- 感受 / Issues
  - 前面字 > 圖
  - 概念清楚
    - $\mathcal{L}$-Lan , $\mathcal{L}$-Struct
    - P.98 free var 同真值
    - P.104 – P.105 難
    - P.127 Discharge Assumption
  - 要細讀*
  - 短而精的書

Remarks

- p.3 { • indepence • consistence
- p.4 { • Formal logic • Decidability • 2^{nd} incomplete
- Deductive validity 的定義
- ND Jaśkowski & Gentzen
- 直覺論證明 Tennant (1990)
- 確定描述詞
- Inductively define $\mathcal{L}_n$ formula

$|\exists x Rxy| = ?$

$|\exists x Rxy|_{\mathcal{E}}$ 有 Struct , R := ... is smaller than ...

α satisfy $\exists x Rxy$ in $\mathcal{E}$

$\alpha(y) =$ Paris

When $|\exists x Rxy|^{\alpha}_{\mathcal{E}} = T$? At least one x smaller than y.

$|\exists x Rxy|^{\alpha}_{\mathcal{E}}$ = There is β $|Rxy|^{\beta}_{\mathcal{E}}$

β is different from α in $\mathcal{E}$ at most in x

i.e. Assigned variable unchanged

點解 id 有特別待遇

$$\frac{a=b \quad Pa}{Pb} =_E \sim \frac{Rab \quad Pa}{Pb} \quad \text{錯}$$

Definitions

5.7

A $\mathcal{L}_n$ formula is logical true iff ϕ is true in all $\mathcal{L}_n$ Structure $\longrightarrow$ Tautology

A $\mathcal{L}_n$ formula is contradiction iff ϕ is false in all $\mathcal{L}_n$ Structure

ϕ and ψ is logically equivalent iff ϕ and ψ are true in exact the same $\mathcal{L}_n$ Structure

2.11 Semantic consistency : There is $\mathcal{L}$-struct s.t. $|\Gamma|_A = T$

2.12 $\Gamma \vDash \phi$ iff $\Gamma \cup \{\neg\phi\}$ is inconsistent

6.12 $\phi[t/v]$ replace all free v by t

$|\phi|_A^\alpha = T \Leftrightarrow \alpha$ satisfies ϕ in $A \Longleftrightarrow A \vDash \phi[a]$ (by some author)

$\mathcal{L}_2$ - Structure $= \langle \Omega, I \rangle$

I : $I(c) \in \Omega$

$I(\text{Sentence}) = T$ or F

$I(\text{Pred\_Letter})$ is n-ary Relation

Variable Assignment : $\alpha : V \to \Omega$

No free variable occurs

Satisfaction and Truth (P.100 - P.105)

5.3 Truth . A sentence ϕ is true in a $\mathcal{L}_2$ structure A iff $|\phi|_A^\alpha = T$ for all var_assi α over A

5.2 (i) $|\Phi t_1, \ldots t_n| = T$ iff $\langle |t_1|_A^\alpha, \ldots, |t_n|_A^\alpha \rangle \in |\Phi|_A^\alpha$

5.2 (ii) $|\neg\phi|_A^\alpha = T$ iff $|\phi|_A^\alpha = F$

5.2 (vii) $|\forall v \phi|_A^\alpha = T$ iff $|\phi|_A^\beta = T$ for all var_assi β over A different from α at most in v

5.2 (viii) $|\exists v \phi|_A^\alpha = T$ iff $|\phi|_A^\beta = T$ At least 1 var_assi β over A different from α at most in v

ND Rules (only important) (changed)

$$\frac{\begin{array}{c}[\phi]^1\\ \vdots \\ \psi\end{array}}{\phi\to\psi}\ \to I, 1 \qquad \frac{\begin{array}{cc}[\phi]^1 & [\psi]^2\\ \vdots & \vdots \\ \psi & \phi\end{array}}{\phi\leftrightarrow\psi}\ \leftrightarrow I \qquad \frac{\begin{array}{ccc} & [\phi]^1 & [\psi]^2\\ \vdots & \vdots & \vdots \\ \phi\vee\psi & \chi & \chi\end{array}}{\chi}\ \vee E, 1, 2 \qquad \frac{\begin{array}{cc}[\phi]^1 & [\phi]^1\\ \vdots & \vdots \\ \psi & \neg\psi\end{array}}{\neg\phi}\ \neg I, 1$$

$$\frac{\begin{array}{c}\vdots\\ \forall v\,\phi\end{array}}{\phi[t/v]}\ \forall E \qquad \frac{\begin{array}{c}\vdots\\ \phi[t/v]\end{array}}{\exists v\,\phi}\ \exists I \qquad \frac{[t=t]^1}{\vdots}\ = \text{Intro}, 1 \qquad \frac{\begin{array}{cc}\vdots & \\ \phi[t/v] & t=s\end{array}}{\phi[s/v]}\ =E$$

$$\frac{\begin{array}{c}\vdots\\ \phi[t/v]\end{array}}{\forall v\,\phi}\ \forall I$$

Given that constant t doesn't occur in ϕ OR any undischarged Asm in the proof $\phi[t/v]$

$$\frac{\begin{array}{cc} & [\phi[t/v]]^1\\ \vdots & \vdots \\ \exists v\,\phi & \psi\end{array}}{\psi}\ \exists I, 1$$

Given that constant t doesn't occur in $\exists v\phi$, or in ψ, or in any undischarged Asm in the proof $\phi[t/v]$

ND Examples

$\neg(P\to Q) \vdash \neg Q$

$$\dfrac{\dfrac{[Q]^1}{P\to Q}\,{\to}I \qquad \neg(P\to Q)}{\neg Q}\,\neg I,1 \qquad \sim \qquad \dfrac{\begin{matrix}[\phi]^1\\ \vdots \\ \psi\end{matrix} \qquad \begin{matrix}\boxed{\cancel{\phi}}\\ \vdots \\ \neg\psi\end{matrix}}{\neg\phi}\,\neg I,1$$

$\vdash P\vee\neg P$

$$\dfrac{\dfrac{\dfrac{\dfrac{[P]^1}{P\vee\neg P}\,\vee I \qquad [\neg(P\vee\neg P)]^2}{\neg P}\,\neg I,1}{P\vee\neg P}\,\vee I \qquad [\neg(P\vee\neg P)]^2}{P\vee\neg P}\,\neg E,2$$

$\vdash \neg(P\vee Q)\to\neg P$

$$\dfrac{\dfrac{\dfrac{[P]^1}{P\vee Q}\,\vee I \qquad [\neg(P\vee Q)]^2}{\neg P}\,\neg I,1}{\neg(P\vee Q)\to\neg P}\,{\to}I,2$$

$\vdash \forall x(Px\to Qx)\ ,\ \exists x Px \ \vdash \exists x Qx$

$$\dfrac{\exists x Px \qquad \dfrac{\dfrac{[Pa]^1 \qquad \dfrac{\forall x(Px\to Qx)}{Pa\to Qa}\,\forall E}{Qa}\,{\to}E}{\exists x Qx}\,\exists I}{\exists x Qx}\,\exists E,1$$

$P \to Q \vdash \neg P \vee Q$

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{[P]^2 \quad P \to Q}{Q}\to E}{\neg P \vee Q} \qquad [\neg(\neg P \vee Q)]^1}{\neg P}\neg I,2}{\neg P \vee Q} \quad [\neg(\neg P \vee Q)]^1}{\neg P \vee Q}\neg E,1$$

1.	$P \to Q$	PI
2.	$\neg(\neg P \vee Q)$	Asm
3.	P	Asm
4.	Q	2,3 $\to$E
5.	$\neg P \vee Q$	4 $\vee$I
6.	$\lightning$	2,5
7.	$\neg P$	3 $\neg$I
8.	$\neg P \vee Q$	7 $\vee$I
9.	$\lightning$	2,8
10.	$\neg P \vee Q$	2 $\neg$E

$$\dfrac{\dfrac{\exists y \forall x\, Rxy \qquad \dfrac{\dfrac{[\forall x\, Rxb]^1}{Rab}\forall E}{\exists y\, Ray}\exists I}{\exists y\, Ray}\exists E,1}{\forall x \exists y\, Rxy}\forall I$$

$$\dfrac{\exists v \phi \qquad \begin{matrix}[\phi[t/v]]^1 \\ \vdots \\ \psi\end{matrix}}{\psi}\exists I,1$$

Given that constant t doesn't occur in $\exists v \phi$, or in ψ, or in any undischarged Asm in the proof $\phi[t/v]$

$$\dfrac{\begin{matrix}\vdots \\ \phi[t/v]\end{matrix}}{\forall v \phi}\forall I$$

Given that constant t doesn't occur in ϕ OR any undischarged Asm in the proof $\phi[t/v]$

$\mathcal{L}_2$ Structure examples

$\Omega_\mathcal{E}$ = All Euro cities $|Q|_\mathcal{E} = \{$ Florence, Stockholm, Barcelona $\}$ $|R|_\mathcal{E}$ = ... is smaller than

$|a|_\mathcal{E}$ = Florence $|b|_\mathcal{E}$ = London

Prove that $|\forall x(Qx \to Rxb)|_\mathcal{E} = T$

let α be arbitrary va

Case 1. $|x|^\alpha_\mathcal{E} \in |R|_\mathcal{E}$ Case 2. skip

$\langle |x|^\alpha_\mathcal{E}, |b|_\mathcal{E} \rangle \in |R|_\mathcal{E}$

$|Rxb|^\alpha_\mathcal{E} = T$

$|Qx \to Rxb|^\alpha_\mathcal{E} = T$

$\therefore\ |\forall x(Qx \to Rxb)|_\mathcal{E} = T$ ($\because$ 5.2 vii)

$\nvDash Qb \to \forall x Qx$

$\Omega_\mathcal{B} = \{1,2\}$ $|b|_\mathcal{E} = 1$ $|Q|_\mathcal{B} = \{1\}$

$|Qb \to \forall x Qx|_\mathcal{B} = F_5$

$|Qb|_\mathcal{B} = T_1$ $|\forall x Qx|_\mathcal{B} = F_4$

$|Qx|^{\beta: x\mapsto 1}_\mathcal{B} = T_2$ $|Qx|^{\alpha: x\mapsto 2}_\mathcal{B} = F_3$

$\forall x(Px \to Qx \vee Rx),\ Pa \nvDash Ra$

$D: \Omega_D = \{1\}$ $|P|_D = |Q_D| = \{1\}$ $|R|_D = \varnothing$

for all α :

$1 \in \{1\}$ $1 \in \{1\}$ $1 \in \varnothing$

$|x|^\alpha_D \in |Q|_D$ $|a|^\alpha_D \in |P|_D$ $|a|_D \notin |R|_D$

$|Qx|^\alpha_D = T$ $|Pa|_D = T$ $|Ra|_D = F$

$\vdots$

$|Px \to Qx \vee Rx|^\alpha_D = T$