Algebraic Geometry II: Exercises for Lecture 6

March 14, 2019

1) Show that a fibre product of separated schemes is separated.

2) Let X, Y, and Z be separated schemes. Assume that $f\colon X \to Y$ is surjective, that $g\colon Y \to Z$ is of finite type, and that $g \circ f$ is proper. Show that g is proper.

3) Assume that X, Y, S, X_1, Y_1, and S_1 are separated schemes. We are given morphisms $X \to S$, $Y \to S$, $X_1 \to S_1$, $Y_1 \to S_1$, $X_1 \to X$, $Y_1 \to Y$, and $S_1 \to S$ that form a commutative diagram. Assume also that $X_1 \to X$ and $Y_1 \to Y$ are closed immersions. Show that the induced morphism from $X_1 \times_{S_1} Y_1$ to $X \times_S Y$ is a closed immersion. (Hint: use that S_1 is separated.)