Definitions

Semantic value = T or F

v satisfied A iff $v \vDash_1 A$

↳ valuation / assignment

- $c \vDash_1 A$ iff A is true in c
- $c \vDash_0 A$ iff A is false in c

Case c is complete iff for all A: $c \vDash_1 A$ xor $c \vDash_0 A$

Case c is consistent iff No A : $c \vDash_1 A$ and $c \vDash_0 A$

Basic Tableau method
(semantical approach)

$c \vDash_1 A : \langle A \ \oplus \rangle$ $\quad$ $c \nvDash_1 A : \langle A \ \ominus \rangle$

Closing conditions :

1. $\langle A \ \oplus \rangle \ \langle A \ \ominus \rangle$
2. $\langle A \ \oplus \rangle \ \langle \neg A \ \oplus \rangle$ both t & f
3. $\langle A \ \ominus \rangle \ \langle \neg A \ \ominus \rangle$ neither

Classical Sentential logic

$\boxed{X \vdash A}$ origin

Classical Sentential Consequence : $X \vDash A$ iff no v s.t. ($v \vDash_1 X$ and $v \nvDash_1 A$)

Rules & Laws (Duality)

Negations (Classical):

- $v \vDash_1 \neg A$ iff $v \vDash_0 A$
- $v \vDash_0 \neg A$ iff $v \vDash_1 A$

Double negation :

- $v \vDash_1 \neg\neg A \rightarrow v \vDash_1 A$
- $v \vDash_0 \neg\neg A \rightarrow v \vDash_0 A$

Conjunctions

$v \vDash_1 A \wedge B$ iff $v \vDash_1 A$ and $v \vDash_1 B$

$v \vDash_0 A \wedge B$ iff $v \vDash_0 A$ or $v \vDash_0 B$

$v \vDash_1 \neg(A \rightarrow B)$ iff $v \vDash_1 A$ and $v \vDash_1 \neg B$

$v \vDash_0 \neg(A \rightarrow B)$ iff $v \vDash_0 A$ or $v \vDash_0 \neg B$

Disjunctions

$v \vDash_1 A \vee B$ iff $v \vDash_1 A$ or $v \vDash_1 B$

$v \vDash_0 A \vee B$ iff $v \vDash_0 A$ and $v \vDash_0 B$

$v \vDash_1 \neg(A \vee B)$ iff $v \vDash_1 \neg A$ and $v \vDash_1 \neg B$

$v \vDash_0 \neg(A \vee B)$ iff $v \vDash_0 \neg A$ or $v \vDash_0 \neg B$

Conditions

$v \vDash_1 A \rightarrow B$ iff $v \vDash_1 \neg A$ or $v \vDash_1 B$

$v \vDash_0 A \rightarrow B$ iff $v \vDash_0 \neg A$ and $v \vDash_0 B$

$v \vDash_1 \neg(A \rightarrow B)$ iff $v \vDash_1 A$ and $v \vDash_1 \neg B$

$v \vDash_0 \neg(A \rightarrow B)$ iff $v \vDash_0 A$ or $v \vDash_0 \neg B$

Classical First order logic

Case / Model in Classical First order logic :

$$c = \langle \underbrace{D}_{\text{Domain}} , \underbrace{\delta}_{\text{Denote function}} \rangle$$

Truthness :

$$\text{Predicate } \Pi \begin{cases} E_{\Pi}^{+} & \text{extension of } \Pi \\ E_{\Pi}^{-} & \text{Anti-extension of } \Pi \end{cases}$$

$c \vDash_1 \Pi \alpha_1 \ldots \alpha_n$ iff $\langle \delta(\alpha_1), \ldots, \delta(\alpha_n) \rangle \in E_{\Pi}^{+}$ under c

$c \vDash_0 \Pi \alpha_1 \ldots \alpha_n$ iff $\langle \delta(\alpha_1), \ldots \delta(\alpha_n) \rangle \in E_{\Pi}^{-}$ under c

Classicality

1. $E_{\Pi}^{+} \cup E_{\Pi}^{-} = D^n$ 2. $E_{\Pi}^{+} \cap E_{\Pi}^{-} = \varnothing$

Identity: $[a] = \{x \mid x = a\}$

Domain D contains $[a]$

e.g. $D = \{[a], [c]\}$

$[a] = \{a, b\}$ $[c] = \{c\}$

$\delta(b) = [a]$

Universal

$c \vDash_1 \forall v A$ iff $c \vDash_1 A[a\backslash v]$ for all a s.t. $\delta(a) \in D$

$c \vDash_0 \forall v A$ iff $c \vDash_0 A[a\backslash v]$ for some a s.t. $\delta(a) \in D$

Existential

$c \vDash_1 \exists v A$ iff $c \vDash_1 A[a\backslash v]$ for all a s.t. $\delta(a) \in D$

$c \vDash_0 \exists v A$ iff $c \vDash_0 A[a\backslash v]$ for some a s.t. $\delta(a) \in D$

QN

$$\begin{array}{c} \langle \neg \forall x A(x) \quad \circ \rangle \\ | \\ \langle \exists x \neg A(x) \quad \circ \rangle \end{array} \qquad \begin{array}{c} \langle \neg \exists x A(x) \quad \circ \rangle \\ | \\ \langle \forall x \neg A(x) \quad \circ \rangle \end{array}$$

Identity

$$\begin{array}{c} \bullet \\ | \\ \langle t=t \;\; \oplus \rangle \end{array} \qquad \begin{array}{c} \bullet \\ | \\ \langle \neg\, t=t \;\; \theta \rangle \end{array} \qquad \begin{array}{c} \langle t=u \;\; \oplus \rangle \\ \langle A(t) \;\; \oplus \rangle \\ | \\ \langle A(u) \;\; \oplus \rangle \end{array} \qquad \begin{array}{c} \langle \neg\, t=u \;\; \theta \rangle \\ \langle A(t) \;\; \theta \rangle \\ | \\ \langle A(u) \;\; \theta \rangle \end{array}$$

$$\begin{array}{c} \langle t=u \;\; \oplus \rangle \\ | \\ \langle \neg\, t=u \;\; \theta \rangle \end{array} \qquad \begin{array}{c} \langle \neg\, t=u \;\; \theta \rangle \\ | \\ \langle t=u \;\; \oplus \rangle \end{array}$$

Non-classical logic

Consistent / Exclusion	Complete / Exhaustion	Logic Theory
✓	✓	Classical
✓	X	Strong Kleene (K3)
X	✓	Logic of paradox (LP)
X	X	First-degree Entailment (FDE)

Classical connection between untruth and falsity

C1: $c \vDash_0 A \rightarrow c \nvDash_1 A$ LP reject

C2: $c \nvDash_1 A \rightarrow c \vDash_0 A$ K3 reject

Para___ theory:

- logical theory is paracomplete if there is c : $c \nvDash_1 A$ and $c \nvDash_0 A$ ($c \nvDash_1 \neg A$) for some A

- logical theory is paraconsistent if there is c : $c \vDash_1 A$ and $c \vDash_0 A$ ($c \vDash_1 A$) for some A

CP1: $X \vDash_{K3} A \Rightarrow X \vDash_{CL} A$ CP2: $X \vDash_{LP} A \Rightarrow X \vDash_{CL} A$

Proof: Any CL case is para___ or classical case

Semantic value:

$$c \vDash_1 A \ \& \ c \vDash_0 A = b$$
$$c \vDash_1 A \ \& \ c \nvDash_0 A = t$$
$$c \nvDash_1 A \ \& \ c \vDash_0 A = f$$
$$c \nvDash_1 A \ \& \ c \nvDash_0 A = n$$

LP accept (b, t, f)

K3 accept (t, f, n)

Counterexample: $A \vee B, \neg B \nvDash_{LP} A$, Disproof:

$$\langle A \vee B \quad \oplus \rangle$$
$$\langle \neg B \quad \oplus \rangle$$
$$\langle A \quad \ominus \rangle$$
$$\begin{array}{cc} \langle A \quad \oplus \rangle & \langle B \quad \oplus \rangle \\ \times_{FDE} & \checkmark_{LP} \end{array}$$

Theorem: Every CL truth is LP truth (?)

Proof by contraposition: A LP case $c \nVdash_1 S$

Define classical case c' as follows

$c'(S) = t$ iff $c(S) = t$

Asm $c' \vDash_1 S$, S is conjunction, disjunction or negation

for any atomic sentence U

$c' \vDash_1 U \Rightarrow c \vDash_1 U \Rightarrow c \vDash_1 S \Rightarrow$ ↯

$\therefore c' \nVdash_1 S$

LEM issues (P.218)

$T(A) :=$ A is true

1. $P \vee \neg P$
2. $T(P) \vee \neg T(P)$ .... Always true
3. $T(P) \vee T(\neg P)$ .... If $v(P) = n$, LEM3 can't be true

Free Logic

$Ia \not\models \exists v(v=a) \iff a$ is imagine / ∴ a exists

counterexample : $D=\{1\}$ $E=\varnothing$ $\delta(a)\in I^+$

$c=\langle D, E, \delta\rangle$ where $D\neq\varnothing$, $E\subseteq D$. E contains obj that exist in given case

Changes :

Universal

$c\models_1 \forall v A$ iff $c\models_1 A[a\backslash v]$ for all a s.t. $\delta(a)\in E$

$c\models_0 \forall v A$ iff $c\models_0 A[a\backslash v]$ for some a s.t. $\delta(a)\in E$

Existential

$c\models_1 \exists v A$ iff $c\models_1 A[a\backslash v]$ for all a s.t. $\delta(a)\in E$

$c\models_0 \exists v A$ iff $c\models_0 A[a\backslash v]$ for some a s.t. $\delta(a)\in E$

Alethic modal logic

(Put all things together)

$\text{Frame} = \langle \mathcal{U}, D, E, \delta \rangle$

$\mathcal{U}$ = universe of world

D = domain

E = a function assign E_w to world w

world-relative domain

δ = $\underbrace{\text{rigid designators}}_{\text{Denote same obj at All world}}$

$\text{Case} = [\mathcal{F}, w]$

frame world

Another approach

$\text{Frame} = \langle \mathcal{U}, R, D, E, \delta \rangle$, R is relation on $\mathcal{U}$

$[\mathcal{F}, w] \models_1 \Box A$ iff every w-accessiable w' s.t. $[\mathcal{F}, w'] \models_1 A$

Rules & Laws

Basic (As example)

$[\mathcal{F}, w] \models_1 \Pi \alpha_1 \ldots \alpha_n$ iff $\langle \delta(\alpha_1), \ldots, \delta(\alpha_n) \rangle \in \Pi_w^+$

$[\mathcal{F}, w] \models_1 \neg A$ iff $[\mathcal{F}, w] \models_0 A$

$[\mathcal{F}, w] \models_0 \neg A$ iff $[\mathcal{F}, w] \models_1 A$

Modal connectives

$$[\mathcal{F}, w] \models_1 \Diamond A \quad \text{iff} \quad \exists w' \in \mathcal{U} \quad \text{s.t.} \quad [\mathcal{F}, w'] \models_1 A$$

$$[\mathcal{F}, w] \models_0 \Diamond A \quad \text{iff} \quad \forall w' \in \mathcal{U} \quad \text{s.t.} \quad [\mathcal{F}, w'] \models_0 A$$

$$[\mathcal{F}, w] \models_1 \Box A \quad \text{iff} \quad \forall w' \in \mathcal{U} \quad \text{s.t.} \quad [\mathcal{F}, w'] \models_1 A$$

$$[\mathcal{F}, w] \models_0 \Box A \quad \text{iff} \quad \exists w' \in \mathcal{U} \quad \text{s.t.} \quad [\mathcal{F}, w'] \models_0 A$$

Modal Tableau rules (Part of)

$$\begin{array}{c} \langle \neg \Box A \quad w \quad \oplus \rangle \\ | \\ \langle \neg A \quad i \quad \oplus \rangle \end{array} \qquad\qquad \begin{array}{c} \langle \neg \Box A \quad w \quad \ominus \rangle \\ | \\ \langle \neg A \quad v \quad \ominus \rangle \end{array}$$

$$\begin{array}{c} \langle \neg \Diamond A \quad w \quad \oplus \rangle \\ | \\ \langle \neg A \quad v \quad \oplus \rangle \end{array} \qquad\qquad \begin{array}{c} \langle \neg \Diamond A \quad w \quad \ominus \rangle \\ | \\ \langle \neg A \quad i \quad \ominus \rangle \end{array}$$

$\forall / \exists$ Tableau rules

$$\begin{array}{ccc} & \langle \forall x A(x) \quad \oplus \rangle & \\ \swarrow & & \searrow \\ \langle \mathbb{E}t \quad \ominus \rangle & & \langle A(t) \quad \oplus \rangle \end{array} \qquad\qquad \begin{array}{c} \langle \forall x A(x) \quad \ominus \rangle \\ | \\ \langle A(a) \quad \ominus \rangle \\ \langle \mathbb{E}(a) \quad \oplus \rangle \end{array}$$

$$\begin{array}{c} \langle \exists x A(x) \quad \oplus \rangle \\ | \\ \langle \mathbb{E}a \quad \oplus \rangle \\ \langle A(a) \quad \oplus \rangle \end{array} \qquad\qquad \begin{array}{ccc} & \langle \exists x A(x) \quad \ominus \rangle & \\ \swarrow & & \searrow \\ \langle \mathbb{E}t \quad \ominus \rangle & & \langle A(t) \quad \ominus \rangle \end{array}$$

心得

- 2nd ed. 多嘢
- 冇 deductive
- 細本，易帶
- 先有基本功
- Gate of non-C logic
- 有 non-classical