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1) $A = \begin{pmatrix} 4 & 6 & 5 & 7 \\ 3 & 1 & 9 & 4 \end{pmatrix}$

$B = \begin{pmatrix} 4 & 6 & 5 & 7 \\ 3 & 1 & 9 & 4 \end{pmatrix}$

a. $A+B = \begin{pmatrix} 4 & 6 & 5 & 7 \\ 3 & 1 & 9 & 4 \end{pmatrix} + \begin{pmatrix} 4 & 6 & 5 & 7 \\ 3 & 1 & 9 & 4 \end{pmatrix}$

$= \begin{pmatrix} 8 & 12 & 10 & 14 \\ 6 & 2 & 18 & 8 \end{pmatrix}$

b. $A-B = \begin{pmatrix} 4 & 6 & 5 & 7 \\ 3 & 1 & 9 & 4 \end{pmatrix} + \begin{pmatrix} 4 & 6 & 5 & 7 \\ 3 & 1 & 9 & 4 \end{pmatrix}$

$= \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

c. $3A+2B = 3\begin{pmatrix} 4 & 6 & 5 & 7 \\ 3 & 1 & 9 & 4 \end{pmatrix} + 2\begin{pmatrix} 4 & 6 & 5 & 7 \\ 3 & 1 & 9 & 4 \end{pmatrix}$

$= \begin{pmatrix} 12 & 18 & 15 & 21 \\ 9 & 3 & 27 & 12 \end{pmatrix} + \begin{pmatrix} 8 & 12 & 10 & 14 \\ 6 & 2 & 18 & 8 \end{pmatrix}$

$= \begin{pmatrix} 20 & 30 & 25 & 35 \\ 15 & 5 & 45 & 20 \end{pmatrix}$

2.) $A = \begin{pmatrix} 4 & 3 \\ 2 & 7 \\ 6 & 1 \end{pmatrix}$ $B = \begin{pmatrix} 5 & 9 & 2 \\ 4 & 0 & 8 \end{pmatrix}$

a. $5A = 5\begin{pmatrix} 4 & 3 \\ 2 & 7 \\ 6 & 1 \end{pmatrix} = \begin{pmatrix} 20 & 15 \\ 10 & 35 \\ 30 & 5 \end{pmatrix}$

b. $A.B = \begin{pmatrix} 4 & 3 \\ 2 & 7 \\ 6 & 1 \end{pmatrix} . \begin{pmatrix} 5 & 9 & 2 \\ 4 & 0 & 8 \end{pmatrix}$

$= \begin{pmatrix} 4.5+3.4 & 4.9+3.0 & 4.2+3.8 \\ 2.5+7.4 & 2.9+7.0 & 2.2+7.8 \\ 6.5+1.4 & 6.9+1.0 & 6.2+1.8 \end{pmatrix}$

$= \begin{pmatrix} 32 & 36 & 32 \\ 38 & 18 & 60 \\ 34 & 54 & 20 \end{pmatrix}$

c. $B.A = \begin{pmatrix} 5 & 9 & 2 \\ 4 & 0 & 8 \end{pmatrix} . \begin{pmatrix} 4 & 3 \\ 2 & 7 \\ 6 & 1 \end{pmatrix}$

$= \begin{pmatrix} 5.4+9.2+2.6 & 5.3+9.7+2.1 \\ 4.4+0.2+8.6 & 4.3+0.7+8.1 \end{pmatrix}$

$= \begin{pmatrix} 20+18+12 & 15+63+2 \\ 16+0+48 & 12+0+8 \end{pmatrix} = \begin{pmatrix} 50 & 80 \\ 64 & 20 \end{pmatrix}$

3. $A = \begin{pmatrix} 2 & 6 \\ 5 & 7 \\ 4 & 1 \end{pmatrix}$ $B = \begin{pmatrix} 3 & 2 \\ 0 & 7 \\ 2 & 3 \end{pmatrix}$

a. Tidak bisa dihitung, karena tidak memenuhi syarat perkalian matriks

b. Tidak bisa dihitung, karena tidak memenuhi syarat perkalian matriks

4. a. Perkalian matriks tidak bersifat kumutatif ($AB \neq BA$)

Kali ini saya memberikan contoh dari nomor 2a dan 2b

$A = \begin{pmatrix} 4 & 3 \\ 2 & 7 \\ 6 & 1 \end{pmatrix}$ $B = \begin{pmatrix} 5 & 9 & 2 \\ 4 & 0 & 8 \end{pmatrix}$

AB BA

$\begin{pmatrix} 4 & 3 \\ 2 & 7 \\ 6 & 1 \end{pmatrix} \begin{pmatrix} 5 & 9 & 2 \\ 4 & 0 & 8 \end{pmatrix}$ $\begin{pmatrix} 5 & 9 & 2 \\ 4 & 0 & 8 \end{pmatrix} \begin{pmatrix} 4 & 3 \\ 2 & 7 \\ 6 & 1 \end{pmatrix}$

$\begin{pmatrix} 4.5+3.4 & 4.9+3.0 & 4.2+3.8 \\ 2.5+7.4 & 2.9+7.0 & 2.2+7.8 \\ 6.5+1.4 & 6.9+1.0 & 6.2+1.8 \end{pmatrix}$ 5.4+9.2+2.6 5.3

$\begin{pmatrix} 20+12 & 36+0 & 8+32 \\ 10+28 & 18+0 & 4+56 \\ 30+4 & 52+0 & 12+8 \end{pmatrix}$

$\begin{pmatrix} 32 & 36 & 32 \\ 38 & 18 & 60 \\ 34 & 54 & 20 \end{pmatrix}$

B.A

$\begin{pmatrix} 5 & 9 & 2 \\ 4 & 0 & 8 \end{pmatrix} \begin{pmatrix} 4 & 3 \\ 2 & 7 \\ 6 & 1 \end{pmatrix}$

$\begin{pmatrix} 5.4+9.2+2.6 & 5.3+9.7+2.1 \\ 4.4+0.2+8.6 & 4.3+0.7+8.1 \end{pmatrix}$

$\begin{pmatrix} 20+18+12 & 15+63+2 \\ 16+0+48 & 12+0+8 \end{pmatrix}$

$\begin{pmatrix} 50 & 80 \\ 64 & 20 \end{pmatrix}$

$\begin{pmatrix} 32 & 36 & 40 \\ 38 & 18 & 60 \\ 34 & 72 & 20 \end{pmatrix} \neq \begin{pmatrix} 50 & 80 \\ 64 & 20 \end{pmatrix}$ Terbukti//

b. $(AB)C = A(BC)$ (sifat asosiatif)

$A = \begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix}$ $B = \begin{pmatrix} -1 & 0 \\ 2 & 1 \end{pmatrix}$ $C = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$

$$AB(C) = \left\{ \begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix} \times \begin{pmatrix} -1 & 0 \\ 2 & 1 \end{pmatrix} \right\} \times \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1.-1+2.2 & 1.0+2.1 \\ 2.-1+4.2 & 2.0+4.1 \end{pmatrix} \times \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 3 & 2 \\ 6 & 4 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 3.1+2.1 & 3.0+2.1 \\ 6.1+4.1 & 6.0+4.1 \end{pmatrix}$$

$$\begin{pmatrix} 3+2 & 0+2 \\ 6+4 & 0+4 \end{pmatrix}$$

$$\begin{pmatrix} 5 & 2 \\ 10 & 4 \end{pmatrix}$$

$$A(BC) = \begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix} \times \left\{ \begin{pmatrix} -1 & 0 \\ 2 & 1 \end{pmatrix} \times \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} \right\}$$

$$\begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix} \times \begin{pmatrix} -1.1+0.1 & -1.0+0.1 \\ 2.1+1.1 & 2.0+1.1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix} \times \begin{pmatrix} -1 & 0 \\ 3 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1.-1+2.3 & 1.0+2.1 \\ 2.-1+4.3 & 2.0+4.1 \end{pmatrix}$$

$$\begin{pmatrix} -1+6 & 0+2 \\ -2+12 & 0+4 \end{pmatrix}$$

$$\begin{pmatrix} 5 & 2 \\ 10 & 4 \end{pmatrix}$$

= Terbukti//

c. $AI = IA = A$ (sifat matriks satuan, identitas perkalian)

$$AI = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1.1+2.0 & 1.0+2.1 \\ 3.1+4.0 & 3.0+4.1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$$

Terbukti//

$$IA = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$$

$$= \begin{pmatrix} 1.1+0.3 & 1.2+0.4 \\ 0.1+1.3 & 0.2+1.4 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$$

$AI = IA$

$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$$

d. $AO = OA$ (sifat matrik nol)

$$AO = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1.0+2.0 & 1.0+2.0 \\ 3.0+4.0 & 3.0+4.0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$OA = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$$

$$= \begin{pmatrix} 0.1+0.3 & 0.2+0.4 \\ 0.1+0.3 & 0.2+0.4 \end{pmatrix}$$

$$\begin{matrix} 0 & 0 \\ 0 & 0 \end{matrix}$$

$AO = OA$

$$\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

Terbukti//

e. $A^n = \begin{cases} AA \dots A, \text{ jika } n = 1, 2, \dots \\ I, \text{ jika } n = 0 \end{cases}$

$A^1 = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = A$

$A^2 = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = A . A$

$A^3 = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = A.A.A.$

Terbukti//

$A^0 = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$

$A^{n-n} = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}^{n-n}$

$= \dfrac{\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}^n}{\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}^n}$

$= \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \times \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}^{-1}$

$= \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \times \dfrac{1}{1.4 - 2.3}\begin{pmatrix} 4 & -2 \\ -3 & 1 \end{pmatrix}$

$= \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \times -\frac{1}{2}\begin{pmatrix} 4 & -2 \\ -3 & 1 \end{pmatrix}$

$= \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \times \begin{pmatrix} -2 & 1 \\ \frac{3}{2} & -\frac{1}{2} \end{pmatrix}$

$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$ (matrix identitas)

Terbukti//

f. $A^r A^s = A^{r+s}$, jika r dan s bilangan asli

$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}^1 \times \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}^2 = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}^3$

$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$

$\begin{pmatrix} 1.1+2.3 & 1.2+2.4 \\ 3.1+4.3 & 3.2+4.4 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} 1.1+2.3 & 1.2+2.4 \\ 3.1+4.3 & 3.2+4.4 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$

$\begin{pmatrix} 7 & 10 \\ 15 & 22 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} 7 & 10 \\ 15 & 22 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$

$\begin{pmatrix} 7.1+10.3 & 7.2+10.4 \\ 15.1+22.3 & 15.2+22.4 \end{pmatrix} = \begin{pmatrix} 7.1+10.3 & 7.2+10.4 \\ 15.1+22.3 & 15.2+22.4 \end{pmatrix}$

$\frac{\overset{12}{32}}{48}$ 16 32 12 32

g. $(kA)B = k(AB) = A(kB)$

$$\left(2\begin{pmatrix}1&2\\3&4\end{pmatrix}\right)\begin{pmatrix}2&2\\4&4\end{pmatrix} = 2\begin{pmatrix}1&2\\3&4\end{pmatrix}\begin{pmatrix}2&2\\4&4\end{pmatrix} = \begin{pmatrix}1&2\\3&4\end{pmatrix}2\begin{pmatrix}2&2\\4&4\end{pmatrix}$$

$$\begin{pmatrix}2&4\\6&8\end{pmatrix}\begin{pmatrix}2&2\\4&4\end{pmatrix} = 2\begin{pmatrix}1.2+2.4 & 1.2+2.4\\3.2+2.4 & 3.2+3.4\end{pmatrix} = \begin{pmatrix}1&2\\3&4\end{pmatrix}\begin{pmatrix}4&4\\8&8\end{pmatrix}$$

$$\begin{matrix}2.2+4.4 & 2.2+4.4\\6.2+8.4 & 6.2+8.4\end{matrix} = 2\begin{pmatrix}10&10\\14&18\end{pmatrix} = \begin{pmatrix}1.4+2.8 & 1.4+2.8\\3.4+4.8 & 3.4+4.8\end{pmatrix}$$

$$\begin{pmatrix}20&20\\48&48\end{pmatrix} = \begin{pmatrix}20&20\\48&48\end{pmatrix} = \begin{pmatrix}20&20\\48&48\end{pmatrix}$$

h. $(A+B)C = AC+BC$

$$\left(\begin{matrix}1&2\\3&4\end{matrix}+\begin{matrix}2&2\\4&4\end{matrix}\right)\begin{pmatrix}1&1\\2&2\end{pmatrix} = \left(\begin{pmatrix}1&2\\3&4\end{pmatrix}\begin{pmatrix}1&1\\2&2\end{pmatrix} + \begin{pmatrix}2&2\\4&4\end{pmatrix}\begin{pmatrix}1&1\\2&2\end{pmatrix}\right)$$

$$\begin{pmatrix}1&1\\2&2\end{pmatrix}\begin{pmatrix}3&4\\7&8\end{pmatrix}\begin{pmatrix}1&1\\2&2\end{pmatrix} = \begin{pmatrix}1.1+2.2 & 1.1+2.2\\3.1+4.2 & 3.1+4.2\end{pmatrix}\begin{pmatrix}2.1+2.2 & 2.1+2.2\\4.1+4.2 & 4.1+4.2\end{pmatrix}$$

$$\cancel{\begin{matrix}1&1\\2&2\end{matrix}}\begin{pmatrix}3.1+4.2 & 3.1+4.2\\7.1+8.2 & 7.1+8.2\end{pmatrix} = \begin{pmatrix}5&5\\11&11\end{pmatrix}\begin{pmatrix}6&6\\12&12\end{pmatrix}$$

$$\cancel{\begin{matrix}1&1\\2&2\end{matrix}}\begin{pmatrix}11&11\\23&23\end{pmatrix} = \begin{pmatrix}5&5\\11&11\end{pmatrix}\begin{pmatrix}6&6\\12&12\end{pmatrix}$$

$$\begin{pmatrix}11&11\\23&23\end{pmatrix} = \begin{pmatrix}5.6+5.12 & 5.6+5.12\\11.6+11.12 & 11.6+11.12\end{pmatrix}$$

$$\begin{pmatrix}11&11\\23&23\end{pmatrix} \neq \begin{pmatrix}90&90\\198&198\end{pmatrix}$$ Tidak terbukti

5 a. $(AB)^T = B^T A^T$ (urutan operasi dibalik)

$$\left[\begin{pmatrix} 2 & 3 \\ 1 & 5 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 1 & 3 \end{pmatrix}\right]^T = \begin{pmatrix} 1 & 2 \\ 1 & 3 \end{pmatrix}^T \begin{pmatrix} 2 & 3 \\ 1 & 5 \end{pmatrix}^T \rightarrow A^T = \begin{pmatrix} 2 & 1 \\ 3 & 5 \end{pmatrix} \quad B^T = \begin{pmatrix} 1 & 1 \\ 2 & 3 \end{pmatrix}$$

$$\begin{pmatrix} 2+3 & 4+9 \\ 1+5 & 2+15 \end{pmatrix} = \begin{pmatrix} 2+2 & 2+3 \\ 3+10 & 3+15 \end{pmatrix}$$

$$\begin{pmatrix} 5 & 6 \\ 13 & 17 \end{pmatrix} \neq \begin{pmatrix} 4 & 5 \\ 13 & 18 \end{pmatrix}$$ Tidak terbukti

b. $(kA)^T = kA^T$

$$\left[2\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}\right]^T = 2\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}^T$$

$$\begin{pmatrix} 2 & 4 \\ 6 & 8 \end{pmatrix}^T = 2\begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix}$$

$$\begin{pmatrix} 2 & 4 \\ 6 & 8 \end{pmatrix} = \begin{pmatrix} 2 & 6 \\ 4 & 8 \end{pmatrix}$$ Terbukti

c. $(A+B)^T = A^T + B^T$ (sifat transpos matriks terhadap penjumlahan)

$$\left[\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} + \begin{pmatrix} 2 & 2 \\ 4 & 4 \end{pmatrix}\right]^T = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}^T + \begin{pmatrix} 2 & 2 \\ 4 & 4 \end{pmatrix}^T$$

$$\begin{pmatrix} 3 & 4 \\ 7 & 8 \end{pmatrix}^T = \begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix} + \begin{pmatrix} 2 & 4 \\ 2 & 4 \end{pmatrix}$$

$$\begin{pmatrix} 3 & 7 \\ 4 & 8 \end{pmatrix} = \begin{pmatrix} 3 & 7 \\ 4 & 8 \end{pmatrix}$$ Terbukti

d. Trase $(A+B)$ = trase (A) + trase (B)

Trase $\left[\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} + \begin{pmatrix} 2 & 2 \\ 4 & 4 \end{pmatrix}\right]$ = trase $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ + trase $\begin{pmatrix} 2 & 2 \\ 4 & 4 \end{pmatrix}$

Trase $\begin{pmatrix} 3 & 4 \\ 7 & 8 \end{pmatrix} = 1+4 + 2+4$

$3+8 = 5+6$

$11 = 11$ Terbukti