Exercise sheet for Algebraic Topology II
Week6

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Exercise 1. Go through the proof of Theorem 1.11 to show that for two reduced cohomology theories $\widetilde{h}$ and $\widetilde{k}$ a natural transformation φ between them (i.e. a collection of natural transformations $\varphi_n\colon \widetilde{h}^n \to \widetilde{k}^n$ that are compatible with the suspension isomorphisms) is a natural isomorphism if and only if $\varphi_n(S^0)$ is an isomorphism for all n.

Exercise 2 (Homework)**.** Show that for a pointed map $f\colon A \to X$, the inclusion $i\colon X \hookrightarrow Cf$ is a based cofibration in the following sense: Let $h\colon X \times I \to Y$ be a *pointed* homotopy (i.e. $h(\{x_0\} \times I) = \{y_0\}$ if $x_0 \in X$ and $y_0 \in Y$ are the basepoints) and $f\colon Cf \times \{0\} \to Y$ be another pointed map agreeing with h on the overlap. Then there exists a pointed map $H\colon Cf \times I \to Y$ extending h and f.

Exercise 3. Show that if X is compact and Y a metric space, then the compact-open topology on $\mathrm{Map}(X,Y)$ coincides with that induces by the metric $d(f,g) = \sup_{x\in X} d(f(x), g(x))$.

Exercise 4. Let X and Y be locally compact and Z be arbitrary. Then there is a homeomorphism between $\mathrm{Map}(X, \mathrm{Map}(Y,Z))$ and $\mathrm{Map}(X \times Y, Z)$.

Exercise 5 (Homework)**.**

(a) Let $f\colon X \to Y$ be a continuous map and Z be a space. Show that the induced map $f^*\colon \mathrm{Map}(Y,Z) \to \mathrm{Map}(X,Z)$ is continuous. Repeat this with the pointed mapping spaces for pointed spaces and maps.

(b) Conclude that the multiplication map $\Omega Z \times \Omega Z \to \Omega Z$ is continuous. Similarly show that the "inverse of loop" map $\Omega Z \to \Omega Z$ is continuous as well.

(c) If X is an H-space, the usual product on $\pi_n(X)$ (for $n \geq 1$) agrees with that induced by X and this is abelian, even for $n = 1$.