Exercise sheet for Algebraic Topology II
Week 16

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Exercise 1. Let A be a finitely generated[1] abelian group such that $\mathrm{Hom}(A, \mathbb{Z}) = 0$ and $\mathrm{Ext}(A, \mathbb{Z}) = 0$. Deduce that $A = 0$.

Exercise 2. Let $u \in H^n(K(\mathbb{Z}, n); \mathbb{Z}_{(p)})$ be the fundamental class and denote by $\Lambda(u)$ the polynomial ring (over $\mathbb{Z}_{(p)}$) on u if n is even and the exterior algebra (over $\mathbb{Z}_{(p)}$) if n is odd. Then $\Lambda(u) \to H^*(K(\mathbb{Z}, n); \mathbb{Z}_{(p)})$ is an isomorphism for $* < 2p + n - 1$ and $H^{2p+n-1}(K(\mathbb{Z}, n); \mathbb{Z}_{(p)}) \cong \mathbb{Z}/p$ if $n \geq 3$.

[1] The statement will be also true without finiteness hypothesis, but harder. The case $A = \mathbb{Q}$ might be instructive.