Introduction to Fuzzy Cognitive Maps

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Abstract This chapter gives a short introduction to Fuzzy Cognitive Maps (FCMs). It starts with the origin and first applications of Cognitive Maps, then describes the theoretical background of FCMs. Based on the cognitive model, simulations can be performed in order to predict the dynamic behavior of the system and support decision making tasks. The widely applied variations of implementation details are also covered, including their effect on model properties and behavior. A simple example is given to help understanding the theoretical parts, and a short outlook is provided to the possible ways of model creation, too.

1 The birth of Fuzzy Cognitive Maps

Cognitive Maps (CM) are used in political analysis and decision making in international relations, foreign policy for a long time. The method was suggested by Robert Axelrod in his book [1] in the late '70s. According to Bart Kosko's description in [17], these maps are signed digraphs. Graphs, as algebraic structures have two components: nodes and edges (arcs). In CM, nodes represent variable *concepts* (eg. social instability) and the causal connections among these concepts are characterized by edges. The edges have a direction and a sign. If concept A causally increases concept B, it is represented by an edge from A to B with positive sign. On the other hand, if A reduces the value of B, the edge has a negative sign. Kosko illustrated CM with an example based on Henry Kissinger's essay "Starting Out in the Direction of Middle East Peace" published in *Los Angeles Times* in 1982 (see Fig. 1). Besides the graph, he also composed the adjacency (connection, weight) matrix (Fig. 2) of the model. Only three different values can be found in this matrix, representing the causal relationship among concepts. If $w_{ij} = w(C_i, C_j)$ is 1, concept C_1 causally increases the value of C_2 (positive edges). On the contrary, if C_1 causally decreases

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the value of C_2, it is represented by -1 (negative edges), and the value 0 indicates the lack of causal connection.

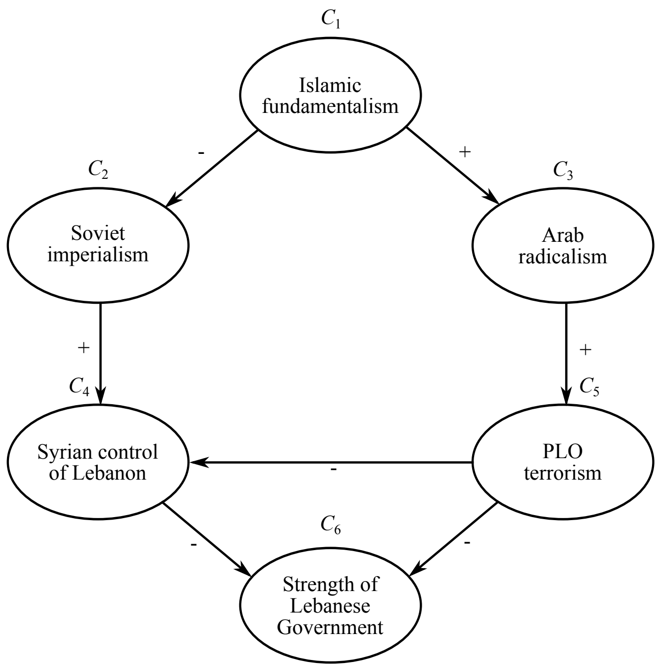

Fig. 1 The Cognitive Map drawn by Kosko based on Kissinger's essay.

$$
\begin{array}{c}
 \\ C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \\ C_6
\end{array}
\begin{array}{c}
\begin{array}{cccccc} C_1 & C_2 & C_3 & C_4 & C_5 & C_6 \end{array} \\
\begin{pmatrix}
0 & -1 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & -1 \\
0 & 0 & 0 & -1 & 0 & -1 \\
0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}
\end{array}
$$

Fig. 2 The adjacency matrix of the CM based on Kissinger's essay.

It became quickly evident that the structure of a Cognitive Map imply too much limitations. The degree of causality, the levels of the causal effects (sometimes–often, little–much, etc.) cannot be expressed with the existing tool, and needs further development. Kosko introduced the Fuzzy Cognitive Map (FCM) [17], where the edges may have several causality values. This way the original CM turned into a bipolar fuzzy graph [31]. Kosko also developed a fuzzy causal algebra for propagating causality in it, making the static analysis of the model possible. The concepts may affect other concepts indirectly because of the cyclic and non-feedforward structure of FCM.

2 Simulations

The visual representation and formal description of models may help experts to review the structure of the studied system, but the real benefit of FCMs is the possibility of running simulations. This can be done by dynamic analysis [18]. Using a simple inference technique, the next state of concepts (also called the *activation values* of concepts, based on the similarity of FCMs and artificial neural networks) can be calculated using their current state and the weight of connections among them [4, 28]. This way what-if analysis can be performed which is very useful for decision makers.

2.1 Inference rules

Kosko defined the first, and still widely applied inference rule (Eq. 1). In this case concepts are not allowed to directly influence themselves. With other words, a concept does not have "memory", technically speaking the edges must connect different concepts (i.e. self-loops are forbidden), but indirect feedbacks are still possible. Accordingly, the main diagonal of the connection matrix contains only zeros. These models are called FCMs of Type I [27].

$$A_i^{t+1} = f\left(\sum_{\substack{j=1 \\ j\neq i}}^{n} w_{ji} A_j^t\right) \tag{1}$$

Here, A_i^{t+1} is the activation value of concept i at time step $t+1$, n is the number of concepts and $f(\cdot)$ is the *threshold* (transformation, squeezing) function. Without this function, the activation values of concepts may exceed or fall below their allowed extreme values.

FCMs of Type III allow concepts to take into account their current states during the calculation of the next state, and apply the inference rule defined by Eq. 2. In this case, the main diagonal of the connection matrix may contain nonzero values, which are often ones in practice.

$$A_i^{t+1} = f\left(w_{ii} A_i^t + \sum_{\substack{j=1 \\ j\neq i}}^{n} w_{ji} A_j^t\right) \tag{2}$$

FCMs of Type II also have a one step memory, but the weight of self-loops (k_1) are the same for all concepts. Moreover, the influence coming from other concepts can be limited as well (k_2, see Eqs. 3, 4).

$$A_i^{t+1} = f\left(k_1 A_i^t + k_2 \sum_{\substack{j=1 \\ j \neq i}}^{n} w_{ji} A_j^t\right) \tag{3}$$

$$0 < k_1, k_2 \leq 1 \tag{4}$$

Papageorgiu suggested the re-scale inference rule (Eq. 5) in [23], because problems emerged in cases where the initial values of concepts were 0 or 0.5, assuming that the initial states are known at all. This modified inference rule yielded more reliable and natural results than other FCM versions.

$$A_i^{t+1} = f\left((2A_i^t - 1) + \sum_{\substack{j=1 \\ j \neq i}}^{n} w_{ji} \cdot (2A_j^t - 1)\right) \tag{5}$$

2.2 Threshold functions

In general, the activation values are in the $A_i \in [0, 1]$ interval. Several threshold functions were published during the years, but the most widely applied one is the *sigmoid* (*logistics*) function (Eq. 6):

$$f_{\text{sigmoid}}(x) = \frac{1}{1 + e^{-\lambda x}} \tag{6}$$

where the $\lambda > 0$ specifies the steepness of the function. It's typical value is 5. With greater values it approximates a binary function, with lower values a linear function (see Fig. 3).

Choosing an appropriate λ value

Is there an "appropriate" value of this parameter? According to Bueno and Salmeron [3], the appropriate value should be found during an experimentation phase. As it was pointed out in [9, 13], the congergence speed and the final, stable values of concepts are heavily affected by the value of parameter λ (see Fig. 4). Despite of the different results, different λ values does not change the result of a simulation qualitatively, because the sigmoid function is strictly monotonically increasing. In practice, it is still worth to find an "optimal" value for λ, because sometimes the otherwise hardly different concept states may become distinguishable, and the order of concepts can be defined, if it is required. A possible method for "lambda optimization" is to maximize the standard deviation of final concept values (Fig. 5) even by a quick local search algorithm, eg. the Golden Section Search.

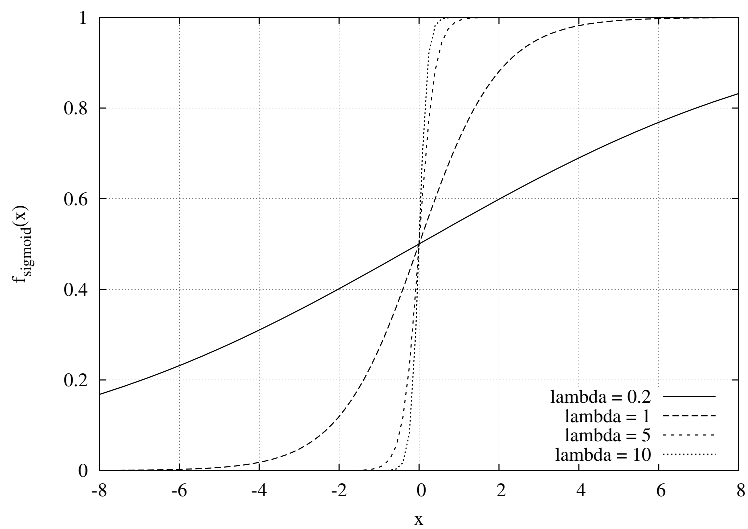

Fig. 3 Steepness of the sigmoid function with various λ parameters.

Even if the value of λ can be considered "optimal", a scalar value is unable to unleash the full modeling potencial of an FCM. In an extreme case, every concept may have its own, specific λ value, in order to get more realistic predictions. This way, the inference rule defined by Eq. 1 can be re-written to Eq. 7, where f_i represents the threshold function of concept C_i that uses λ_i. This approach increases the number of model parameters, however. One of the benefits of using FCM was its simple structure, which can be understood easily by stakeholders, experts of different fields, and the values (initial states of concepts, connection weights) can be matched to real values or effects in a straightforward way. The values of λ do not have trivial connection with any real world objects, however, and their values can be set with trial and error technique or computationally. It also increases the required computational power of applications because it increases the number of model parameters. A good balance can be found if some concepts have their own parameters, the others share a common value.

$$A_i^{t+1} = f_i\left(\sum_{j=1}^{n} w_{ji} A_j^t\right) \tag{7}$$

Concepts may be classified in three groups: if a concept influences the state of other concepts, but not affected by other concepts, it can be called an *input* concept. Depending on the applied inference method, these concepts keep their initial values or

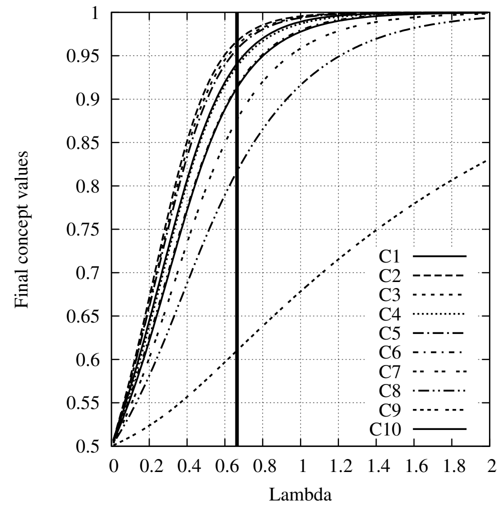

Fig. 4 The final concept values of a Stakeholder Relationship Management System as function of parameter λ. $\lambda = 0.664$ maximized the standard deviation of concept values (marked with the thick vertical line).

their values can be sustained artificially (like "sustained inputs" in [4]), therefore they do not need threshold function and parameter λ neither. Another group of concepts

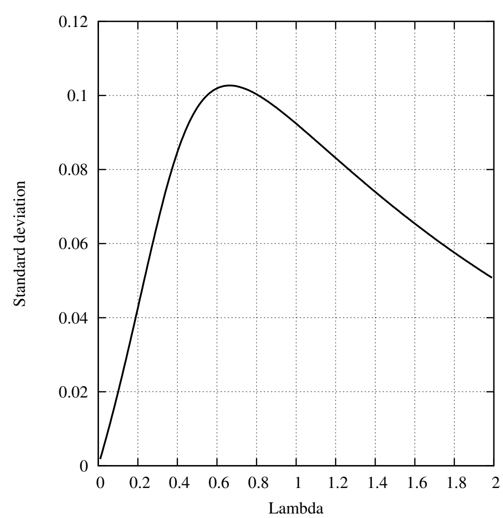

Fig. 5 Standard deviation of final concept values of a Stakeholder Relationship Management System calculated with different λ values.

is affected by other concepts but its elements do not influence the state of other concepts. In many applications, only these concepts are used for decision making, and they are called *output* concepts. The remaining concepts are both affected by and they also affect other concepts, and are called *intermediate* concepts [2].

If all output concepts have specific λ values, and intermediate concepts have a common λ, the complexity of the model remains moderate but its forecasting ability can be increased as it happened in [12]. If historical time series data are available, λs can be optimized with eg. the Big Bang – Big Crunch algorithm [29].

The *sign* function (Eq. 8) is also a popular threshold function, but because of the two available values, a concept can only be activated (1) or deactivated (0).

$$f_{\text{sign}}(x) = \begin{cases} 1, \ x > 0, \\ 0, \ x \leq 0 \end{cases} \tag{8}$$

In some cases the system to be modeled requires the activation values to be in $A_i \in [-1, +1]$, and different threshold functions have to be applied. If continuous states are allowed, the *hyperbolic tangent* function (Eq. 9) is a possible choice:

$$f_{\text{ht}}(x) = tanh(\lambda x) = \frac{e^{\lambda x} - e^{-\lambda x}}{e^{\lambda x} + e^{-\lambda x}} \tag{9}$$

The λ parameter can be used here as well, and has similar effect on the steepness of the function than on sigmoid function's (Fig. 6).

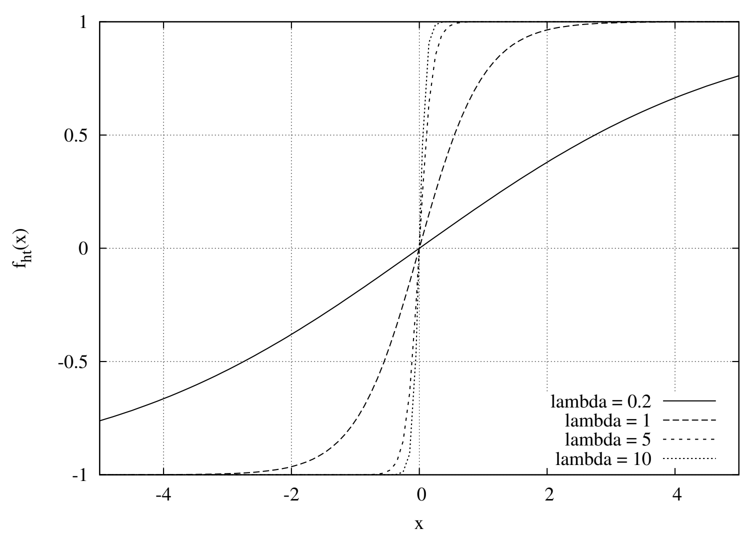

Fig. 6 Steepness of the hyperbolic tangent function with various λ parameters.

If only discrete states have to used, the *trivalent* function (Eq. 10) can be the solution. The value 1 expresses the increasing, the value −1 expresses the decreasing and 0 means the stable state of a concept. A slightly modified version of this function can be found in [30] (Eq. 11).

$$f_{\text{trivalent1}}(x) = \begin{cases} 1, & x > 0, \\ 0, & x = 0, \\ -1, & x < 0. \end{cases} \tag{10}$$

$$f_{\text{trivalent2}}(x) = \begin{cases} 1, & x \geq 0.5, \\ 0, & -0.5 < x < 0.5, \\ -1, & x \leq -0.5. \end{cases} \tag{11}$$

2.3 Simulation outcomes

During simulations, the states of concepts continuously evolve, until one of the three possible outcomes happens [28]:

1. In most cases, the activation values converge very quickly to a stable state (equilibrium point, see Eq. 12). These states are also called *fixed points* (FPs). Depending on the connection matrix, initial state, inference rule, threshold function and its λ parameter (if applied), a model may have multiple FPs.

$$\exists t_{\mathbf{FP}} \in \mathbb{N} : A_i^{t+1} = A_i^t, \forall t \geq t_{\mathbf{FP}}, i = 1, \ldots, n \tag{12}$$

2. Sometimes the system enters a *limit cycle* (LC). In these cases after several time steps, a T steps long sequence of state vectors starts to repeat (see Eq. 13).

$$\exists t_{\mathbf{LC}}, T \in \mathbb{N} : A_i^{t+T} = A_i^t, \forall t \geq t_{\mathbf{LC}}, i = 1, \ldots, n \tag{13}$$

3. The system never stabilizes, behaves chaotically.

It's easy to prove that bivalent and trivalent systems cannot behave chaotically. The process of simulations are deterministic, and the number of different state vectors is 2^n or 3^n, respectively, where n is the number of concepts. According to this, if the system is unstable, its states are going to repeat after 2^n or 3^n time steps, or earlier. FPs and LCs are related to the hidden patterns described in Kosko's work [18].

In contrast, continuous state FCMs may enter practically infinite different states, thus chaotic behavior is possible. It happens especially often in case of Type III FCMs, which enter LCs frequently as well [21]. Martchenko et al. also defined the condition of stability, too, which depends on the connection matrix.

Harmati et al. went further [6] and determined the conditions of convergence and the number of FPs. According to their results, if the FCM uses sigmoid threshold function and the inequality

$$||W||_F < \frac{4}{\lambda} \tag{14}$$

holds, then the FCM has exactly one FP, without any LCs or chaotic patterns. Here W is the connection matrix, $||\cdot||_F$ is the Frobenius-norm of the matrix defined by Eq. 15, and λ is the steepness of the threshold function.

$$||W||_F = \left(\sum_i \sum_j w_{ij}^2\right)^{1/2} \tag{15}$$

Similarly, if the hyperbolic tangent function is applied and Eq. 16 holds, the FCM has one and only one FP. These theorems are valid even if the connection matrix contains direct feedbacks.

$$||W||_F < \frac{1}{\lambda} \tag{16}$$

Moreover, if the sigmoid or hyperbolic tangent threshold functions are used, and W contains only non-negative elements than the FCM has FP, regardless of the value of λ. Similar contexts were also explored in case of some extended versions of FCM, the Grey FCMs [7] and Fuzzy Set Valued Sigmoid FCMs [8].

These are remarkable achievements because some of the behavioral properties of an FCM can be revealed without executing an exhaustive set of simulations, which is computationally very intensive. Unfortunately it is often unavoidable. If the model is made by experts, it may contain many subjective elements, eg. it is really hard to define the exact weight of a relationship. Even a small difference in the weights may lead to different behaviour, however. It is worth investigate the effect of small model modifications on behavioural properties, but it means a huge job which must be automated like in [11, 10]. The search space is huge for an exhaustive search hence Bacterial Evolutionary Algorithm [22] was applied to generate and analyse the behavioral difference caused by slightly modified model versions and λ values. If modified models with significantly different dynamic behaviour are found, experts are suggested to consider some changes on the model.

In most decision making tasks, stable states are the desired results. In these cases the future state of a system can be determined, and if that equilibrium state is undesirable, measures can be taken. There are some situations however, eg. time series predictions [15, 14, 20], when LCs or chaotic behavior seems very useful. Often extended versions of the original FCM are suggested to get better predictions.

2.4 An example simulation

Finally, an example FCM is given for illustration. The model has 5 concepts, their allowed values are in the interval [0, 1]. Kosko's original inference rule is applied (Eq. 1), the threshold function is the sigmoid type (Eq. 6) with parameter $\lambda = 5$.

Table 1 Connection matrix of the example FCM.

	C_1	C_2	C_3	C_4	C_5
C_1	0	0.5	0.3	0	0
C_2	0	0	0	−0.1	0.35
C_3	0	−0.1	0	−0.2	0.4
C_4	0.4	0	0	0	0
C_5	−0.7	0	0	0	0

Table 2 State vectors of the example FCM.

Time	C_1	C_2	C_3	C_4	C_5
t_0	0.00	0.50	1.00	0.50	0.00
t_1	0.94	0.44	0.32	0.50	0.50
t_2	0.83	0.65	0.57	0.87	0.04
t_3	0.92	0.41	0.25	0.84	0.05
t_4	0.80	0.42	0.28	0.86	0.04
t_5	0.81	0.41	0.27	0.83	0.06
t_6	0.81	0.42	0.28	0.84	0.05
t_7	0.81	0.42	0.28	0.83	0.06
t_8	0.81	0.42	0.28	0.84	0.05
t_9	0.81	0.42	0.28	0.84	0.05

Formally, this model can be defined by a 4-tuple (C, W, A, f), where $C = C_1, C_2, \dots, C_n$ is the set of $n = 5$ concepts, function $W : (C_i,\ C_j) \to w_{ij}$ associates the causal value $w_{ij} \in [-1,\ +1]$ to each pair of concepts $(C_i,\ C_j)$. These weights of directed edges are collected in the connection matrix denoted by W_{nxn} (see Table 1 and Fig. 7). Another function, $A : (C_i) \to A_i$ associates the activation value $A_i \in \mathbb{R}$ to each node C_i at each time step $t = 1, 2, \dots, m$. The (sigmoid) threshold function $f : \mathbb{R} \to [0,\ 1]$ forces the activation values in their allowed range.

These are very general settings, and according to this, the dynamic behavior of the model is also very typical: the concepts' values converge after some time steps to their final values, to an equilibrium point (see Table 2 and Fig. 8).

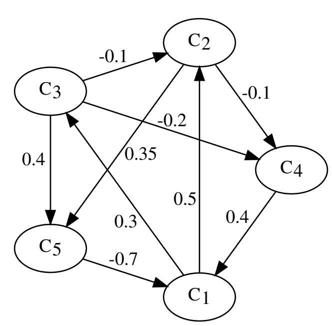

Fig. 7 Graphical representation of the example FCM.

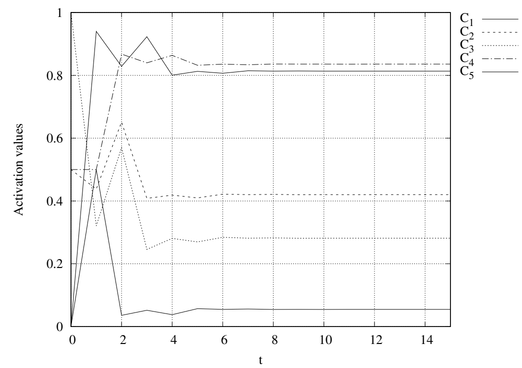

Fig. 8 Convergence of concept activations during simulation.

3 Creating a Fuzzy Cognitive Map model

There are two main ways of model creation: the first one is called *deductive modeling*, when experts are asked to identify the main driving factors (concepts) and the cause-effect relationships among them (connection matrix). These techniques do not require the existence of recorded historical data, because they utilize the domain-specific knowledge of experts. Unfortunately, these experiences are often limited, contain subjective opinions as well, and only in case of relatively small systems can be modeled this way with high confidence.

The other way is called *inductive modeling*, when objective, measured historical data are used to generate the model with learning algorithms. These algorithms do not require human interaction in general, but the concepts are still often defined by experts.

It is often hard to define which concepts play an important role in the model. According to the strategy described in [26] and originally suggested in [17, 16] all available concepts should be included in a list, and later remove the insignificant ones. A similar approach is suggested, but executed automatically in [12]. The hardest job for experts is the definition of connection weights among concepts, however, because the number of connections is the quadratic function of the number of concepts.

In many cases the capability of exact re-generation of the time series used for learning is not a requirement. If the model assigns initial states and their corresponding fixed points correctly, that is enough in most decision making tasks.

Some possible methods of model creation are suggested by Kosko in his early papers (eg. in [18, 4]). In order to increase the credibility of experts, he advised to ask several experts, then combine and differentiate their suggestions based on their credibility. Groumpos described this technique and provided an alternative one in [5]. Kosko also suggested a simple learning technique based on the Hebbian law (DHL). Later several authors improved his method (eg. NHL, AHL, DD-NHL). A good summary of these are available in [24], which presents some population-based methods as well. The book [25] gives a wider insight into model creation methods, including hybrid techniques. Even today are new methods born, eg. [19] takes a new approach.

Acknowledgements If you want to include acknowledgments of assistance and the like at the end of an individual chapter please use the `acknowledgement` environment – it will automatically be rendered in line with the preferred layout.

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