**ELEE 4200/5200: Autonomous Mobility Robotics**
**Term I, 2018**
**Homework 3: Numerical Problems on Kinematic Model**

<u>Note:</u>
This homework is made up of some sample problems on the kinematic model topic, based on the equations derived in class. The purpose is to familiarize you with the meaning and use of the equations. You should be aware that the actual problems appearing in Test 1 might require simple extensions of the concepts highlighted here, based on the solid understanding you develop in solving this problem set!

1. The estimate of a *differentially-steered* robot's instantaneous motion with respect to the global frame is given below. **theta=135**

$$\dot{\xi}_I = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} -5\ m/sec \\ 5\ m/sec \\ -0.5\ rad/sec \end{bmatrix}$$

Determine the robot's instantaneous velocity motion vector, $\dot{\xi}_R$, with respect to its own frame of reference <u>using formal techniques, as established in class</u>.

2. A differentially-steered robot has the following parameters:

$$v = -2m/s; \quad \omega = \frac{\pi}{3} rad/s$$

$$Radius\ of\ wheels = 0.4m$$

$$Wheel\ spacing\,(2L) = 0.8m$$

$$Pose: \begin{bmatrix} 4m \\ 6m \\ 30^0 \end{bmatrix}$$

a) Calculate the two drive wheel velocities that will produce the required robot forward velocity and turn rate.

b) What will the radius of the resultant circular path be?

c) Using the forward velocity of the robot, v, estimate the time it will take for the robot to move along this circle so that its pose angle changes by $60^0$?

d) Re-estimate the above, this time using the turn rate, $\omega$.

e) What is the location of the Instantaneous Center of Rotation (ICR) with respect to the global axis?

3. A differentially-steered robot is currently positioned at the origin of the global reference and is pointed in the direction of the $X_I$ axis. It is necessary to drive it to a goal whose range and bearing is (70m, $30^0$). The robot's construction requires that the maximum turn rate that it can be run at is 0.25 rad/s (in magnitude).

a) If the robot is required to follow a single circular path to the goal, what should the radius of this path be?

b) Where is the ICC located with respect to the global axes?

c) What is the shortest time that the robot will take to arrive at the goal?

d) What will its orientation (pose angle) be when it gets to the goal?

e) Reevaluate part (a) if the robot is required to have a pose angle of $90^0$ at the destination.

4. At a certain instant in time, Robot A and Robot B have poses as given below:

$$pose_A = \begin{bmatrix} -175m \\ 90m \\ 45^0 \end{bmatrix} \qquad pose_B = \begin{bmatrix} 150m \\ 60m \\ -30^0 \end{bmatrix}$$

Robot B spots a bomb at a range of 215 meters and bearing of $-60^0$. Using this information, determine the co-ordinates of the bomb with respect to Robot A's local coordinate frame, <u>using mapping to the global frame as an intermediate stage</u>.

Note: The result needs to be obtained using formal matrix analysis using the technique discussed in class, not through intuitive unstructured application of geometrical concepts!

5. Figure 1 below shows two sets of axes which are displaced and rotated with respect to each other; consider (x, y, z) to be the original set of axes.
Obtain the transformation equations between the axes using:
a) The fixed axis method
b) The moving axis method
c) A point is located at (-5, 15, -10) with respect to the (x, y, z) axes. Obtain the coordinates of the point with respect to the (x′, y′, z′) axes.

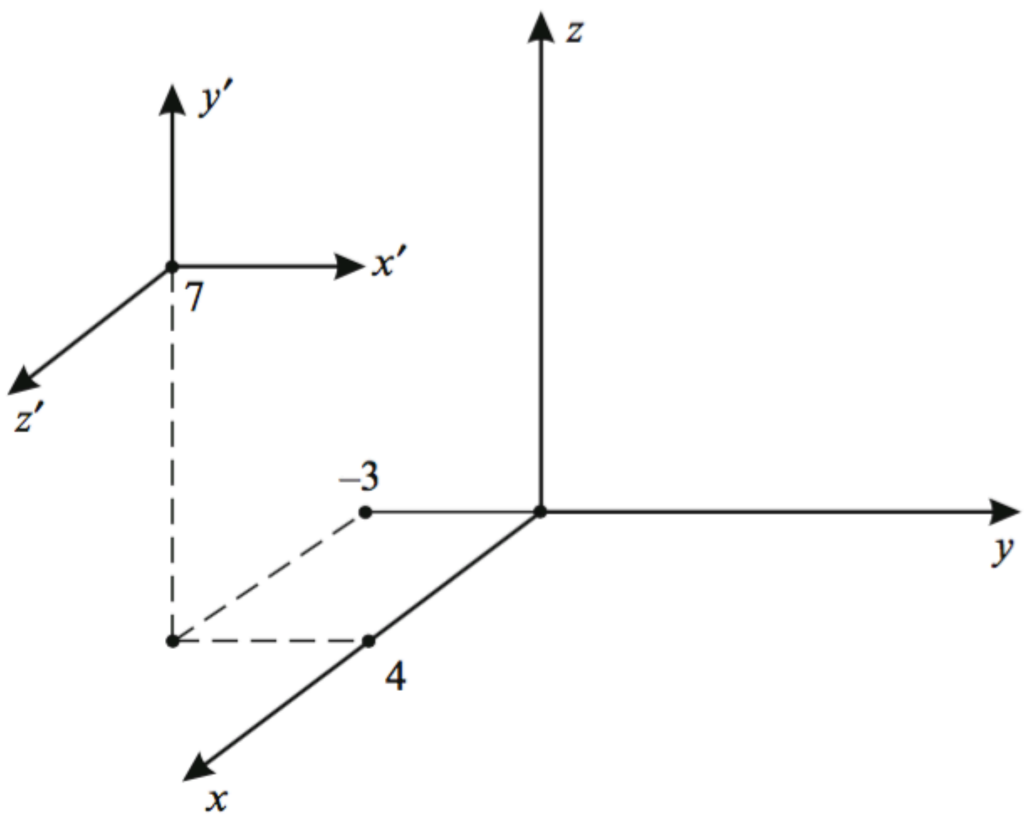


*Figure 1*

6. A differentially-steered robot has the initial pose given by:

$$pose = \begin{bmatrix} 2m \\ 4m \\ 0^0 \end{bmatrix}$$

It is moving with constant (but different) left and right wheel velocities. After a certain interval of time, wheel encoder calculations indicate a forward motion of 2 m and a turn angle of +$10^0$ with respect to the starting position. If the Borenstein odometry approximation is applied to estimate robot position:

a) What is the predicted location of the robot?

b) What is the actual location of the robot?

<u>Additional Reference</u>

- http://rossum.sourceforge.net/papers/CalculationsForRobotics/CirclePath.htm