Exercise sheet for Algebraic Topology II
Week 8

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Exercise 1. Compute $\mathbb{Z}/k \otimes \mathbb{Z}/m$ and $\mathrm{Tor}(\mathbb{Z}/k, \mathbb{Z}/m)$ for all natural numbers k, m.

Exercise 2 (Homework)**.** In Theorem 11.6 we proved that $\pi_n X \cong H_n X$ for any $(n-1)$-connected space and $n \geq 2$. On the other hand, we claimed in the previous statement Theorem 8.7 that a *specific* homomorphism (called the *Hurewicz map* h_X) between these groups is an isomorphism. We will rectify the situation by the technique of *universal example*.

1. Show without recourse to a Hurewicz theorem that $h_{S^n}\colon \pi_n S^n \to H_n S^n$ is a surjection.
2. Convince yourself that the proof of Theorem 11.6 provides a *natural* isomorphism $g_X\colon \pi_n X \cong H_n X$ on the category of $(n-1)$-connected pointed topological spaces. (You are allowed to use without proof that the Serre spectral sequence is natural in a suitable sense.)
3. Show that h_{S^n} and g_{S^n} agree up to sign and deduce the analogous statement for h_X and g_X for every $(n-1)$-connected space X. Deduce the statement of Theorem 8.7 from Theorem 11.6.

Exercise 3 (Homework)**.** Let $n \geq 2$ and X be the space obtain from S^n by attaching an $(n+1)$-cell along a degree-k map $S^n \to S^n$ for a nonzero integer k. Compute $\pi_* X \otimes \mathbb{Q}$.