Exam Algebraic Number Theory 2014
Tuesday 27 January 2014, 10:00 – 13:00

During this exam the use of handwritten or printed lecture notes, graded homework sets and a **non-graphing non-programmable** calculator is allowed. The use of any other electronic equipment and books is not allowed.

Exercise 1.

How many ideals of norm 100 does the ring of integers of $\mathbb{Q}(\sqrt{-7})$ have?

Exercise 2.

Determine all pairs $(X, Y) \in \mathbb{Z}^2$ such that $X^2 + 11 = Y^3$.

Exercise 3.

Let $K = \mathbb{Q}(\alpha)$ be the field obtained by adjoining a root α of the irreducible polynomial $X^3 - 3X - 27$ to $\mathbb{Q}$. Give an explicit finite set of generators of its maximal order $\mathcal{O}_K$ as a ring.

Exercise 4.

Let $L = \mathbb{Q}(\sqrt{33})$ and let $\mathcal{O}_L$ be its maximal order.

(a) Calculate the class group of L.

(b) Calculate the unit group of $\mathcal{O}_L$, i.e., calculate its rank, give a generator of the torsion subgroup and give a set of fundamental units.