Crash Course on

Laws of Motion

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Newton's laws of motion

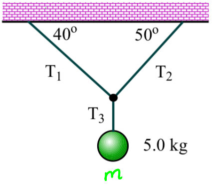
- Newton's First Law: Consider a body on which no force is acting. Then if it is at rest it will remain at rest, and if it is moving with constant velocity it will continue to move at that velocity.
- Newton's Second Law: Newton's Second Law is a relation between the net force (F) acting on a mass **m** and its acceleration **a**. It says: $\sum \vec{F} = m\vec{a}$. Note: At equilibrium, $\sum \vec{F} = 0$. (Also, we can state as $\Sigma \vec{F} = \frac{d\vec{p}}{dt}$, $\vec{p} = m\vec{v}$)
- Newton's Third Law(law of action and reaction): Consider two objects A and B. The force which object A exerts on object B is equal and opposite to the force which object B exerts on object A i.e., $\vec{F}_{AB} = -\vec{F}_{BA}$.

Warm up problems

1. A $3.0kg$ mass undergoes an acceleration given by $\vec{a} = (2\hat{i} + 5\hat{j}\,)\, m/s^2$. Find the resultant force $\vec{F}$ and its magnitude.
2. Two forces act on a particle of mass $m = 3.2kg$ and move continuously with velocity $3\hat{i}$ - 4 $\hat{j}$ m/s. One of the forces is $\vec{F}_1$=$2\hat{i}$ -6 $\hat{j}$ N. What is the other force?
3. A $4.0kg$ object has a velocity of 3.0 $\hat{i}$ m/s at one instant. Eight seconds later, its velocity is (8 $\hat{i}$+10 $\hat{j}$) m/s . Assuming the object was subject to a constant net force, find (a) the components of the force and (b) its magnitude.
4. If a man weighs 875 N on Earth, what would he weigh on Jupiter, where the free–fall acceleration is 25.9 m/s^2 ?

Practice Problem

Q.1. Find the tension in each cord for the systems shown in figure below. (Neglect the mass of the cords.).

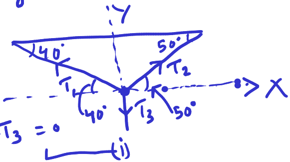


Tension T_3, using Newton's 2^{nd} law. Since the system is in equilibrium, acceleration $= 0$

$\therefore$ For mass 'm', $\Sigma F_y = 0$

$\Rightarrow T_3 - mg = 0 \Rightarrow T_3 = mg$

To calculate T_1 & T_2,



$\Sigma F_y = 0$

$\Rightarrow T_1 \sin 40^\circ + T_2 \sin 50^\circ - T_3 = 0$ —— (i)

$\Sigma F_x = 0$

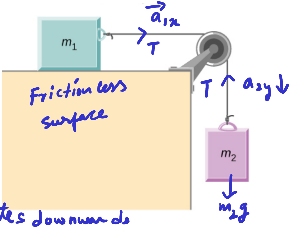
$\Rightarrow T_2 \cos 50^\circ - T_1 \cos 40^\circ = 0$ —— (ii)

Free body diagram of mass 'm'

Now, we can solve for T_1 & T_2 from equation (i) & (ii) easily.

Practice Problem

Q.2. Figure below shows a block of mass m_1 on a frictionless, horizontal surface. It is pulled by a light string that passes over a frictionless and massless pulley. The other end of the string is connected to a block of mass m_2 . Find the acceleration of the blocks and the tension in the string in terms of m_1, m_2, and g.



Since m_2 accelerates downwards, it will pull m_1 with the same accelerat^n, $a = a_{2y} = a_{1x}$.

For Block - 1, $\sum F_x = m_1 a_{1x} = m_1 a$

$\Rightarrow \quad T = m_1 a \quad \text{——(1)}$

For Block - 2, $\sum F_y = m_2 a_{2y}$

$\Rightarrow \quad T - m_2 g = m_2(-a)$ ($\because$ m_2 accelerates downwards

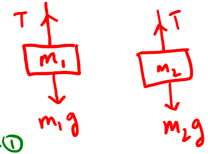
$\Rightarrow \quad T - m_2 g = -m_2 a \quad \text{——(2)}$

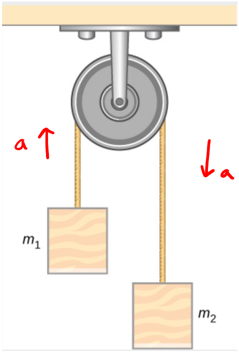
Solving eqn (1) & (2) we get, $a = \dfrac{m_2 g}{m_1 + m_2}$ and $T = \dfrac{m_1 m_2}{m_1 + m_2} g$

Practice Problem

Q.3. Consider the Atwood's machine, which consists of a rope running over a pulley (assumed to be frictionless), with two objects of mass m_1 and m_2 attached($m_2 > m_1$). If m_2 is released, what will its acceleration be? (express in terms of m_1, m_2, and g)

Free body diagram,





For Block of mass m_1

$$T - m_1 g = m_1 a \quad \text{—— (1)}$$

For Block of mass m_2

$$T - m_2 g = -m_2 a \quad \text{—— (2)}$$

Solving for 'T' & 'a' from eqn (1) & (2) we get–

$$a = \frac{m_2 - m_1}{m_1 + m_2} g$$

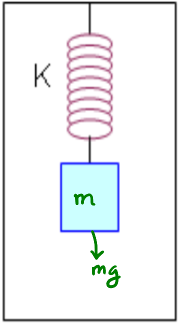
Tension of the 1st block, $T = m_1 a + m_1 g = \left(\frac{(m_2 - m_1) m_1}{m_1 + m_2} g + m_1 g\right) = \left(\frac{2 m_1 m_2}{m_1 + m_2}\right) g$

Practice Problem

Q.4. A 10kg block is hanging from a spring with spring constant K=1000 N/m. The spring is attached to the ceiling of an elevator. The elevator is currently moving upwards at 10 m/s and slowing down at 1.0 m/s^2. How much is the spring stretched?

Force on the spring, $\boxed{F_s = -Kx}$

Since the elevator is slowing down, the accl^n. is taken to be negative.



$$\sum F_y = ma_y$$

$$F_s - mg = -ma$$

$$-Kx = m(g-a)$$

$$x = -\frac{m(g-a)}{K} = \frac{10\times(9.8-1)}{1000}$$

$$= \frac{10\times 8.8}{1000} = \frac{8.8}{100} = 0.088\,m$$

$$= 8.8\ cm$$ //

Practice Problem

Given, $x = 20\,cm = 0.02\,m$
$K = 15\,N/m$, $m = 0.3\,kg$

Using $F = ma$

$a = F/m = \frac{-kx}{m} = -\frac{15 \times 0.02}{0.3} = -10\,m/s^2$

Q.5a. A particle of mass 0.3 kg is subjected to a force $F = -kx$ with $k = 15$ N/m. What will be its initial acceleration if it is released from a point $x = 20$ cm ?

Q.5a. A spring is stretched by 5 cm by a force 10 N. The time period of the oscillations when a mass of 2 kg is suspended by it is : **NEET-2021**

1. 0.0628 s
2. 6.28 s
3. 0.628 s ✓
4. 3.14 s

Given, $x = 0.05\,m$, $F = 10\,N$, $m = 2\,kg$

We know, $F = kx$, $\Rightarrow k = \frac{F}{x} = \frac{100}{0.05} = 200\,N/m$

Frequency $f = \frac{1}{2\pi}\sqrt{\frac{K}{m}}$

$\therefore$ Time period, $T = \frac{1}{f} = 2\pi\sqrt{\frac{m}{K}}$

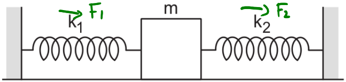
$\Rightarrow T = 2\pi\sqrt{\frac{2}{200}} = 2\pi \times \sqrt{\frac{1}{100}} = \frac{2\pi}{10} = 0.628\,s$

Practice Problem

Q.6. Both the springs shown in figure are unstretched. If the block is displaced by a distance x and released, what will be the initial acceleration?

Since we displaced both the springs by a distance 'x'

The forces F_1 & F_2 will be



$$F_1 = -k_1 x, \quad F_2 = -k_2 x$$

Using $\sum F = ma$

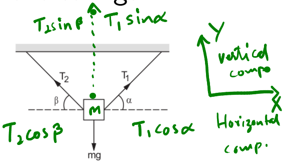
$$F_1 + F_2 = ma$$

$$-k_1 x - k_2 x = ma$$

$$a = \frac{-(k_1 + k_2)}{m} x$$

Practice Problem

Q.7.A body of mass m is suspended by two strings making angles α and β with the horizontal. Find the tensions in the strings.



Resolving the components of forces along the horizontal & vertical directions we have

$$T_1 \cos\alpha - T_2 \cos\beta = 0 \quad (\because \text{there is no acceleration } (a=0))$$

or, $$T_1 \cos\alpha = T_2 \cos\beta \quad \text{--- (1)}$$

Again, $$T_1 \sin\alpha + T_2 \sin\beta - mg = 0$$

$\Rightarrow$ $$T_1 \sin\alpha + T_2 \sin\beta = mg$$

or, $$T_2 \frac{\cos\beta \sin\alpha}{\cos\alpha} + T_2 \sin\beta = mg$$

or, $$T_2 \left(\frac{\cos\beta \sin\alpha + \sin\beta \cos\alpha}{\cos\alpha} \right) = mg$$

$\Rightarrow$ $$T_2 \sin(\alpha+\beta) = mg\cos\alpha$$

$$T_2 = \frac{mg\cos\alpha}{\sin(\alpha+\beta)}$$

putting T_2 in eqn (1) we get

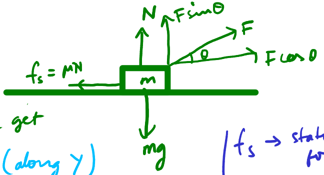
$$T_1 \cos\alpha = T_2 \cos\beta$$

$$T_1 \cancel{\cos\alpha} = \frac{mg \cancel{\cos\alpha} \times \cos\beta}{\sin(\alpha+\beta)}$$

$$T_1 = \frac{mg\cos\beta}{\sin(\alpha+\beta)}$$

Practice Problem

Q.8. A block of mass m rest on a horizontal floor with which it has a coefficient of static friction μ. It is desire to make the block move by applying the minimum possible force F. The magnitude of F which has to be applied is



1. $mg\sin\theta$
2. $mg\cos\theta$
3. $mg\cot\theta$
4. $mg\tan\theta$

Resolving the forces along Y & X directions, we get

| $f_s \to$ static friction force

$N + F\sin\theta - mg = 0$ (along Y)

$F\cos\theta - \mu N = 0$ (along X)

$\Rightarrow$ $N + F\sin\theta = mg$ —— (1)

and, $F\cos\theta = \mu N$ —— (2) $\Rightarrow N = \dfrac{F\cos\theta}{\mu}$ —— (*)

Eliminating N from (1) & (2) we get by using (*) in eqn (1) as

$F(\cos\theta + \mu\sin\theta) = \mu mg$

$\Rightarrow F = \dfrac{\mu mg}{(\cos\theta + \mu\sin\theta)}$

F will be minimum if $\cos\theta + \mu\sin\theta$ is maximum.

Let us maximise the denominator;

let $y = \cos\theta + \mu\sin\theta$, $\Rightarrow \dfrac{dy}{d\theta} = 0$ (condition for maximum)

$-\sin\theta + \mu\cos\theta = 0$

$\Rightarrow \mu = \tan\theta$

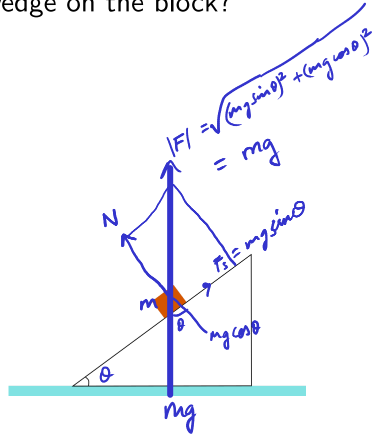
$\therefore F = \dfrac{mg\tan\theta}{\cos\theta + \tan\theta\sin\theta} = mg\sin\theta$ //

Practice Problem

Q.9. A block of mass m is at rest on a rough wedge as shown in figure. What is the force exerted by the wedge on the block?

1. Vertically upwards
2. $mg\sin\theta$
3. mg
4. Parallel to incline

$\therefore$ Vertically upwards and mg

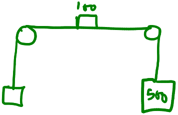


Practice Problem

Q.10. The acceleration of the 500 gm block in the figure below is

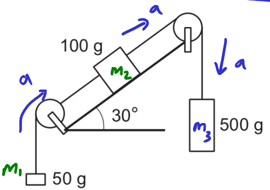
1. $\frac{8}{13} g$ upward
2. $\frac{8}{9} g$ downward
3. $\frac{8}{13} g$ downward
4. $\frac{1}{2} g$ upward

Note: If $\theta = 0$, and then we have



$$\Rightarrow a = \frac{m_3 g - m_1 g}{m_1 + m_2 + m_3} = \frac{450}{650} g$$

$$a = \frac{9}{10} g \text{ downward}$$



$$F_{net} = m_T a$$

$$a = \frac{F_{net}}{m_T} = \frac{m_3 g - m_1 g - m_2 g \sin\theta}{m_1 + m_2 + m_3}$$

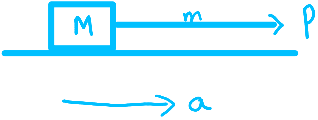
$$= \frac{(500 - 50 - \frac{100}{2})/1000 \; g}{650/1000}$$

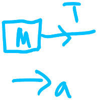
$$= \frac{400}{650} g = \frac{40}{65} g = \frac{8}{13} g \text{ downward}$$

Practice Problem

Q.11. A block of mass M is pulled along a horizontal frictionless surface by a rope of mass m. If a force P is applied at the free end of the rope, the force exerted by the rope on the block is

1. $\frac{Pm}{M+m}$
2. $\frac{P}{M+m}$
3. $\frac{Pm}{2}$
4. $\frac{PM}{M+m}$ ✓

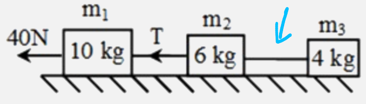




$$\sum F = ma + Ma$$
$$P = a(m+M)$$
$$a = \frac{P}{m+M}$$
$$T = Ma$$
$$= \frac{MP}{m+M}$$

Practice Problem

Q.12. Three blocks of masses m_1, m_2 and m_3 are placed on a horizontal frictionless surface. A force of 40 N pulls the system then calculate the value of T, if $m_1 = 10kg$, $m_2 = 6kg$, $m_3 = 4kg$.



1. 10 N
2. 20 N ✓
3. 30 N
4. 40 N

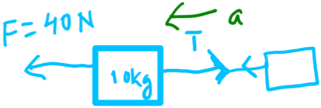
Sol^n

$$a = \frac{F_{net}}{m_{total}}$$

$$= \frac{40\,N}{20\,kg}$$

$$a = 2\,m/s^2$$

To find T,



$$F - T = m_1 a$$

$$T = F - m_1 a$$

$$= 40 - 10 \times 2 = 20\,N$$

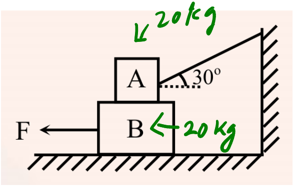
Practice Problem

Q.13. In the figure given below, if all surface are assumed to be smooth and the force F = 100N. If acceleration of block B of mass 20kg is 'a' and tension in string connecting block A of mass 20 kg is T then just after when the force F is applied.

1. $T = 0$ and $a = 5\ m/s^2$ ✓
2. $T = 100N$ and $a = 0\ m/s^2$
3. $T = 200N$ and $a = 5\ m/s^2$
4. *None of the above*

Given $F = 100\ N$

$$a = \frac{F}{m} = \frac{100}{20} = 5\ m/s^2$$



Practice Problem

Q.14. The force required to just prevent a body from sliding down the plane is one third of the force required to just move it up the plane. If the coefficient of friction is μ, then the angle of inclination θ is given by

1. $\theta = \tan^{-1}(3\mu)$
2. $\theta = \tan^{-1}(\mu)$
3. $\theta = \tan^{-1}(2\mu)$
4. $\theta = \tan^{-1}(4\mu)$

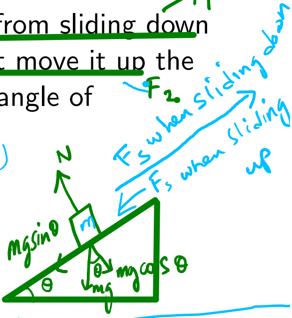
F_1

F_2

$$F_1 = \frac{1}{3} F_2 \quad (1)$$

$$F_1 = mg\sin\theta - \mu mg\cos\theta$$

$$F_2 = mg\sin\theta + \mu mg\cos\theta$$



$$F_s = \mu N = mg\mu\cos\theta$$

using (1) we have,

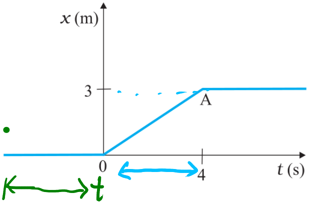
$$3mg\sin\theta - 3\mu mg\cos\theta = mg\sin\theta + \mu mg\cos\theta$$

$$\cancel{2}\,\cancel{mg}\sin\theta = \cancel{4}^{\,2}\mu\,\cancel{mg}\cos\theta$$

$$\frac{\sin\theta}{\cos\theta} = 2\mu \Rightarrow \tan\theta = 2\mu$$

Practice Problem

Q.15. Figure shows the position-time graph of a particle of mass 4 kg. What is the (a) force on the particle for $t < 0, t > 4s, 0 < t < 4s$? (b) impulse at t = 0 and t = 4 s ? (Consider one-dimensional motion only). **NCERT**



$$F = ma$$
$$= m\frac{dv}{dt}$$
$$= \frac{d(mv)}{dt} = \frac{dp}{dt}$$

$\boxed{\text{Impulse}, F \times dt} = dp$ (change in momentum)

$$= P_f - P_i$$

$$\therefore p_f - p_i = m(v - u)$$
$$= 4 \times (3/4) = 3 \text{ kgm/s}$$

$u = 0$

$v = \frac{3}{4}$

Practice Problem

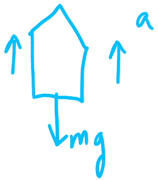
Q.16. A rocket with a lift-off mass 20,000 kg is blasted upwards with an initial acceleration of 5.0 ms^{-2}. The initial thrust (force) of the blast is **NCERT**

1. $1.5 \times 10^3 N$
2. $2.0 \times 10^5 N$
3. $3.0 \times 10^5 N$
4. $4.1 \times 10^4 N$

$m = 2 \times 10^4$ kg

$a = 5$ m/s^2

$g = 10$ m/s^2



$F - mg = ma$

$F = m(g + a)$

$= 3 \times 10^5$ N

Practice Problem

Q.17. If the force on a rocket moving with a velocity of 300 m/s is 210 N, then rate of combustion of fuel is

1. $0.7 kg/s$ ✓
2. $1.4 kg/s$
3. $7.0 kg/s$
4. $10.7 kg/s$

$$F = \frac{dp}{dt} = \frac{d(mv)}{dt}$$

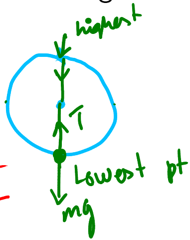
$$F = v\frac{dm}{dt}$$

$$\frac{dm}{dt} = \frac{F}{v} = \frac{21\cancel{0}}{30\cancel{0}} = \frac{7}{10} = 0.7$$

Practice Problem

Q.18. A stone tied to a string is rotated with a uniform speed in a vertical plane. If mass of the stone is m, the length of the string is r and the linear speed of the stone is v, when the stone is at its lowest point,then the tension on the string will be:

1. $\frac{mv^2}{r} + mg$ ✓
2. $\frac{mv^2}{r} - mg$
3. $\frac{mv^2}{r}$
4. mg



$$T - mg = \frac{mv^2}{r}$$

$$T = \frac{mv^2}{r} + mg$$

At highest point

$$T = \frac{mv^2}{r} - mg$$

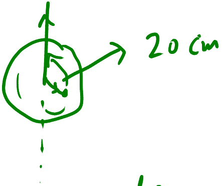
Practice Problem

Q.19. A disc revolves in a horizontal plane at a steady rate of 3 rad/s. A coin, when placed on the disc at 20 cm from the axis of rotation, just starts to slip. The coefficient of friction $(g = \pi^2 m/s^2)$ is

1. 0.50
2. 0.30
3. 0.20
4. 0.72 ✓

$$\frac{mv^2}{r} = \mu N = \mu m g$$

$(N = mg)$



$$\frac{v^2}{r} = \mu g$$

$$v = \omega r \qquad \frac{d\theta}{dt}$$

$$\frac{\omega^2 r}{g} = \mu$$

$$\frac{dL}{dt} = \frac{r\,d\theta}{dt}$$

$$\frac{4\pi^2 \nu^2 r}{\pi^2} = \mu \quad (\omega = 2\pi\nu)$$

$$v = r\omega$$

$$\mu = 0.72$$

Practice Problem

Q.20. A shell of mass 0.020 kg is fired by a gun of mass 100 kg. If the muzzle speed of the shell is 80 m/s, the recoil speed of the gun is (**NCERT**)

1. 0.51 m/s
2. 0.20 m/s
3. 0.016 m/s ✓
4. 0.02 m/s

Given , mass of the shell m =0.02 kg

velocity of the shell v =80 m/s

mass of the gun M=100kg

recoil velocity of the gun V=?

From the laws of momentum conservation,
Final momentum p_f = Initial momentum p_i

Since both the gun & shell initially are at rest, therefore $p_i = 0$

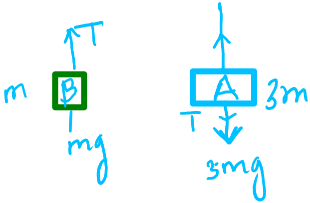
$\Rightarrow$ $p_f = 0$

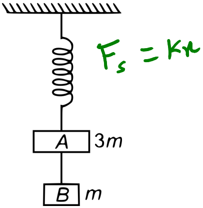
$\Rightarrow$ $mv - MV = 0$

$$V = \frac{mv}{M} = \frac{0.02 \times 80}{100} = 0.016$$

Practice Problem

Q.21. Two blocks A and B of masses 3m and m respectively are connected by a massless and inextensible string. The whole system is suspended by a massless spring as shown in figure. The magnitudes of acceleration of A and B immediately after the string is cut, are respectively (**NEET-2017**)





1. g, $g/3$
2. $g/3$, g
3. g, g
4. $g/3$, $g/3$

For body A, $T + 3mg = kx$ —(1)

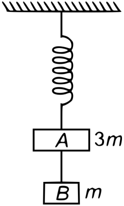
" B, $T - mg = 0$

$\Rightarrow T = mg$

Put $T = mg$ in (1) $\Rightarrow kx = 4mg$

Practice Problem

Q.21. Two blocks A and B of masses 3m and m respectively are connected by a massless and inextensible string. The whole system is suspended by a massless spring as shown in figure. The magnitudes of acceleration of A and B immediately after the string is cut, are respectively (**NEET-2017**)



When the string is cut, $T = 0$

The accelaration a of A is

$$a = \frac{kx - 3mg}{3m} = \frac{4mg - 3mg}{3m} = \frac{g}{3}$$

Accl^n of B $= g$

1. $g, g/3$
2. $g/3, g$
3. g, g
4. $g/3, g/3$