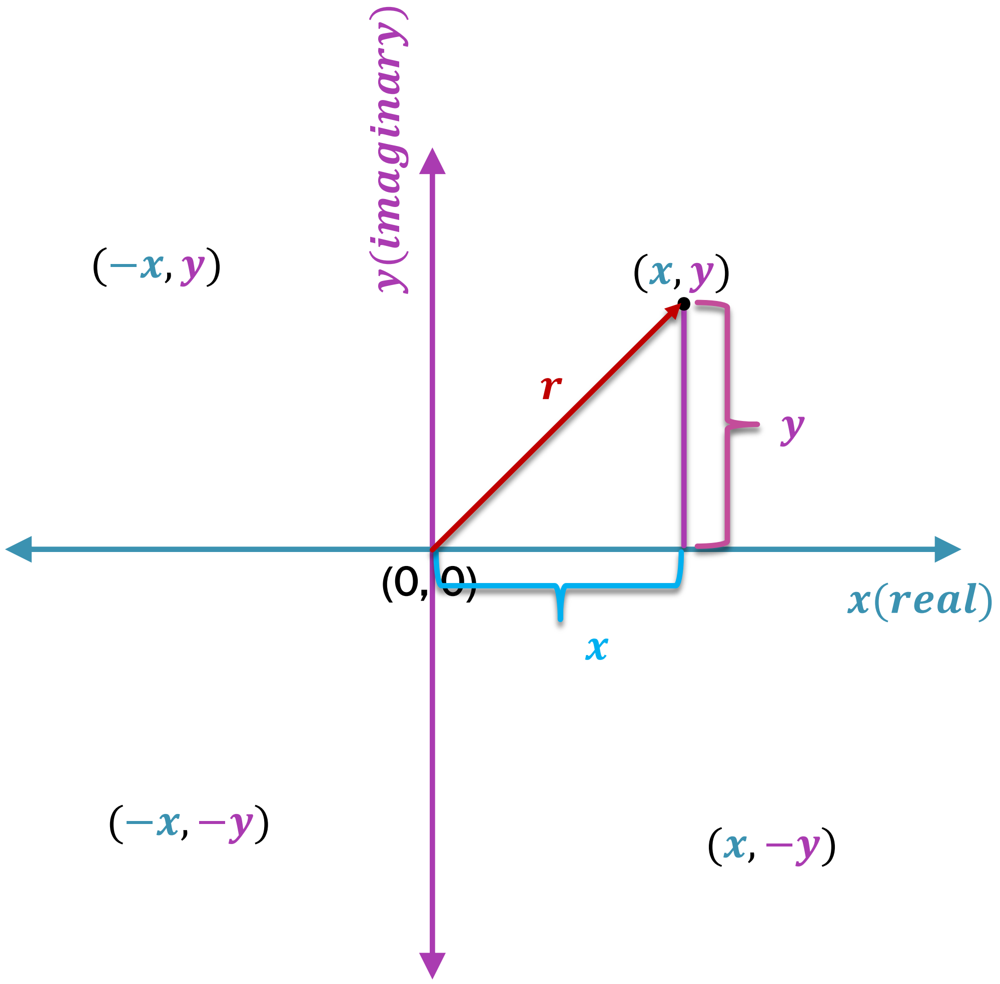
Examples :

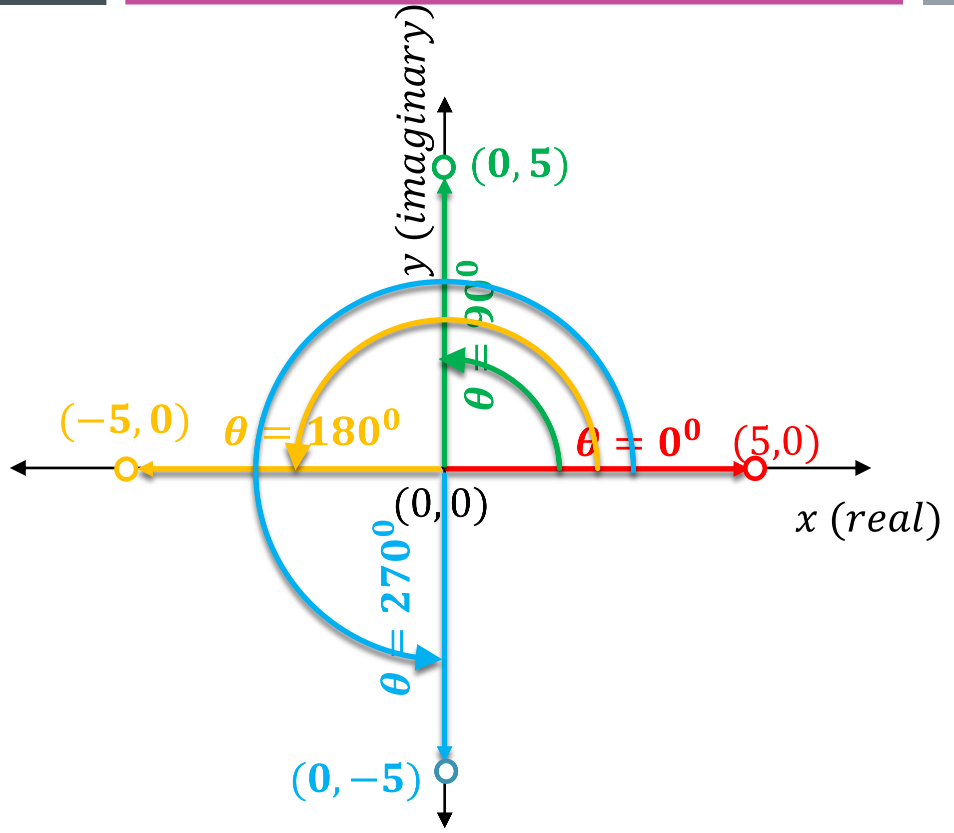
$z = 5 + 5\,i \qquad \therefore |\, z \,| = \sqrt{5^2 + 5^2} = \sqrt{50}$

$z = -5 + 5\,i \qquad \therefore |\, z \,| = \sqrt{(-5)^2 + 5^2} = \sqrt{50}$

$z = -5 - 5\,i \qquad \therefore |\, z \,| = \sqrt{(-5)^2 + (-5)^2} = \sqrt{50}$

$z = 5 - 5\,i \qquad \therefore |\, z \,| = \sqrt{5^2 + (-5)^2} = \sqrt{50}$



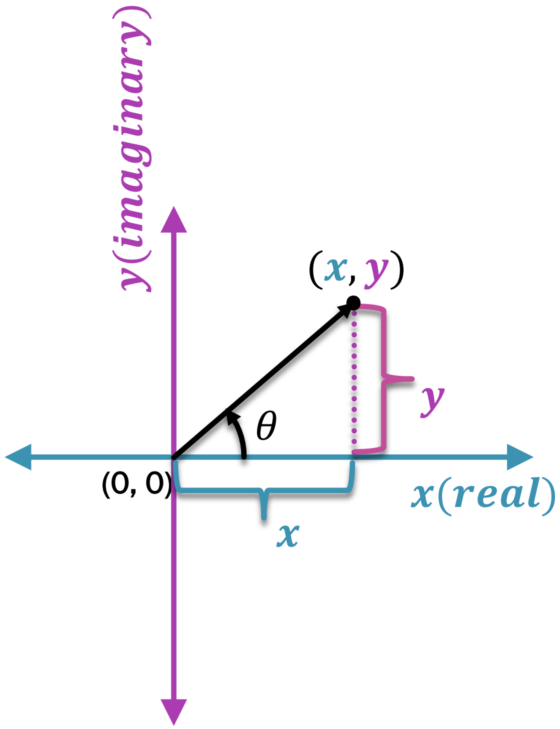


ARGUMENT OF Z

z	θ	θ
$(5,0)$	0^0	0
$(-5,0)$	180^0	π

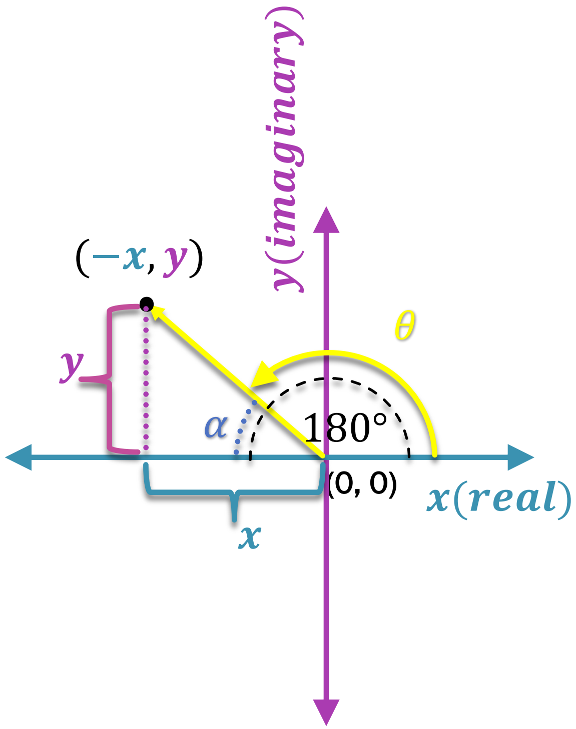
z	θ	θ
$(0,5)$	90^0	$\pi/2$
$(0,-5)$	270^0	$3\pi/2$

$z = x + iy$



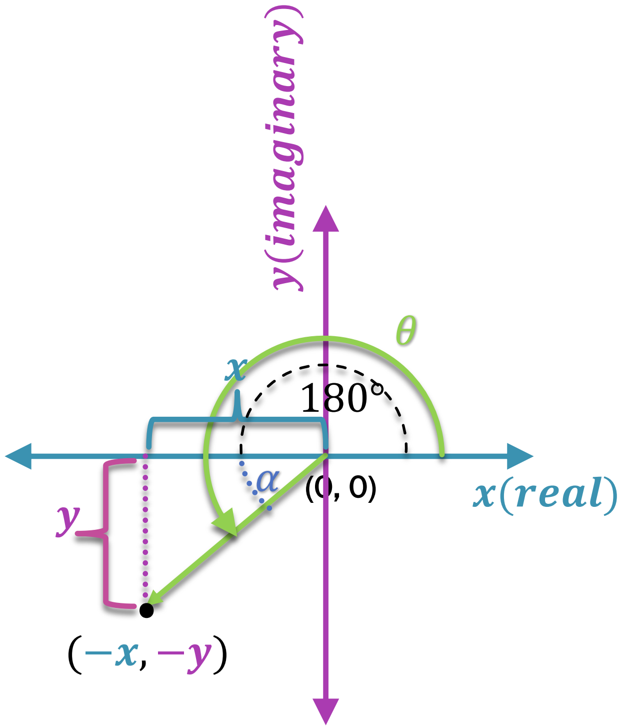
$$\theta = \tan^{-1}\left|\frac{y}{x}\right|$$

$z = -x + iy$



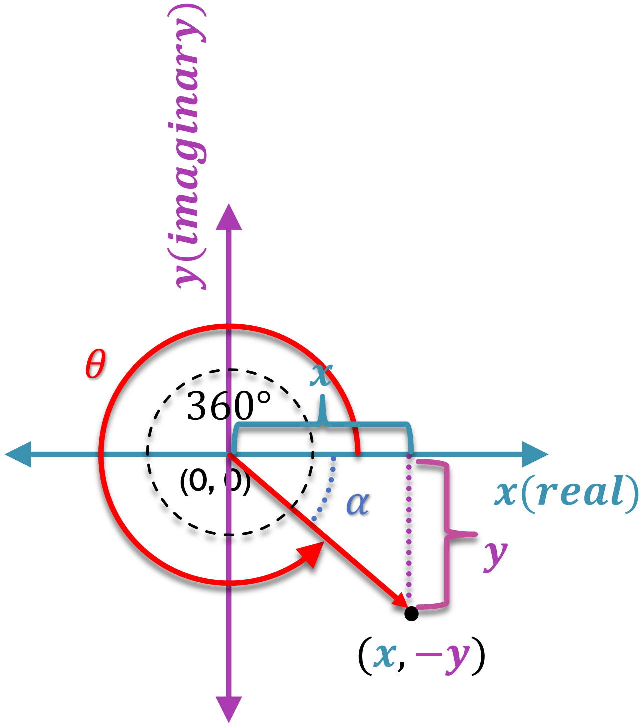
$$\theta = 180° - \alpha$$
$$\therefore \theta = 180° - tan^{-1}\left|\frac{y}{x}\right|$$

$z = -x - iy$



$$\theta = 180° + \alpha$$
$$\therefore \theta = 180° + tan^{-1}\left|\frac{y}{x}\right|$$

$z = x - iy$



$$\theta = 360° - \alpha$$
$$\therefore \theta = 360° - tan^{-1}\left|\frac{y}{x}\right|$$

$\boldsymbol{\theta} =?$

- Plot $z = (x, y)$ in the coordinate axes. The following cases will arise:
- $\tan^{-1}\theta = \arctan\theta$
- To find θ in radians, use:
  - radian mode ;
  - π for 180^0 & 2π for 360^0

$\theta = 180^0 - \tan^{-1}\left|\frac{y}{x}\right|$

$\theta = \tan^{-1}\left|\frac{y}{x}\right|$

$\theta = 180^0 + \tan^{-1}\left|\frac{y}{x}\right|$

$\theta = 360^0 - \tan^{-1}\left|\frac{y}{x}\right|$

$\boldsymbol{\theta} =?$

$z_1 = 5 + 5\,i$

$$\theta_1 = \tan^{-1}\left|\frac{5}{5}\right| = \tan^{-1} 1 = 45^0$$

$z_2 = -5 + 5\,i$

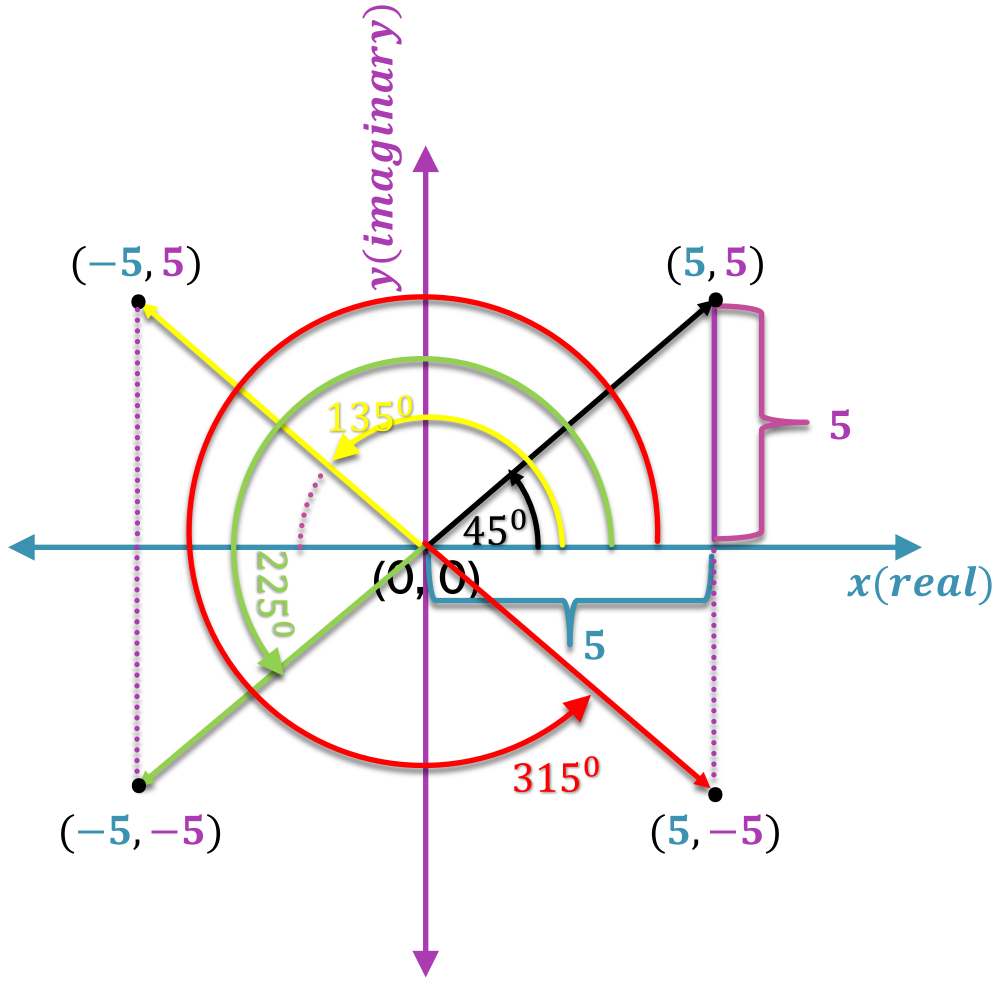
$$\theta_2 = 180^0 - \tan^{-1}\left|\frac{5}{-5}\right| = 180^0 - \tan^{-1} 1 = 180^0 - 45^0 = 135^0$$

$z_3 = -5 - 5\,i$

$$\theta_3 = 180^0 + \tan^{-1}\left|\frac{-5}{-5}\right| = 180^0 + \tan^{-1} 1 = 180^0 + 45^0 = 225^0$$

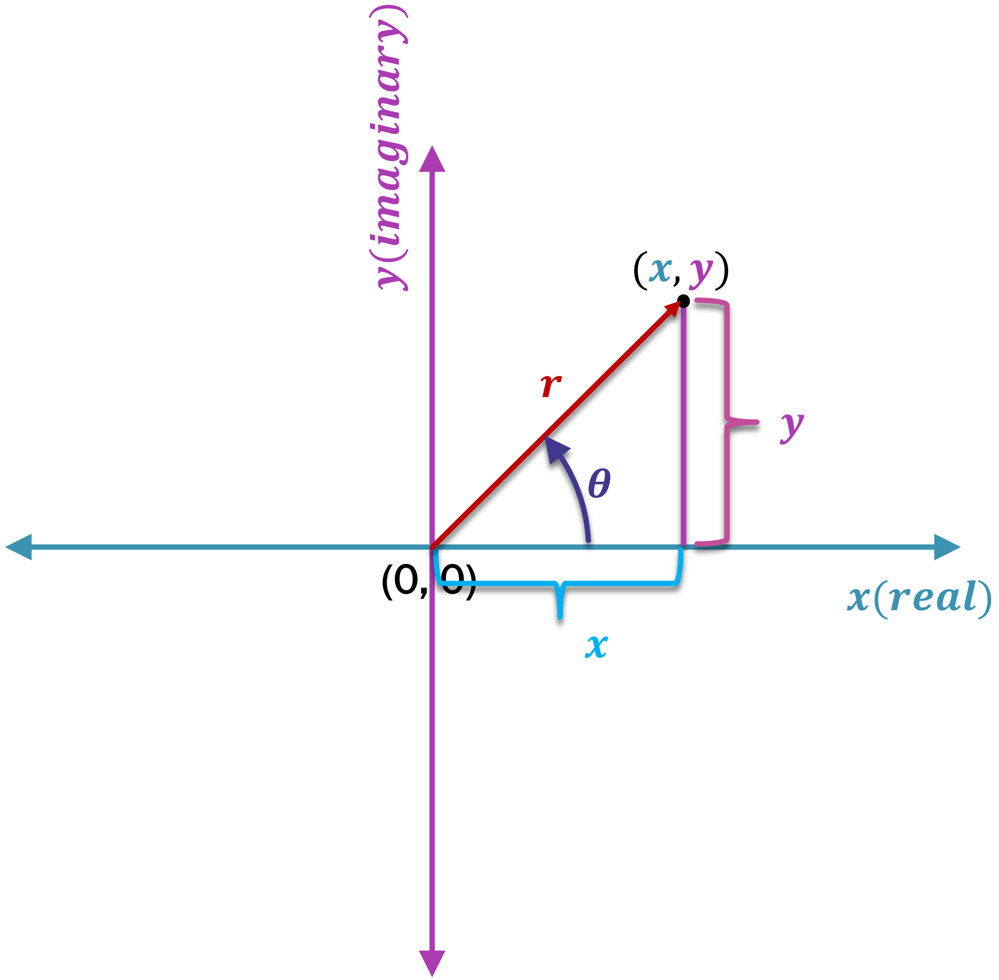
$z_4 = 5 - 5\,i$

$$\theta_4 = 360^0 - \tan^{-1}\left|\frac{-5}{5}\right| = 360^0 - \tan^{-1} 1 = 360^0 - 45^0 = 315^0$$



POLAR FORM OF COMPLEX NUMBER

$z = x + iy$



- Based on the figure:

$$\cos\theta = \frac{x}{r}$$

$$\therefore x = r\cos\theta$$

Again,

$$\sin\theta = \frac{y}{r}$$

$$\therefore y = r\sin\theta$$

- The polar form is defined by:

$$z = r\cos\theta + ir\sin\theta$$

$$z = r(\cos\theta + i\sin\theta)$$

$$z = (r, \theta)$$

Where $r = \sqrt{x^2 + y^2}$ and $\theta = tan^{-1}\left(\frac{y}{x}\right)$

EXPONENTIAL FORM OF z

$$\boxed{z = re^{i\theta}}$$

where, $r = |z|$

$\theta = \arg(z)$

θ must be in radian.