Proving the Quadratic Formula

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1 Introduction

The quadratic formula states that:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \tag{1}$$

when given a quadratic equation such as:

$$ax^2 + bx + c = 0 \tag{2}$$

2 Proof

$$ax^2 + bx + c = 0 \tag{1}$$

$$ax^2 + bx = -c \tag{2}$$

$$x^2 + \frac{b}{a}x = -\frac{c}{a} \tag{3}$$

$$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = -\frac{c}{a} + \frac{b^2}{4a^2} \tag{4}$$

$$(x + \frac{b}{2a})^2 = -\frac{c}{a} + \frac{b^2}{4a^2} \tag{5}$$

$$x + \frac{b}{2a} = \pm\sqrt{-\frac{c}{a} + \frac{b^2}{4a^2}} \tag{6}$$

$$x = -\frac{b}{2a} \pm \sqrt{-\frac{c}{a} + \frac{b^2}{4a^2}} \tag{7}$$

$$x = -\frac{b}{2a} \pm \frac{\sqrt{-\frac{c}{a} \cdot (2a)^2 + \frac{b^2}{4a^2} \cdot (2a)^2}}{2a} \tag{8}$$

$$x = -\frac{b}{2a} \pm \frac{\sqrt{-4ac + b^2}}{2a} \tag{9}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \tag{10}$$

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