**Homework exercises Algebraic topology, hand in before class on 19-9-2018**

Exercise 1.
Compute $H_0(X; \mathbb{Z}/2\mathbb{Z})$ where $X$ is the finite set $\{a, b\}$ whose topology consists of the following set subsets $\{\emptyset, \{a\}, X\}$.

Exercise 2.

1. For any $p, q \in \mathbb{Z}$ give an element $s_{p,q} \in C_2(\mathbb{R}^2; \mathbb{Z})$ such that $s_{p,q}$ is non-zero on only two singular 2-simplices $\alpha, \beta$ and $im(\alpha) \cup im(\beta) = [p, p+1] \times [q, q+1]$ and $\partial_2(s_{p,q}) \in \mathbb{Z}[S(\mathbb{R}^2)_1]$ is the function that sends all singular 1-simplices in $\mathbb{R}^2$ to 0 except the simplices $a, b, c, d : \Delta^1 \to \mathbb{R}^2$, defined below. $\partial_2(s_{p,q})$ should send $a, b, c$ to 1 and $d$ to $-1$. Define

$$\begin{aligned} a(te_0 + (1-t)e_1) &= pe_0 + qe_1 + te_0 \\ b(te_0 + (1-t)e_1) &= pe_0 + qe_1 + e_0 + te_1 \\ c(te_0 + (1-t)e_1) &= pe_0 + qe_1 + e_0 + e_1 - te_0 \\ d(te_0 + (1-t)e_1) &= pe_0 + qe_1 + e_1 + te_1 \end{aligned}$$

2. Identifying elements of $\mathbb{Z}[S(\mathbb{R}^2)_n]$ with formal $\mathbb{Z}$-linear combination of $n$-simplices, how would you write $s_{0,0}$ and its boundary $\partial_2 s_{0,0}$?

3. Finally express $\partial_2 \sum_{p,q=0}^{7} s_{p,q}$ as a function that takes only four non-zero values.
*Hint. Draw a picture.*