Proving the Converse of Pythagorean Theorem

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1 Introduction

The Pythagorean Theorem is defined as: if $a^2 + b^2 = c^2$, then $\theta = 90°$. By definition, then the converse of the Pythagorean Theorem is if $\theta = 90°$, then $a^2 + b^2 = c^2$.

2 Proof

Let $\triangle ABC$ be a triangle with sides a (AB), b (BC), c (AC) such that:

$$a^2 + b^2 = c^2 \tag{1}$$

Let $\triangle DEF$ be a triangle with sides a (DE), b (EF), d (DF). Let $\angle DEF = 90°$, such that side d is the hypotenuse. By the Pythagorean Theorem, we have that:

$$a^2 + b^2 = d^2 \tag{2}$$

Now we can substitute a^2+b^2 in (1) for d^2 from (2), so we have that $c^2 = d^2$. Since $c, d > 0$, $c = d$.

By SSS Congruence, we have that $\triangle ABC \cong \triangle DEF$, so $\angle ABC = \angle DEF = 90°$. Hence, $\triangle ABC$ contains a right angle. $\square$