Commutative algebra 2018-2019
Assignment 2 (marked)

- Deadline is **1am on November 17th.** Do not use computer programs, etc.
- Please list your *full name*, *student id number*, *university*, and *e-mail address* at the top of your work.
- This assignment must be handed as a pdf through the ELO website of mastermath, as a pdf produced by a latex file based on the template latex file provided.
- There is a page limit of two pages using the provided latex template. This is to help you to give the correct amount of detail in your solutions (our model solution is 1 page).

(1) Consider the polynomial $f = (x^2 - 7)^2(x^2 + 7) \in \mathbb{Z}[x]$ and the ring $A = \mathbb{Z}[x]/(f)$. We write φ for the natural mapping $\varphi \colon \mathbb{Z} \to \mathbb{Z}[x]/(f)$.

    (a) Give a primary decomposition of the ideal $(f) \subset \mathbb{Z}[x]$. Is the primary decomposition of the ideal (f) unique?

    (b) Describe each irreducible component of $\mathrm{Spec}(A)$ by giving generators of the corresponding prime ideal in A.

    (c) Show that for $\mathfrak{p} \in \mathrm{SpecMax}(A)$ not above (2) or (7) the ring $A_{\mathfrak{p},\mathrm{red}} = A_{\mathfrak{p}}/\mathrm{nil}(A_{\mathfrak{p}})$ is integrally closed.

    (Hint: Write $A_{\mathfrak{p}} = S^{-1}(\mathbb{Z}[x]/(f))$ for a suitable multiplicatively closed subset $S \subset \mathbb{Z}[x]$, and then use properties of localization that we have studied, like Exercise 3.3, exactness and behavior with respect to primary decompositions).