Exercise problems, lecture 1

Note. These are just for practice and need not be handed in!

Exercise 1. Using the cellular cochain complex, compute the cohomology groups of spheres $H^*(S^n; A)$ for an arbitrary abelian group A.

Exercise 2. Similarly, compute the cohomology groups of complex projective spaces $H^*(\mathbb{C}P^n; A)$.

Exercise 3 Compute the singular cohomology groups of real projective spaces $H^*(\mathbb{R}P^n; A)$ in the following cases:

(a) $A = \mathbb{Z}/2$,
(b) $A = \mathbb{Z}$
(c) $A = \mathbb{Z}/p$ for an odd prime p.

Using (b), demonstrate that it is *not* the case that $H^*(\mathbb{R}P^n; \mathbb{Z})$ is isomorphic to $\mathrm{Hom}(H_*(\mathbb{R}P^n), \mathbb{Z})$ for all values of n and $*$. What about the cases (a) and (b)?