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Q2 a)

```
def QUOTREM(x,y)
    if x<y:
        return (0,x)
    else:
        (q,r) = QUOTREM(x-y,y)
        return (q+1,r)
```

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Q6 a)

$\boxed{\text{Count Palindrome}}$

```
def isPalindrome(s,l,r):
    if l>r:
        return True
    if s[l] != s[r]:
        return False
    else:
        return isPalindrome(s,l+1,r-1)

def CountPalindrome(s,0,n-1):
    count = 0
    for i in range(n):
        for j in range(i+1,n):
            if isPalindrome(i,j)== True:
                count = count + 1

    return count
```

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Q4 Claim: Let $f(n), g(n), h(n)$ be functions from $\mathbb{Z}_{\geq 0} \to \mathbb{R}$

If $f(n) = O(g(n))$ and $g(n) = O(h(n))$ then $f(n) = O(h(n))$

Proof: Given $f(n) = O(g(n))$

c_1, c_2 are positive integers

$\Rightarrow \exists\ c_1 > 0,\ n_0 \in \mathbb{N}$ such that $f(n) \leq c_1 g(n)$ for all $n \geq n_0$

Also, $g(n) = O(h(n))$

$\Rightarrow \exists\ c_2 > 0,\ n_1 \in \mathbb{N}$ such that $g(n) \leq c_2 h(n)$ for all $n \geq n_1$

$\Rightarrow f(n) \leq c_1 g(n)$ for all $n \geq \max\{n_0, n_1\}$

$g(n) \leq c_2 h(n)$ for all $n \geq \max\{n_0, n_1\}$

$\Rightarrow f(n) \leq c_1 g(n)$

$\Rightarrow f(n) \leq c_1 c_2 h(n)$

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take $c = c_1 c_2$, $K = \max\{n_0, n_1\}$

hence $\exists\ c, K \in \mathbb{N}$ such that $f(n) \leq c\,h(n)\ \forall\ n \geq K$

$\Rightarrow f(n) = O(h(n))$

Hence proved

Q5 a) $\log_2 3n = O(\log_2 2n)$

if it is true then, $\exists\ c, n_0 \in \mathbb{N}$ s.t. $\log_2 3n \leq c \log_2 2n$ $\forall\ n \geq n_0$

$\Rightarrow \log_2 3n \leq \log_2 (2n)^c$

since $3n > 1, 2n > 1\ \forall\ n \in \mathbb{N}$
also the bases $2 > 0$ ~~also~~ $\Rightarrow \log_2 3n, \log_2 (2n)^c > 0$
we raise the expression by 2

$\Rightarrow 2^{\log_2 3n} \leq 2^{\log_2 (2n)^c}$

$\Rightarrow 3n \leq (2n)^c$

for $c > 1 \Rightarrow 3n \leq 2^c n^c$

$\Rightarrow 2^c n^c - 3n \geq 0$

$\Rightarrow n(2^c n^{c-1} - 3) \geq 0$

$n > 0$ as $n \in \mathbb{N}$

$\Rightarrow 2^c n^{c-1} - 3 \geq 0$

$\Rightarrow n^{c-1} \geq \frac{3}{2^c}$

$\Rightarrow n \geq \left(\frac{3}{2^c}\right)^{\frac{1}{c-1}}$

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so for any $c > 1$, $n_0 = \left\lceil \left(\frac{3}{2^c}\right)^{\frac{1}{c-1}} \right\rceil$ the inequality holds

hence proved a) is True

5 b) $2^{3n} = O(2^{2n})$

if this is true then $\exists\ c, n_0 \in \mathbb{N}$ s.t. $2^{3n} \leq c2^{2n}\ \forall\ n \geq n_0$

since $2^{3n} > 0,\ c2^{2n} > 0$ when $c, n_0 \in \mathbb{N}$

we can put log both sides

$\Rightarrow \log_2 2^{3n} \leq \log_2 c2^{2n}$

$\Rightarrow 3n \leq \log_2 c + \log_2 2^{2n}$

$\Rightarrow 3n \leq \log_2 c + 2n$

take $K = \log_2 c$ for any $c \in \mathbb{N}$

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$3n \leq K + 2n$

for $n > K$ the inequality doesn't hold

say $n = K+1$

$3K+3 > K + 2K + 2 = 3K+2$

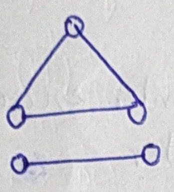
hence for any $c \in \mathbb{N}\ \exists\ n_0 = \log_2 c + 1$ such that

inequality $2^{3n} \leq c2^{2n}$ doesn't hold

Hence, the statement b) is false

Q1 a) Take the graph G

G:



G doesn't have multiple edges
nor self loops
no isolated vertex
hence G is special

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$|V| = 5$

$|E| = 4$

$\Rightarrow |V| = |E| + 1 \geq 2$

b) The proof only considers those special graphs G which can be induced from the base case $|V|=2$, $|E|=1$

as given in the counter example
the graph cannot be induced by the base case
but is a special graph

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Hence the proof is wrong.

6 b)

for the function isPalindrome()

the recursion ends when $l>r$, the left pointer exceeds the right pointer

since in every recursion step we are increasing l by 1 and decreasing r by 1

eventually the condition $l>r$ will hit and algorithm will come to an end with True

also if at any point the extreme values are not equal the algorithm terminates with False

You haven't explained why isPalindrome() always return the correct answer.

for the base case $l=0$, $r=0$

algorithm returns True as $l>r$ after the first iteration

For the main function countPalindrome()

i goes from 0 to $n-1$

j goes from i to $n-1$

$i \leq j$ in every loop

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since isPalindrome gives correct answer

we increase count by 1 if it returns True

eventually when $i=j=n-1$

after this iteration loop exits and return the count.

0 c) Analysis of Count Palindrome

1	$T(n)$
2	1
3	n
4	$n * \sum_{i=0}^{n-1} n-i$
5	
6	1
7	1

Incomplete

Analy

is Palindrome (s, l, r)

1	$T(n)$
2	1
3	1
4	1
5	1
6	1
7	$T(n-2)$

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$$T(n) = T(n-2) + 5$$
$$= T(n-4) + 2 \cdot 5$$
$$= T(n-6) + 3 \cdot 5$$

③ a) A = [1, 1, 1, 1, 1] ✓

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