CSE250 - A5

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1

$$T(n) = 4T(\frac{n}{2}) + n^2 lg(n) \tag{1}$$
$$T(\frac{n}{2}) = 4T(\frac{n}{4}) + (\frac{n}{2})^2 lg(\frac{n}{2}) \tag{2}$$
$$T(\frac{n}{4}) = 4T(\frac{n}{8}) + (\frac{n}{4})^2 lg(\frac{n}{4}) \tag{3}$$

$$\begin{aligned}
T(n) &= 4T(\frac{n}{2}) + n^2 lg(n) \\
&= 4\left(4T(\frac{n}{4}) + (\frac{n}{2})^2 lg(\frac{n}{2})\right) + n^2 lg(n) && \text{insert (2) into (1)} \\
&= 4^2 T(\frac{n}{4}) + n^2 lg(n) + 2^2 (\frac{n}{2})^2 lg(\frac{n}{2}) \\
&= 4^3 T(\frac{n}{8}) + n^2 lg(n) + 2^2 (\frac{n}{2})^2 lg(\frac{n}{2}) + 4^2 (\frac{n}{4})^2 lg(\frac{n}{4}) && \text{insert (3) into T(n/4)} \\
&= ... \\
&= \text{T(n/2) term reaches to the base case}
\end{aligned}$$

$$
\begin{aligned}
T(n) &= \sum_{i=0}^{lg(n)} 4^i (\frac{n}{2^i})^2 lg(\frac{n}{2^i}) \\
&= \sum_{i=0}^{lg(n)} n^2 lg(\frac{n}{2^i}) \\
&= n^2 \sum_{i=0}^{lg(n)} lg(n) - lg(2^i) \\
&= n^2 \sum_{i=0}^{lg(n)} lg(n) - i \\
&< n^2 \left(lg^2(n) - \frac{lg(n)lg(n+1)}{2} \right) \\
&< n^2 \left(\frac{1}{2} lg^2(n) \right) \\
&= O\left(n^2 lg^2(n) \right)
\end{aligned}
$$

2

$$
\begin{aligned}
T(n) &= 5T(\frac{n}{3}) + O(1) && (4) \\
T(\frac{n}{3}) &= 5T(\frac{n}{9}) + O(1) && (5) \\
T(\frac{n}{9}) &= 5T(\frac{n}{27}) + O(1) && (6)
\end{aligned}
$$

$$
\begin{aligned}
T(n) &= 5T(\frac{n}{3}) + O(1) \\
&= 5\left(5T(\frac{n}{9}) + O(1) \right) + O(1) && \text{insert (5) into (4)} \\
&= 5^2 T(\frac{n}{9}) + O(1) + 5O(1) \\
&= 5^3 T(\frac{n}{27}) + O(1) + 5O(1) + 5^2 O(1) && \text{insert (6) into T(n/9)} \\
&= ... \\
&= \text{T(n/3) term reaches to the base case}
\end{aligned}
$$

$$
\begin{aligned}
T(n) &= \sum_{i=0}^{\log_3(n)} 5^i \\
&= \frac{5^{\log_3(n)} - 1}{5-1} && \text{geometric series} \\
&= O(5^{\log_3(n)}) \\
&= O(n^{\log_3(5)})
\end{aligned}
$$

3

$$
T(n) = 6T(\frac{n}{2}) + n^3 \tag{7}
$$

$$
T(\frac{n}{2}) = 6T(\frac{n}{4}) + (\frac{n}{2})^3 \tag{8}
$$

$$
T(\frac{n}{4}) = 6T(\frac{n}{8}) + (\frac{n}{4})^3 \tag{9}
$$

$$
\begin{aligned}
T(n) &= 6T(\frac{n}{2}) + n^3 \\
&= 6\left(6T(\frac{n}{4}) + (\frac{n}{2})^3\right) + n^3 && \text{insert (8) into (7)} \\
&= 6^2T(\frac{n}{4}) + n^3 + 6(\frac{n}{2})^3 \\
&= 6^3T(\frac{n}{8}) + n^3 + 6(\frac{n}{2})^3 + 6^2(\frac{n}{4})^3 && \text{insert (6) into T(n/9)} \\
&= ... \\
&= \text{T(n/2) term reaches to the base case}
\end{aligned}
$$

$$
\begin{aligned}
T(n) &= \sum_{i=0}^{lg(n)} 6^i \frac{n^3}{2^{3i}} \\
&= n^3 \sum_{i=0}^{lg(n)} \left(\frac{6}{8}\right)^i && \text{geometric series - infinity} \\
&< n^3 \left(\frac{1}{1-\frac{6}{8}}\right) = 4n^3 \\
&= O(n^3)
\end{aligned}
$$

4

$$T(n) = 4T(\frac{n}{4}) + n \tag{10}$$

$$T(\frac{n}{4}) = 4T(\frac{n}{16}) + \frac{n}{4} \tag{11}$$

$$T(\frac{n}{16}) = 4T(\frac{n}{64}) + \frac{n}{16} \tag{12}$$

$$\begin{aligned}
T(n) &= 4T(\frac{n}{4}) + n \\
&= 4\left(4T(\frac{n}{16}) + \frac{n}{4}\right) + n && \text{insert (11) into (10)} \\
&= 4^2T(\frac{n}{16}) + n + 4(\frac{n}{4}) \\
&= 4^3T(\frac{n}{64}) + n + 4(\frac{n}{4}) + 4^2(\frac{n}{4^2}) \\
&= ... \\
&= \text{T(n/4) term reaches to the base case}
\end{aligned}$$

$$\begin{aligned}
T(n) &= \sum_{i=0}^{\log_4(n)} n \\
&= \theta(n \log_4(n))
\end{aligned}$$

5

$$T(n) = T(\frac{2}{3}n) + n^2 \tag{13}$$

$$T(\frac{2}{3}n) = T(\frac{4}{9}n) + (\frac{4}{9})n^2 \tag{14}$$

$$T(\frac{4}{9}n) = T(\frac{8}{27}n) + (\frac{4}{9})^2n^2 \tag{15}$$

$$
\begin{aligned}
T(n) &= T(\frac{2}{3}n) + n^2 \\
&= \left(T(\frac{4}{9}n) + (\frac{4}{9})n^2\right) + n && \text{insert (14) into (13)} \\
&= T(\frac{4}{9}n) + n^2 + (\frac{4}{9})n^2 \\
&= T(\frac{8}{27}) + n^2 + (\frac{4}{9})n^2 + (\frac{4}{9})^2 n^2 \\
&= ... \\
&= \text{T(2n/3) term reaches to the base case}
\end{aligned}
$$

$$
\begin{aligned}
T(n) = & \sum_{i=0}^{\log_{\frac{3}{2}}(n)} n^2 \left(\frac{4}{9}\right)^i \\
&= n^2 \sum_{i=0}^{\log_{\frac{3}{2}}(n)} \left(\frac{4}{9}\right)^i \\
&< n^2 \left(\frac{1}{1-\frac{4}{9}}\right) && \text{infinite geometric series} \\
&= O(n^2)
\end{aligned}
$$

6

$$T(n) = 2T(\frac{n}{3}) + n \tag{16}$$

$$T(\frac{n}{3}) = 2T(\frac{n}{9}) + \frac{n}{3} \tag{17}$$

$$T(\frac{n}{9}) = 2T(\frac{n}{27}) + \frac{n}{9} \tag{18}$$

$$
\begin{aligned}
T(n) &= 2T(\frac{n}{3}) + n \\
&= 2\left(2T(\frac{n}{9}) + \frac{n}{3}\right) + n && \text{insert (17) into (16)} \\
&= 2^2T(\frac{n}{9}) + n + 2(\frac{n}{3}) \\
&= 2^3T(\frac{n}{27}) + n + 2(\frac{n}{3}) + 2^2(\frac{n}{3^2}) \\
&= ... \\
&= \text{T(n/3) term reaches to the base case}
\end{aligned}
$$

$$
\begin{aligned}
T(n) &= \sum_{i=0}^{\log_3(n)} (\frac{2}{3})^i n \\
&= n \sum_{i=0}^{\log_3(n)} (\frac{2}{3})^i \\
&< n \left(\frac{1}{1-\frac{2}{3}}\right) && \text{infinite geometric series} \\
&= O(n)
\end{aligned}
$$

7

$$
\begin{aligned}
T(n) &= 5T(\frac{n}{3}) + nlg(n) && (19) \\
T(\frac{n}{3}) &= 5T(\frac{n}{9}) + (\frac{n}{3})lg(\frac{n}{3}) && (20) \\
T(\frac{n}{9}) &= 5T(\frac{n}{27}) + (\frac{n}{9})lg(\frac{n}{9}) && (21)
\end{aligned}
$$

$$
\begin{aligned}
T(n) &= 5T(\frac{n}{3}) + nlg(n) \\
&= 5\left(5T(\frac{n}{9}) + (\frac{n}{3})lg(\frac{n}{3})\right) + nlg(n) && \text{insert (20) into (19)} \\
&= 5^2T(\frac{n}{9}) + nlg(n) + 5(\frac{n}{3})lg(\frac{n}{3}) \\
&= 5^3T(\frac{n}{27}) + nlg(n) + 5(\frac{n}{3})lg(\frac{n}{3}) + 5^2(\frac{n}{9})lg(\frac{n}{9}) \\
&= ... \\
&= \text{T(n/3) term reaches to the base case}
\end{aligned}
$$

$$
\begin{aligned}
T(n) &= \sum_{i=0}^{\log_3(n)} n(\frac{5}{3})^i lg(\frac{n}{3^i}) \\
&= n \sum_{i=0}^{\log_3(n)} (\frac{5}{3})^i \left(lg(n) - lg(3^i)\right) \\
&< n \sum_{i=0}^{\log_3(n)} (\frac{5}{3})^i lg(n) \\
&= nlg(n) \sum_{i=0}^{\log_3(n)} (\frac{5}{3})^i \\
&= nlg(n) \frac{\frac{5}{3}^{\log_3(n)+1} - 1}{\frac{5}{3} - 1} = nlg(n)\frac{3}{2}\frac{5}{3}(\frac{5}{3}^{\log_3(n)} - 1) = nlg(n)\frac{15}{6}(\frac{5^{\log_3(n)}}{3^{\log_3(n)}} - 1) \\
&= nlg(n)\frac{15}{6}(\frac{n^{\log_3(5)}}{n} - 1) = nlg(n)\frac{15}{6}(\frac{n^{\log_3(5)} - n}{n}) \\
&< \frac{15}{6} lg(n)(n^{\log_3(5)}) \\
&= O\left(lg(n)(n^{\log_3(5)}\right)
\end{aligned}
$$

8

$$T(n) = 2T(n-1) + lg(n) \tag{22}$$
$$T(n-1) = 2T(n-2) + lg(n-1) \tag{23}$$
$$T(n-2) = 2T(n-3) + lg(n-2) \tag{24}$$
$$T(n-3) = 2T(n-4) + lg(n-3) \tag{25}$$

$$
\begin{aligned}
T(n) &= 2T(n-1) + lg(n) \\
&= 2\left(2T(n-2) + lg(n-1)\right) + lg(n) && \text{insert (23) into (22)} \\
&= 2^2T(n-2) + lg(n) + 2lg(n-1) \\
&= 2^3T(n-3) + lg(n) + 2lg(n-1) + 2^2lg(n-2) \\
&= ... \\
&= \text{T(n-1) term reaches to the base case}
\end{aligned}
$$

$$
\begin{aligned}
T(n) &= \sum_{i=0}^{n} 2^i lg(n-i) \\
&< \sum_{i=0}^{n} 2^i lg(n) \\
&= lg(n) \sum_{i=0}^{n} 2^i \\
&= lg(n) \frac{2^{n+1}-1}{2-1} \\
&= O\left(2^n lg(n)\right)
\end{aligned}
$$

9

$$
\begin{aligned}
T(n) &= 3T(n-2) + lg^2(n) && (26) \\
T(n-2) &= 3T(n-4) + lg^2(n-2) && (27) \\
T(n-4) &= 3T(n-6) + lg^2(n-4) && (28)
\end{aligned}
$$

$$
\begin{aligned}
T(n) &= 3T(n-2) + lg^2(n) \\
&= 3\left(3T(n-4) + lg^2(n-2)\right) + lg^2(n) && \text{insert (27) into (26)} \\
&= 3^2T(n-4) + lg^2(n) + 3lg^2(n-2) \\
&= 3^3T(n-6) + lg^2(n) + 3lg^2(n-2) + 3^2lg^2(n-4) \\
&= ... \\
&= \text{T(n-2) term reaches to the base case}
\end{aligned}
$$

$$
\begin{aligned}
T(n) &= \sum_{i=0}^{n/2} 3^i lg^2(n-2i) \\
&< \sum_{i=0}^{n/2} 3^i lg^2 n \\
&= lg^2 n \frac{3^{\frac{n+2}{2}}-1}{3-1} \\
&= O\left(3^{n/2} lg^2 n\right)
\end{aligned}
$$

10

$$T(n) = 2T(n-2) + lg(n^2) \tag{29}$$
$$T(n-2) = 2T(n-4) + lg((n-2)^2) \tag{30}$$
$$T(n-4) = 2T(n-6) + lg((n-4)^2) \tag{31}$$

$$\begin{aligned}
T(n) &= 2T(n-2) + lg(n^2) \\
&= 2\left(2T(n-4) + lg((n-2)^2\right) + lg^2(n) && \text{insert (29) into (30)} \\
&= 2^2T(n-4) + lg^2(n) + 2lg^2(n-2) \\
&= 2^3T(n-6) + lg^2(n) + 2lg^2(n-2) + 2^2lg(n-4) \\
&= ... \\
&= \text{T(n-2) term reaches to the base case}
\end{aligned}$$

$$\begin{aligned}
T(n) &= \sum_{i=0}^{n/2} 2^i lg(n-2i)^2 \\
&< \sum_{i=0}^{n/2} 2^i lgn^2 \\
&= lgn^2 \frac{2^{\frac{n+2}{2}} - 1}{2-1} \\
&= O\left(2^{\frac{n}{2}} lgn^2\right)
\end{aligned}$$