Exercise problem

Exercise 1. Here is an alternative way to compute $\pi_4 S^3$. Start with the fibration sequence

$$S^3\langle 3\rangle \to S^3 \to K(\mathbb{Z}, 3).$$

You computed part of the homology of $K(\mathbb{Z}, 3)$ last week. The resulting groups $H_n(K(\mathbb{Z}, 3))$ should look as follows:

n=0	1	2	3	4	5	6	7
$\mathbb{Z}$	0	0	$\mathbb{Z}$	0	$\mathbb{Z}/2$	0	$\mathbb{Z}/3$

Now apply the homological Serre spectral sequence to the fibration sequence above to prove

$$H_4(S^3\langle 3\rangle) \cong H_5(K(\mathbb{Z}, 3) \cong \mathbb{Z}/2.$$

Homework problem, to be handed in May 9

Exercise 2. Use the Serre spectral sequence and the fact that $K(\mathbb{Z}/2, 1) \cong \mathbb{R}P^\infty$ to compute the cohomology ring $H^*(K(\mathbb{Z}/2, 2); \mathbb{Z}/2)$ up to degree 6. Note that the coefficients for cohomology are $\mathbb{Z}/2$. Your answer should list not only the groups but include the cup product structure! Below is a description of the answer to guide your calculation, where n is the degree and the bottom row lists generators of copies of $\mathbb{Z}/2$.

n=0	1	2	3	4	5	6
1	0	x	y	x^2	xy, z	x^3, y^2