

LATIHAN

1. Tentukan solusi PD $y''' - 2y'' - y' + 2y = 0$
2. Tentukan solusi PD $y^{(4)} - 2y''' + 2y' - y = 0$

JAWABAN

$$\begin{aligned} 1. \quad & y''' - 2y'' - y' + 2y = 0 \\ & r^3 - 2r^2 - r + 2 = 0 \\ & r^2(r-2) - (r-2) = 0 \\ & (r-2)(r^2-1)(r-1) = 0 \\ & r_1 = 2 \quad r_2 = -1 \quad r_3 = 1 \end{aligned}$$

Cara 1

Sehingga solusi umumnya :

$$y = C_1 e^{r_1 t} + C_2 e^{r_2 t} + C_3 e^{r_3 t}$$

$$y = C_1 e^{2t} + C_2 e^{-t} + C_3 e^t //$$

Cara II

$$\begin{array}{c|cccc} & r^3 & -2r^2 & -r & +2 \\ 2 & 1 & -2 & -1 & 2 \\ & & 2 & 0 & -2 \\ \hline & 1 & 0 & -1 & 0 & + \end{array}$$

didapatkan $r_1 = 2$

$$\begin{array}{c|ccc} & 1 & 0 & -1 \\ & & 1 & 1 \\ \hline 1 & 1 & 0 & -1 & + \end{array}$$

didapatkan $r_2 = 1$

$$\begin{array}{c|cc} & 1 & 1 \\ & & -1 \\ \hline -1 & 1 & 0 & + \end{array}$$

didapatkan $r_3 = -1$

Sehingga solusi umumnya adalah

$$y = C_1 e^{r_1 t} + C_2 e^{r_2 t} + C_3$$

$$y = C_1 e^{2t} + C_2 e^t + C_3 e^{-t}$$

$$2. \quad y^{(4)} - 2y''' + 2y' - y = 0$$

$$r^4 - 2r^3 + 2r - 1 = 0$$

$$1 \left| \begin{array}{cccc|c} 1 & -2 & 0 & 2 & -1 \\ & & 1 & -1 & 1 \\ \hline & 1 & -1 & -1 & 1 \\ & & & 1 & 0 \end{array} \right| + \text{didapatkan } r_1 = 1$$

$$1 \left| \begin{array}{ccc|c} 1 & -1 & -1 & 1 \\ & & 1 & 0 \\ \hline & 1 & 0 & -1 \\ & & & 1 \end{array} \right| + \text{didapatkan } r_2 = 1$$

$$1 \left| \begin{array}{cc|c} 1 & 0 & -1 \\ & & 1 \\ \hline & 1 & 1 \\ & & 1 \end{array} \right| + \text{didapatkan } r_3 = 1$$

$$-1 \left| \begin{array}{cc|c} 1 & & 1 \\ & & -1 \\ \hline & 1 & 0 \end{array} \right| + \text{didapatkan } r_4 = -1$$

Sehingga solusi umumnya adalah

$$y = C_1 e^{r_1 t} + C_2 e^{r_1 t} + C_3 e^{r_2 t} + C_4 e^{r_4 t}$$

$$C_1 e^t + C_2 e^t + C_3 e^t + C_4 e^{-t}$$