Algebraic Geometry II: Exercises for Lecture 5

March 7, 2019

1) Show that the closed subschemes of $\mathbb{P}^n_k$ correspond bijectively with the homogeneous ideals $A \subseteq k[X_0, \dots, X_n]$ with the property that $f \in A$ if $X_i \cdot f \in A$ for all i (for $f \in k[X_0, \dots, X_n]$).

2) Let X be a scheme. Denote by $X \times X$ the fibre product over $\operatorname{Spec} \mathbb{Z}$. Let $Z = \{y \in X \times X \mid p_1(y) \equiv p_2(y)\}$. Show that Z equals $\Delta(X)$, where $\Delta\colon X \to X \times X$ is the diagonal. Conclude that $\Delta(X)$ is closed if and only if X is separated.

3) Let X and K be schemes and let f and g be morphisms from K to X (i.e., K-valued points of X). Assume that K is reduced. Show that $f = g$ if and only if $f(x) \equiv g(x)$ for all $x \in K$.

4) Let T and U be open affine subsets of a scheme Y. Show that $T \cap U$ is the union of open sets that are distinguished both in T and in U.