Crash Course on

GRAVITATION

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Newton's law of Gravitation

Newton's law of gravitation states that every body in this universe attracts every other body with a force, which is directly proportional to the product of their masses and inversely proportional to the square of the distance between their centres of mass. The direction of the force is along the line joining the particles.

Thus the magnitude of the gravitational force F that two particles of masses m_1 and m_2 separated by a distance r exert on each other is given by

$$F \propto \frac{m_1 \times m_2}{r^2}$$

$$\text{or} \qquad F = G\,\frac{m_1 \times m_2}{r^2}$$

where G is constant of proportionality which is called 'Universal gravitational constant'. Setting $m_1 = m_2 = 1$ and $r = 1$, we have $G = F$. Therefore, the universal gravitational constant is equal to the force of attraction between two bodies each of unit mass whose centres are placed unit distance apart. For vector form see NCERT textbook .

Note:

(i) The value of G in S.I unit is $6.67 \times 10^{-11}\ Nm^2kg^{-2}$ and in $c.g.s$ is $6.67 \times 10^{-8}\ dyne\,cm^2g^{-2}$.

(ii) Dimensional formula of G is $[M^{-1}L^3T^{-2}]$.

(iii) The value of G does not depend upon the nature and size of the bodies.

(iv) As G is very small hence gravitational forces are very small, unless one (or both) of the masses is huge.

Example 1: Mass M is divided into two parts xM and $(1–x)M$. For a given separation, what is the value of x for which the gravitational attraction between the two pieces becomes maximum. Answer:1/2

Example 2: The mass of the moon is about 1.2% of the mass of the earth. Compared to the

gravitational force the earth exerts on the moon, the gravitational force the moon exerts on earth.
(a) Is the same (b) Is smaller (c) Is greater (d) Varies with its phase

Example 3: Four particles of masses m, 2m, 3m and 4m are kept in sequence at the corners of a square of side a. What will be the magnitude of gravitational force acting on a particle of mass m placed at the centre of the square. Answer: $\frac{4\sqrt{2}Gm^2}{a^2}$

Acceleration Due to Gravity.

The force of attraction exerted by the earth on a body is called gravitational pull or gravity. We know that when force acts on a body, it produces acceleration. Therefore, a body under the effect of gravitational pull must accelerate.

The acceleration produced in the motion of a body under the effect of gravity is called acceleration due to gravity, it is denoted by g. Consider a body of mass m is lying on the surface of earth of mass M, then gravitational force on the body is given by

$$F = G\frac{M\,m}{R^2} \tag{1}$$

where, $R =$ is the radius of the earth. If g is the acceleration due to gravity, then the force on the body due to earth is given by

$$\begin{aligned} & F = Force \times accelaration \\ \text{or} \quad & F = mg \end{aligned} \tag{2}$$

Equating equations (1) and (2) we have

$$g = \frac{GM}{R^2} \tag{3}$$

Let ρ be the earth's density and V $(= \frac{4}{3}\pi R^3)$ as its volume. Then g can be expressed as

$$\begin{aligned} g &= \frac{G}{R^2} \times \frac{4}{3}\pi R^3 \rho \\ &= G \times \frac{4}{3}\pi R \rho \end{aligned} \tag{4}$$

Example 4: A spherical planet far out in space has a mass M_0 and diameter D_0 . A particle of mass m falling freely near the surface of this planet will experience an acceleration due to gravity which is equal to

(a) GM_0/D_0^2 (b) $4GM_0/D_0^2$ (c) $4mGM_0/D_0^2$ (d) GmM_0/D_0^2

Example 5: The moon's radius is 1/4 that of the earth and its mass is 1/80 times that of the earth. If g represents the acceleration due to gravity on the surface of the earth, that on the surface of the moon is

(a) $g/3$ (b) $g/4$ (c) $g/5$ (d) $g/6$

Example 6: If the radius of the earth were to shrink by 1% its mass remaining the same, the acceleration due to gravity on the earth's surface would

(a) Decrease by 2% (b) Remain unchanged (c) Increase by 2% (d) Increase by 1%

Example 7: A planet has mass 1/10 of that of earth, while radius is 1/3 that of earth. If a person can throw a stone on earth surface to a height of $90m$, then he will be able to throw the stone on that planet to a height

(a) $90\,m$ (b) $40\,m$ (c) $100\,m$ (d) $45\,m$

Variation in g Due to Shape of the Earth.

Earth is elliptical in shape. It is flattened at the poles and bulged out at the equator. The equatorial radius is about 21 km longer than polar radius. Let R_e and R_p be the radius at the equator and the pole. Then,

$$g_e = \frac{GM}{R_e^2} \tag{5}$$

$$\text{and} \quad g_p = \frac{GM}{R_p^2} \tag{6}$$

Since, $R_e > R_p$, therefore $g_p > g_e$ and $g_p = g_e + 0.018.m/s^2$. Thus the weight of body increases as it is taken from equator to the pole.

Example 8: Force of gravity is least at

(a) The equator (b) The poles (c) A point in between equator and any pole (d) None of these

Variation in g with Height.

Acceleration due to gravity at the surface of the earth

$$g = \frac{GM}{R^2} \tag{7}$$

The acceleration due to gravity at height h above the surface of the earth

$$g' = \frac{GM}{(R+h)^2} \tag{8}$$

From (7) and (8), we have

$$g' = g\frac{R^2}{(R+h)^2} \tag{9}$$

Note:

(i) When $h \sim \infty$, $g' = 0$ i.e., at infinite distance from the earth, the value of g becomes zero.

(ii) If $h << R$ i.e., height is small in comparison to the radius, then from equation (9) we get

$$g' = g\frac{1}{(1+h/R)^2} = g(1+h/R)^{-2} \tag{10}$$

Using Binomial theorem for negative or fractional index, for $x < 1$ given by $(1+x)^n = 1 + nx + \frac{n(n-1)}{1*2}x^2 + \cdot$, we get

$$g' = g\left(1 - \frac{2h}{R}\right) \tag{11}$$

(iii) If $h << R$ then decrease in the value of g with height :

$$\text{Absolute decrease } \Delta g = g - g' = \frac{2hg}{R} \tag{12}$$

$$\text{Fractional decrease } \frac{\Delta g}{g} = \frac{g-g'}{g} = \frac{2h}{R} \tag{13}$$

$$\text{Percentage decrease } \frac{\Delta g}{g} * 100\% = \frac{2h}{R} * 100\% \tag{14}$$

Example 9: The acceleration of a body due to the attraction of the earth (radius R) at a distance $2R$ from the surface of the earth is (g = acceleration due to gravity at the surface of the earth)

(a) $g/9$ (b) g (c) $g/3$ (d) $g/5$

Example 10: The height of the point vertically above the earth's surface, at which acceleration due to gravity becomes 1% of its value at the surface is (Radius of the earth = R)

(a) $8R$ (b) $9R$ (c) $10R$ (d) $16R$

Example 11: At surface of earth weight of a person is 72 N then his weight at height $R/2$ from surface of earth is (R = radius of earth)[**CBSE PMT 2000; AIIMS 2000**]

(a) 28 N (b) 16 N (c) 32 N (d) 20 N

Variation in g with depth.

Acceleration due to gravity at the surface of the earth

$$g = \frac{4}{3}\pi\rho G R \tag{15}$$

The acceleration due to gravity at depth d inside the surface of the earth

$$g' = \frac{4}{3}\pi\rho G(R-d) \tag{16}$$

From (15) and (16), we have

$$g' = g\left[1 - \frac{d}{R}\right] \tag{17}$$

Note:

(i) The value of g decreases as $g' \propto (R-d)$ on going below the surface of the earth. So it is clear that if d increase, the value of g decreases.

(ii) At the centre of earth $d = R$. Therefore $g' = 0$, i.e., the acceleration due to gravity at the centre of earth becomes zero.

(iii) Decrease in the value of g with depth :

$$\text{Absolute decrease } \Delta g = g - g' = \frac{dg}{R} \tag{18}$$

$$\text{Fractional decrease } \frac{\Delta g}{g} = \frac{g-g'}{g} = \frac{d}{R} \tag{19}$$

$$\text{Percentage decrease } \frac{\Delta g}{g} * 100\% = \frac{d}{R} * 100\% \tag{20}$$

(iv) The rate of decrease of gravity outside the earth (if $h << R$) is double to that of inside the earth.

Example 12: Weight of a body of mass m decreases by 1% when it is raised to height h above the earth's surface. If the body is taken to a depth h in a mine, change in its weight is **[KCET 2003; MP PMT 2003]**

(a) 2% decrease (b) 0.5% decrease (c) 1% increase (d) 0.5% increase

Example 13: The depth at which the effective value of acceleration due to gravity is $g/4$ is (R = radius of the earth)

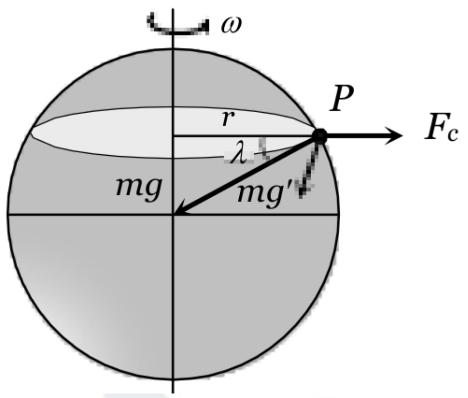
(a) $R/3$ (b) $R/4$ (c) $3R/4$ (d) $2R/3$

Variation in g due to Earth's Rotation.

As the earth rotates, a body placed on its surface moves along the circular path and hence experiences centrifugal force, due to it, the apparent weight of the body decreases.

Since the magnitude of centrifugal force varies with the latitude of the place, therefore the apparent weight of the body varies with latitude due to variation in the magnitude of centrifugal force on the body.

If the body of mass m is at point P, whose latitude is λ (the latitude at a point on the surface of the earth is defined as the angle, which the line joining that point to the centre of earth makes with equatorial plane), then the net force is



$$
\begin{aligned}
F_{net} &= mg' \qquad (21)\\
\implies g' &= \frac{F_{net}}{m}\\
\implies g' &= \frac{mg - m\omega^2 r\cos\lambda}{m}\\
\implies g' &= \frac{mg - m\omega^2 R\cos\lambda\cos\lambda}{m}\\
\implies g' &= g - R\omega^2\cos^2\lambda
\end{aligned}
$$

Note:

(i) At the equator, $\lambda = 0^0$, which implies that $g_{equator} = g - R\omega^2$ and this shows that the effect of rotation of earth on the value of g at the equator is maximum.

(ii) At the pole, $\lambda = 90^0$, which implies that $g_{pole} = g$ and it shows that the rotation of earth has no effect on the value of g at the poles.

(iii) When a body of mass m is moved from the equator to the poles, its weight increases by an amount

$$m(g_p - g_e) = m\omega^2 R \qquad (22)$$

(iv) Weightlessness due to rotation of earth : As we know that apparent weight of the body decreases due to rotation of earth. If ω is the angular velocity of rotation of earth for which a body at the equator will become weightless

$$
\begin{aligned}
g' &= g - R\omega^2\cos^2\lambda\\
0 &= g - R\omega^2\cos^2\lambda\\
\implies \omega &= \sqrt{\frac{g}{R\cos\lambda}}
\end{aligned}
$$

At the equator, $\lambda = 0^o$, we have $\omega = \sqrt{\frac{g}{R}}$. The time period of rotation of the earth is $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{R}{g}}$. If we substitute the value of $R = 6400 \times 10^3 m$ and take $g = 10\ m/s^2$, we get $\omega = 1/800$ and $T = 1.40\ hr$

This time is about 1/17 times the present time period of earth. Therefore if earth starts rotating 17 times faster then all objects on equator will become weightless.

If earth stops rotation about its own axis then at the equator the value of g increases by $\omega^2 R$ and consequently the weight of body lying there increases by $m\omega^2 R$.

Example 14: If the angular speed of earth is increased so much that the objects start flying from the equator, then the length of the day will be nearly

(a) 1.5 hours (b) 6.0 hours (c) 18 hours (d) 22 hours

Example 15: Calculate the speed of rotation of the Earth so that the apparent g at the equator becomes half of its value at the surface. Also calculate the length of the day in this situation.(Earth's radius = 6400 km)

Gravitational field and gravitational field intensity

The space surrounding a material body in which gravitational force of attraction can be experienced is called its gravitational field.

Gravitational field intensity : The intensity of the gravitational field of a material body at any point in its field is defined as the force experienced by a unit mass (test mass) placed at that point, provided the unit mass (test mass) itself does not produce any change in the field of the body. So if a test mass m at a point in a gravitational field experiences a force $\vec{F}$ then

$$\vec{I} = \frac{\vec{F}}{m}$$

Intensity of gravitational field with $F = GMm/r^2$, we have $I = GM/r^2$. At the Earth's surface when $r = R$, $I = g$.

Example 16: Two point masses of mass $10kg$ and $1000kg$ are at a distance $1m$ apart. At which points on the line joining them, will the gravitational potential intensity be zero?

The work done in bringing a unit mass from infinity to a given point in the gravitational field, is called the gravitational potential at that point.

Mathematically, it can be expressed as

$$V = \frac{W}{m}$$

The S.I unit of gravitational potential is $J\,kg^{-1}$. Its mass dimension is $[M^0\,L^2\,T^{-2}]$.

Potential due to point mass: Suppose we want to find the gravitational potential due to a point mass M. Consider bringing a test mass m from infinity to a point P at a distant r from the point mass M. Then the work done in doing so will be

$$W = \int_{\infty}^{r} F dr = \int_{\infty}^{r} \frac{GMm}{r^2} dr = GMm \int_{\infty}^{r} r^{-2} dr = GMm\left[-\frac{1}{r}\right]\Big|_{\infty}^{r} = -\frac{GMm}{r}$$

Therefore, the gravitational potential is

$$V = \frac{W}{m} = \frac{-\frac{GMm}{r}}{m} = -\frac{GM}{r}$$

Gravitational potential due to a system of more than one point masses: Consider a system with point masses $M_1, M_2, M_3, ..., M_{n-1}, M_n$ situated at distances $r_1, r_2, r_3, ..., r_{n-1}, r_n$. Then the potential is given by

$$V = -\left[\frac{GM_1}{r_1} + \frac{GM_2}{r_2} + \frac{GM_3}{r_3} + ... + \frac{GM_n}{r_n}\right]$$

Example 17: Infinite number of bodies, each of mass 2 kg are situated on $x-$axis at distance $1\,m,\ 2\,m,\ 4\,m,\ 8\,m, ...$ respectively from the origin. The resulting gravitational potential due to this system at the origin will be (NEET-2013)

(a) $-G$ (b) $-(8/3)\,G$ (c) $-(4/3)\,G$ (d) $-4G$

Gravitational potential energy

The gravitational potential energy of a body at a point is defined as the amount of work done in bringing the body from infinity to that point against the gravitational force.

$$W = \int_{\infty}^{r} F dr = \int_{\infty}^{r} \frac{GMm}{r^2} dr = GMm \int_{\infty}^{r} r^{-2} dr = GMm\left[-\frac{1}{r}\right]\Big|_{\infty}^{r} = -\frac{GMm}{r}$$

This work done is stored inside the body as its gravitational potential energy

$$\therefore \quad U = -\frac{GMm}{r}$$

In case of discrete distribution of masses Gravitational potential energy

$$U = \sum_i U_i = -\left[\frac{Gm_1m_2}{r_{12}} + \frac{Gm_2m_3}{r_{23}} + ...\right]$$

If the body of mass m is moved from a point at a distance r_1 to a point at distance r_2 ($r_1 >> r_2$) then change in potential energy $\Delta\,U$

$$\Delta U = \int_{r_1}^{r_2} \frac{GMm}{r^2} dr = GMm \int_{r_1}^{r_2} r^{-2} dr = -GMm\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$

Work done against gravity: If the body of mess m is moved from the surface of earth to a point at distance h above the surface of earth, then change in potential energy or work done against gravity will be

$$\begin{aligned} W = \Delta U &= GMm\left[\frac{1}{r_1} - \frac{1}{r_2}\right] \\ &= GMm\left[\frac{1}{R} - \frac{1}{R+h}\right] \\ &= \frac{GMmh}{R}\left[\frac{1}{(R+h)}\right] \\ &= \frac{GMmh}{R^2}\left[\frac{1}{(1+h/R)}\right] \\ \therefore \quad \Delta U &= \frac{mgh}{(1+h/R)} \qquad\qquad \because g = \frac{GM}{R^2} \end{aligned}$$

Example 18: A body of mass m is taken from earth surface to the height h equal to radius of earth, the increase in potential energy will be [**CPMT 1971, 97; IIT-JEE 1983; CBSE PMT 1991; Haryana CEE 1996; CEET Bihar 1995; MNR 1998; RPET 2000**]
(a) mgR (b) $(1/2)\, mgR$ (c) $2mgR$ (d) $(1/4)mgR$

Example 19: Three particles each of mass 100 gm are brought from a very large distance to the vertices of an equilateral triangle whose side is 20 cm in length. Calculate the work done of this system. Answer: -10^{11} J

Escape velocity

The minimum velocity with which a body must be projected up so as to enable it to just overcome the gravitational pull, is known as escape velocity.

The work done to displace a body from the surface of earth ($r = R$) to infinity ($r = \infty$) is

$$W = \int_R^{\infty} F dr = \int_R^{\infty} \frac{GMm}{r^2} dr = GMm\left[-\frac{1}{r}\right]\Bigg|_R^{\infty} = \frac{GMm}{R}$$

This work required to project the body so as to escape the gravitational pull is performed on the body by providing an equal amount of kinetic energy to it at the surface of the earth.
If v_e is the required escape velocity, then kinetic energy which should be given to the body is $= \frac{1}{2}mv_e^2$.

$$
\begin{aligned}
\therefore \quad \frac{1}{2}mv_e^2 &= \frac{GMm}{R} \\
\implies \quad v_e^2 &= \frac{2GM}{R} \\
\implies \quad v_e &= \sqrt{\frac{2GM}{R}} \\
\implies \quad v_e &= \sqrt{2gR} \quad \because\ g = \frac{GM}{R^2} \\
\implies \quad v_e &= R\sqrt{\frac{8}{3}\pi\rho G} \quad \because\ g = \frac{4}{3}\pi\rho GR
\end{aligned}
$$

For Earth, g=9.8 m/s and $R = 6400$ km, we get $v_e = 11.2$ km/s.

Maximum height attained by body: Let a projection velocity of body (mass m) is v, so that is attains a maximum height h . At maximum height, the velocity of particle is zero, so kinetic energy is zero. By the law of conservation of energy,

$$
\begin{aligned}
&\text{Total energy at surface} = \text{Total energy at height } h. \\
\implies \quad & -\frac{GMm}{R} + \frac{1}{2}mv^2 = -\frac{GMm}{R+h} + 0 \\
\implies \quad & h = \frac{R}{\left(\frac{2GM}{Rv^2} - 1\right)} \\
\implies \quad & h = R\left(\frac{v^2}{v_e^2 - v^2}\right) \quad \because\ v_e^2 = \frac{2GM}{R}
\end{aligned}
$$

Energy to be given to a stationary object on the surface of earth so that its total energy becomes zero, is called escape energy.

$$
\begin{aligned}
&\text{Total energy at surface} = 0 \\
\implies \ & \text{escape energy} = KE + PE = 0 - \frac{GMm}{R} = \frac{GMm}{R}
\end{aligned}
$$

Example 20: The escape velocity from the earth is about 11 km / s . The escape velocity from a planet having twice the radius and the same mean density as the earth, is [**MP PMT 1987; UPSEAT 1999; AIIMS 2001; MP PET 2001, 2003**]

(a) 22 km/s (b) 11 km/s (c) 5.6 km/s (d) 15.5 km/s

Hint: $v_e \propto R$

Example 21:A projectile is projected with velocity kv_e in vertically upward direction from the ground into the space. (v_e is escape velocity and $k < 1$). If air resistance is considered to be negligible then the maximum height from the centre of earth to which it can go, will be (R = radius of earth)**[Roorkee 1999; RPET-1]**

(a) $\frac{R}{k^2+1}$ (b) $\frac{R}{k^2-1}$ (c) $\frac{R}{1-k^2}$ (d) $\frac{R}{k+1}$

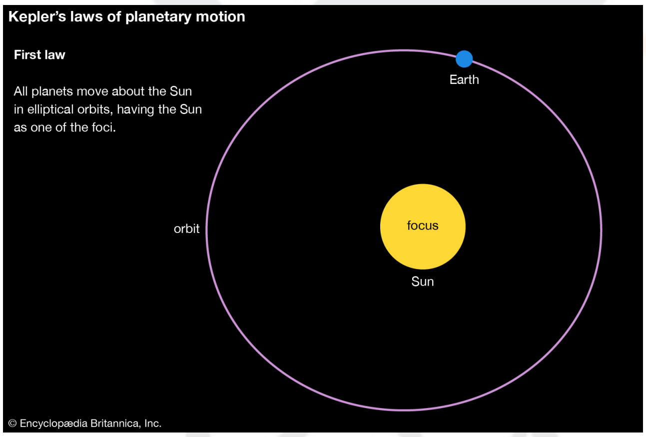
Example 22: If the radius of earth reduces by 4% and density remains same then escape velocity will **[MP PET 1991; MP PMT 1995]**;

(a) Reduce by 2% (b) Increase by 2% (c) Reduce by 4% (d) Increase by 4%

Hint: $v_e \propto R$

Kepler's Laws of Planetary Motion.

1. Kepler's first law states that all planets move in elliptical orbits with the sun situated at one of the foci of the ellipse.

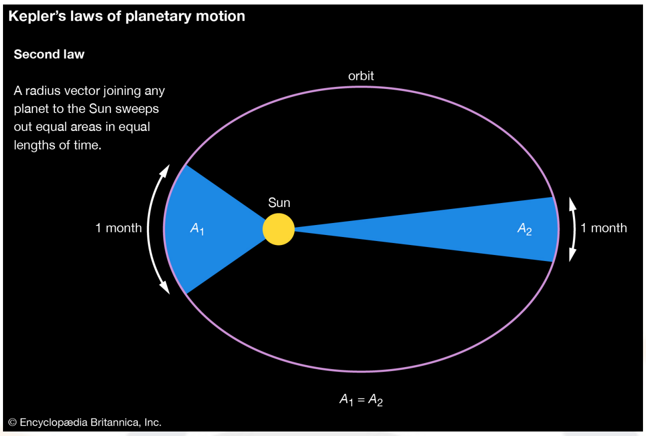


2. Kepler's second law states that a planet moves in its ellipse so that the line between it and the Sun placed at a focus sweeps out equal areas in equal times.

<u>The law of Area :</u> The line joining the sun to the planet sweeps out equal areas in equal interval of time. i.e. areal velocity is constant. According to this law planet will move slowly when it is farthest from sun and more rapidly when it is nearest to sun. It is similar to law of conservation of angular momentum.

$$\begin{aligned}\text{Areal velocity} &= \frac{dA}{dt} \\ &= \frac{\frac{1}{2}rvdt}{dt} = \frac{1}{2}rv = \frac{1}{2m}mvr = \frac{L}{2m}\end{aligned}$$

3. Kepler's third law



Kepler's laws of planetary motion

Third law

The squares of the sidereal periods (P) of the planets are directly proportional to the cubes of their mean distances (d) from the Sun.

$$P \times P = k\,(d \times d \times d)$$

$$P^2 = kd^3$$

$$\frac{P^2}{d^3} = k$$

where k is a constant

planet	period (P, year)	period squared	mean distance (d, AU)	mean distance cubed	P^2/d^3
Mercury	0.24	0.06	0.39	0.06	0.99
Venus	0.62	0.38	0.72	0.38	1.02
Earth	1.00	1.00	1.00	1.00	1.00
Mars	1.88	3.53	1.52	3.51	1.01
Jupiter	11.86	140.66	5.20	140.61	1.00
Saturn	29.46	867.89	9.58	879.22	0.99
Uranus	84.01	7057.68	19.20	7077.89	1.00
Neptune	164.80	27159.04	30.10	27270.90	1.00

Example 23: The distance of a planet from the sun is 5 times the distance between the earth and the sun. The Time period of the planet is

(a) $5^{\frac{3}{2}}$ years (b) $5^{\frac{2}{3}}$ years (c) $5^{\frac{1}{2}}$ years (d) $5^{\frac{1}{3}}$ years

Example 24: In planetary motion the areal velocity of position vector of a planet depends on angular velocity ω and the distance of the planet from sun (r). If so the correct relation for areal velocity is

(a) $\frac{dA}{dt} \propto \omega r$ (b) $\frac{dA}{dt} \propto \omega^2 r$ (c) $\frac{dA}{dt} \propto \omega r^2$ (d) $\frac{dA}{dt} \propto \sqrt{\omega r}$

Example 25: The distance of Neptune and Saturn from sun are nearly 10^{13} and 10^{12} meters respectively. Assuming that they move in circular orbits, their periodic times will be in the ratio

(a) 100 (b) $\sqrt{10}$ (c) $10\sqrt{10}$ (d) $1/\sqrt{10}$

Example 26: A satellite A of mass m is at a distance of r from the centre of the earth. Another satellite B of mass $2m$ is at distance of $2r$ from the earth's centre. Their time periods are in the ratio of

(a) $1:2$ (b) $1:\sqrt{2}$ (c) $1:2\sqrt{2}$ (d) $1:8$

Hint: $T \propto r^{3/2}$

Motion of Satellites

Orbital Velocity of Satellite: Satellites are natural or artificial bodies describing orbit around a planet under its gravitational attraction. Moon is a natural satellite while INSAT-1B is an artificial satellite of earth. Condition for establishment of artificial satellite is that the centre of orbit of satellite must coincide with centre of earth or satellite must move around great circle of earth. Orbital velocity of a satellite is the velocity required to put the satellite into its orbit around the earth. For revolution of satellite around the earth, the gravitational pull provides the required centripetal force.

$$\frac{mv^2}{R} = \frac{GMm}{r^2}$$
$$\implies v = \sqrt{\frac{GM}{r}}$$

At the height h above the earth's surface, we have

$$v = \sqrt{\frac{gR^2}{R+h}} \qquad (\because g = \frac{GM}{R^2})$$
$$\implies v = R\sqrt{\frac{g}{R+h}}$$

Relationship between th orbital speed v_o and Escape speed v_e: We learnt above that $v_e = \sqrt{2gR}$ and $v_o = \sqrt{\frac{gR^2}{R+h}}$. If h is small compare to R i.e., close to earth's surface, $v_o \approx \sqrt{gR}$. Therefore

$$\frac{v_e}{v_o} \approx \sqrt{\frac{2gR}{gR}}$$
$$\implies \frac{v_e}{v_o} \approx \sqrt{2}$$

Time period of revolution of sattelites: The time taken by a satellite to complete one revolution around the earth, is known as the time period of revolution of satellite.

The time period of revolution (T) is given by

$$\begin{aligned} T = \frac{2\pi}{\omega} &= \frac{2\pi r}{v_o} \\ &= \frac{2\pi r}{\sqrt{GM/r}} \\ &= 2\pi\sqrt{\frac{r^3}{GM}} \\ &= 2\pi\sqrt{\frac{r^3}{gR^2}} \qquad (\because g = \frac{GM}{R^2}) \\ T &= 2\pi\sqrt{\frac{(R+h)^3}{gR^2}} \end{aligned}$$

Also we can write

$$T = \frac{2\pi r}{\sqrt{GM/r}} = \frac{2\pi(R+h)}{v_o}$$

to get Kepler's third law $T^2 \propto r^3$.

Height of satellite: We can easily derive the height of the satellite from the time period of the satellite, which is given by

$$h = \left(\frac{gT^2R^2}{4\pi^2}\right)^{\frac{1}{3}} - R$$

Example 27: Two satellites A and B go round a planet P in circular orbits having radii $4R$ and R respectively. If the speed of the satellite A is 3V, the speed of the satellite B will be

(a) $12V$ (b) $6V$ (c) $8V$ (d) $2V$

Hint: Orbital velocity $v = \sqrt{GM/r} \propto \frac{1}{\sqrt{r}}$

Example 28: A satellite is moving around the earth with speed v in a circular orbit of radius r. If the orbit radius is decreased by 1%, its speed will

(a) increase by 1% (b) increases by 0.5% (c) decrease by 1% (d) decreases by 0.5%

Hint: $v \propto \frac{1}{\sqrt{r}}$

Example 29: A satellite is launched into a circular orbit of radius "R" around earth while a second satellite is launched into an orbit of radius $1.02R$. The percentage difference in the time periods of the two satellites is

(a) 0.7 (b) 1 (c) 2 (d) 3

Hint: $T \propto r^{3/2}$

Example 30: Time period of a satellite revolving above Earth's surface at a height equal to R, where R the radius of Earth, is

(a) $2\pi\sqrt{\frac{2R}{g}}$ (b) $2\pi\sqrt{\frac{R}{g}}$ (c) $4\pi\sqrt{\frac{2R}{g}}$ (d) $4\sqrt{2}\pi\sqrt{\frac{R}{g}}$

Hint: $T = 2\pi\sqrt{\frac{(R+h)^3}{GM}} = 2\pi\sqrt{\frac{(R+h)^3}{gR^2}}$

Example 31: An earth satellite S has an orbit radius which is 4 times that of a communication satellite C. The period of revolution of S is Earth, is

(a) 4 days (b) 8 days (c) 16 days (d) 12 days

Hint: Given $r_s = 4r_c$ and $T \propto r^{3/2}$

Geostationary Satellite: The satellite which appears stationary relative to earth is called geostationary or geosynchronous satellite, communication satellite. A geostationary satellite always stays over the same place above the earth such a satellite is never at rest. Such a satellite appears stationary due to its zero relative velocity w.r.t. that place on earth. The orbit of a geostationary satellite is known as the parking orbit.

Period of revolution around the earth should be same as that of earth about its own axis. Angular Momentum of Satellite: We know $L = mvr$ and $v = \sqrt{\frac{GM}{r}}$

$$\implies L = \sqrt{m^2GMr}$$

i.e., Angular momentum of satellite depend on both the mass of orbiting and central body as well as the radius of orbit.

Example 32: The orbital angular momentum of a satellite revolving at a distance r from the centre is L. If the distance is increased to $16r$, then new angular momentum will be

(a) $16L$ (b) $64L$ (c) $4L$ (d) $8L$

Hint: $L \propto \sqrt{r}$