Algebraic Geometry II: Exercises for Lecture 7

March 21, 2019

1) As mentioned, when $\{F_\alpha\}$ is a collection of $\mathcal{O}_X$-modules, then $\oplus F_\alpha$ is the sheaf associated to the presheaf $U \mapsto \oplus\Gamma(U, F_\alpha)$.

Let $U = \operatorname{Spec} A$ be an affine scheme and let M_α be A-modules. Show that $\oplus\widetilde{M_\alpha} \cong \widetilde{\oplus M_\alpha}$.

2) Let X and Y be noetherian schemes and let $f\colon X \to Y$ be an affine morphism. Show that f is finite if and only if $f_*(\mathcal{O}_X)$ is coherent.

3) Can you find a scheme X and an $f \in \Gamma(X, \mathcal{O}_X)$ such that $\Gamma(X, \mathcal{O}_X)_f$ is not isomorphic to $\Gamma(X_f, \mathcal{O}_{X_f})$? Can the natural map (which is an isomorphism when X is a finite union of open affines U_i such that $U_i \cap U_j$ is quasicompact) fail to be injective, resp. surjective?