TUGAS MATEMATIKA

(PDB) Orde satu dgn subtitusi $y = vx$

1. $(x^2 - y^2)\,dx + (xy - x^2)\,dy = 0$

$(x^2 - (v.x)^2)\,dx + (x.v.x - x^2)\,dy = 0$

$(x.v.x - x^2)\,dy = -(x^2 - (v.x)^2)\,dx$

$\frac{dy}{dx} = \frac{-(x^2 - (v.x^2))}{(x^2.v - x^2)}$

$\frac{dy}{dx} = \frac{-(x^2 - v^2.x^2)}{x^2.v - x^2)}$

$\frac{dy}{dx} = \frac{-x^2 + v^2.x^2}{x^2.v - x^2}$

$\frac{dy}{dx} = \frac{x^2(-1 + v^2)}{x^2.v - x^2}$

$\frac{dy}{dx} = \frac{-1 + v^2}{v - 1}$

$v + x\frac{dv}{dx} = \frac{-1 + v^2}{v - 1}$

$\frac{1 + v^2}{v - 1} = v + x\frac{dv}{dx}$

$\frac{-1 + v^2}{v - 1} - v = x\frac{dv}{dx}$

$\frac{(-1 + v^2) - (v^2 - v)}{v - 1} = x\frac{dv}{dx}$

$\frac{v - 1}{v - 1} = x\frac{dv}{dx}$

$\frac{1}{dv} = \frac{x}{dx}$

$\int dv = \int \frac{dx}{dx}$

$v = \ln|x| + c$

$\frac{y}{x} = \ln|x|.e^c$

$y = x\ln|x|.C$

$y = x\ln(x.C)$

2. $\frac{dy}{dx} + \frac{2}{x} y = \frac{\sin 3x}{x^2}$

$$\frac{2}{x} \downarrow P$$

$P(x) = \frac{2}{x}$

Faktor integrasi

$$P(x) = e^{\int P\,dx}$$
$$= e^{\int \frac{2}{x} dx}$$
$$= e^{2\ln(x)}$$
$$= e^{\ln(x^2)} \rightarrow e^{\ln(x)} = x$$
$$= x^2$$

$$x^2 \cdot \frac{dy}{dx} \cdot x^2 \cdot y = x^2 \cdot \frac{\sin 3x}{x^2}$$

$$\frac{d(x^2 \cdot y)}{dx} = x^2 \cdot \frac{\sin 3x}{x^2}$$

$$d(x^2 \cdot y) = x^2 \cdot \frac{\sin 3x}{x^2} \cdot dx$$

$$\int d\left(x^2 \cdot y\right) = \int x^2 \cdot \frac{\sin 3x}{x^2} \cdot dx$$

$$x^2 \cdot y = \int \sin 3x \cdot dx$$

$$x^2 \cdot y = \int \sin(u) \cdot \frac{du}{3}$$

$$x^2 \cdot y = \frac{1}{3} \int \sin(u) \cdot du$$

$$y = \frac{\frac{1}{3} \cos(3x) + c}{x^2}$$

$$y = \left(-\frac{1}{3} \cos 3x + c\right)^{x^{-2}}$$

3. $\quad {}^{2}+9)\dfrac{dy}{dx} + xy = 0$

$$\frac{(x^2+9)}{(x^2+9)}\frac{dy}{dx} + \frac{xy}{(x^2+9)} = \frac{0}{(x^2+9)}$$

$$\frac{dy}{dx} + \frac{x}{(x^2+9)}y = 0$$

$$P(x) = \frac{x}{(x^2+9)}$$

Faktor Integrasi

$$p(x) = \frac{x}{(x^2+9)}$$

$$P(x) = e^{\int P dx}$$

$$= e^{\int \frac{x}{(x^2+9)} dx} \longrightarrow du.2x$$

$u = x^2+9$

$$= e^{\frac{1}{2}\int \frac{2x\, dx}{(x^2+9)}}$$

$$= e^{\frac{1}{2}\ln(x^2+9)}$$

$$= e^{\ln(x^2+9)^{1/2}} \rightarrow e^{\ln(a)} = a$$

$$= \sqrt{x^2+9}$$

$$\sqrt{x^2+9} \cdot \frac{dy}{dx} - \sqrt{x^2+9}.y = \sqrt{x^2+9} \cdot 0$$

$$\frac{d(\sqrt{x^2+9}.y)}{dx} = 0$$

$$\int d(\sqrt{x^2+9}.y = \int 0 \, dx$$

$$\sqrt{x^2+9}.y = c$$

$$y = \frac{c}{\sqrt{x^2+9}} \; //$$