Introduction to Fuzzy Cognitive Maps

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1 The birth of Fuzzy Cognitive Maps

Cognitive Maps (CM) are used in political analysis and decision making in international relations, foreign policy for a long time. The method was suggested by Robert Axelrod in his book [1] in the late '70s. According to Bart Kosko's description in [3], these maps are signed digraphs. Graphs, as algebraic structures have two components: nodes and edges (arcs). In CM, nodes represent variable *concepts* (eg. social instability) and the causal connections among these concepts are characterized by edges. The edges have a direction and a sign. If concept A causally increases concept B, it is represented by an edge from A to B with positive sign. On the other hand, if A reduces the value of B, the edge has a negative sign. Kosko illustrated CM with an example based on Henry Kissinger's essay "Starting Out in the Direction of Middle East Peace" published in *Los Angeles Times* in 1982 (see Fig. 1). Besides the graph, he also composed the adjacency (connection, weight) matrix (Fig. 2) of the model. Only three different values can be found in this matrix, representing the causal relationship among concepts. If $w_{ij} = w(C_i, C_j)$ is 1, concept C_1 causally increases the value of C_2 (positive edges). On the contrary, if C_1 causally decreases

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the value of C_2, it is represented by -1 (negative edges), and the value 0 indicates the lack of causal connection.

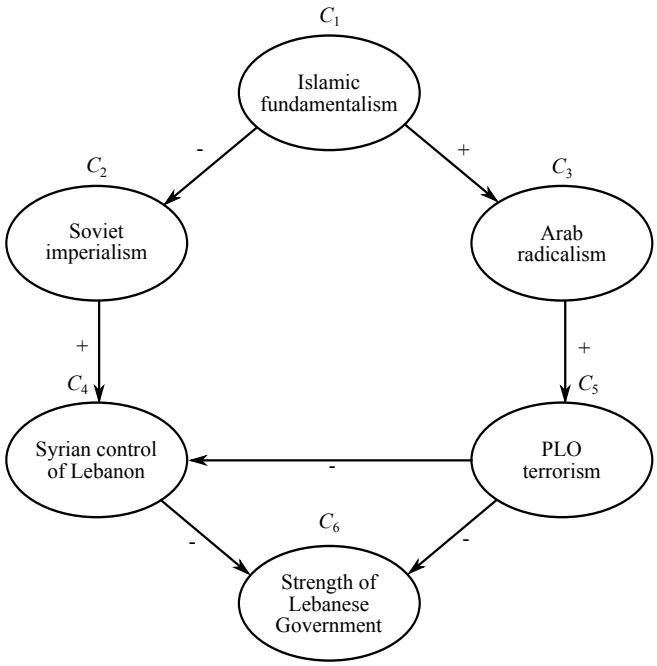

Fig. 1 The Cognitive Map drawn by Kosko based on Kissinger's essay.

Fig. 2 The adjacency matrix of the CM based on Kissinger's essay.

$$
\begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \\ C_6 \end{array}
\begin{array}{c}
\begin{array}{cccccc} C_1 & C_2 & C_3 & C_4 & C_5 & C_6 \end{array} \\
\left(\begin{array}{cccccc}
0 & -1 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & -1 \\
0 & 0 & 0 & -1 & 0 & -1 \\
0 & 0 & 0 & 0 & 0 & 0
\end{array} \right)
\end{array}
$$

It became quickly evident that the structure of a Cognitive Map imply too much limitations. The degree of causality, the levels of the causal effects (sometimes–often, little–much, etc.) cannot be expressed with the existing tool, and needs further development. Kosko introduced the Fuzzy Cognitive Map (FCM) [3], where the edges may have several causality values. This way the original CM turned into a bipolar fuzzy graph [6]. Kosko also developed a fuzzy causal algebra for propagating causality in it, making the static analysis of the model possible. The concepts may affect other concepts indirectly because of the cyclic and non-feedforward structure of FCM.

2 Simulations

The visual representation and formal description of models may help experts to review the structure of the studied system, but the real benefit of FCMs is the possibility of running simulations. This can be done by dynamic analysis [4]. Using a simple inference technique (see Eq. 1), the next state of concepts (also called the *activation values* of concepts, based on the similarity of FCMs and artificial neural networks) can be calculated using their current state and the weight of connections among them [2, 5]. This way what-if analysis can be performed which is very useful for decision makers.

$$A_i^{t+1} = f\left(\sum_{j=1}^{n} w_{ji} A_j^t\right) \tag{1}$$

Here, A_i^{t+1} is the activation value of concept i at time step $t+1$, n is the number of concepts and $f(.)$ is the *threshold* (transformation) function. Without this function, the activation values of concepts may exceed or fall below their allowed extreme values, which is in general $A_i \in [0, 1]$. Several threshold functions were published during the years, but the most widely applied one is the *sigmoid* or *logistics* function (Eq. 2):

$$f_{\text{sigmoid}}(x) = \frac{1}{1 + e^{-\lambda x}} \tag{2}$$

where the $\lambda > 0$ specifies the steepness of the function. It's typical value is 5, but with greater values it approximates a binary function, with lower values a linear function (see Fig. 3).

The *sign* function (Eq. 3) is also a popular choice, but because of the two available values, a concept can only be activated (1) or deactivated (0).

$$f_{\text{sign}}(x) = \begin{cases} 1, & x > 0, \\ 0, & x \leq 0 \end{cases} \tag{3}$$

In some cases the system to be modeled requires the activation values to be in $A_i \in [-1, +1]$, and different threshold functions have to be applied. If continuous states are allowed, the *hyperbolic tangent* function (Eq. 4) is a possible choice:

$$f_{\text{ht}}(x) = tanh(\lambda x) = \frac{e^{\lambda x} - e^{-\lambda x}}{e^{\lambda x} + e^{-\lambda x}} \tag{4}$$

The λ parameter can be used here as well, and has similar effect on the steepness of the function than on sigmoid function's (Fig. 4).

If only discrete states have to used, the *trivalent* function (Eq. 5) can be the solution. The value 1 expresses the increasing, the value -1 expresses the decreasing and 0 means the stable state of a concept.

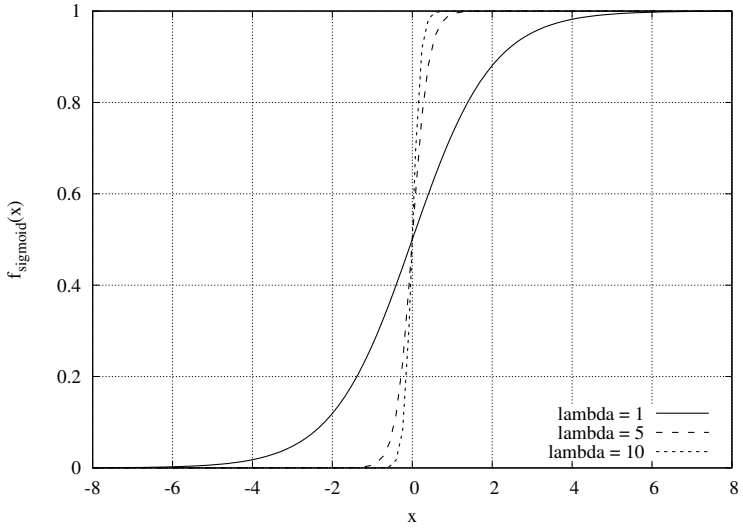

Fig. 3 Steepness of the sigmoid function with various λ parameters.

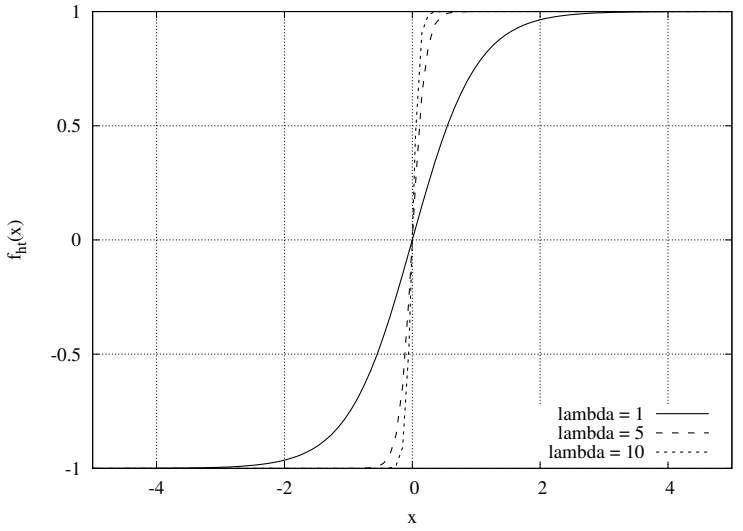

Fig. 4 Steepness of the hyperbolic tangent function with various λ parameters.

$$f_{\text{trivalent}}(x) = \begin{cases} 1, & x > 0, \\ 0, & x = 0, \\ -1, & x < 0. \end{cases} \tag{5}$$

2.1 Subsection Heading

Instead of simply listing headings of different levels we recommend to let every heading be followed by at least a short passage of text. Further on please use the LaTeX automatism for all your cross-references and citations as has already been described in Sect. 2.

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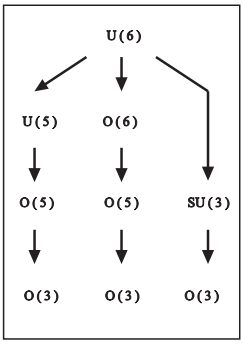

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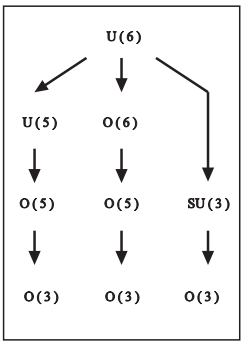

Fig. 6 If the width of the figure is less than 7.8 cm use the `sidecapion` command to flush the caption on the left side of the page. If the figure is positioned at the top of the page, align the sidecaption with the top of the figure – to achieve this you simply need to use the optional argument `[t]` with the `sidecaption` command

For typesetting numbered lists we recommend to use the `enumerate` environment – it will automatically rendered in line with the preferred layout.

1. Livelihood and survival mobility are oftentimes coutcomes of uneven socioeconomic development.
   a. Livelihood and survival mobility are oftentimes coutcomes of uneven socioeconomic development.
   b. Livelihood and survival mobility are oftentimes coutcomes of uneven socioeconomic development.
2. Livelihood and survival mobility are oftentimes coutcomes of uneven socioeconomic development.

Subparagraph Heading

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For unnumbered list we recommend to use the `itemize` environment – it will automatically be rendered in line with the preferred layout.

- Livelihood and survival mobility are oftentimes coutcomes of uneven socioeconomic development, cf. Table 1.
  – Livelihood and survival mobility are oftentimes coutcomes of uneven socioeconomic development.
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Table 1 Please write your table caption here

Classes	Subclass	Length	Action Mechanism
Translation	mRNA[a]	22 (19–25)	Translation repression, mRNA cleavage
Translation	mRNA cleavage	21	mRNA cleavage
Translation	mRNA	21–22	mRNA cleavage
Translation	mRNA	24–26	Histone and DNA Modification

[a] Table foot note (with superscript)

Run-in Heading Boldface Version Use the LaTeX automatism for all your cross-references and citations as has already been described in Sect. 2.

Run-in Heading Boldface and Italic Version Use the LaTeX automatism for all your cross-references and citations as has already been described in Sect. 2.

Run-in Heading Displayed Version
Use the LaTeX automatism for all your cross-references and citations as has already been described in Sect. 2.

3 Section Heading

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If you want to list definitions or the like we recommend to use the enhanced `description` environment – it will automatically rendered in line with the preferred layout.

Type 1 That addresses central themes pertainng to migration, health, and disease. In Sect. 1, Wilson discusses the role of human migration in infectious disease distributions and patterns.

Type 2 That addresses central themes pertainng to migration, health, and disease. In Sect. 2.1, Wilson discusses the role of human migration in infectious disease distributions and patterns.

3.1 Subsection Heading

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for all your cross-references and citations citations as has already been described in Sect. 2.

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If you want to emphasize complete paragraphs of texts we recommend to use the newly defined class option `graybox` and the newly defined environment `svgraybox`. This will produce a 15 percent screened box 'behind' your text.

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Please note that the first line of text that follows a heading is not indented, whereas the first lines of all subsequent paragraphs are.

Theorem 1 *Theorem text goes here.*

Definition 1 Definition text goes here.

Proof Proof text goes here. □

Paragraph Heading

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Note that the first line of text that follows a heading is not indented, whereas the first lines of all subsequent paragraphs are.

Theorem 2 *Theorem text goes here.*

Definition 2 Definition text goes here.

Proof Proof text goes here. □

Trailer Head

If you want to emphasize complete paragraphs of texts in an `Trailer Head` we recommend to use

```
\begin{trailer}{Trailer Head}
...
\end{trailer}
```

? Questions

If you want to emphasize complete paragraphs of texts in an `Questions` we recommend to use

```
\begin{question}{Questions}
...
\end{question}
```

> Important

If you want to emphasize complete paragraphs of texts in an `Important` we recommend to use

```
\begin{important}{Important}
...
\end{important}
```

! Attention

If you want to emphasize complete paragraphs of texts in an `Attention` we recommend to use

```
\begin{warning}{Attention}
...
\end{warning}
```

Program Code

If you want to emphasize complete paragraphs of texts in an `Program Code` we recommend to use

```
\begin{programcode}{Program Code}
\begin{verbatim}...\end{verbatim}
\end{programcode}
```

Tips

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```
\begin{tips}{Tips}
...
\end{tips}
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Overview

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```
\begin{overview}{Overview}
...
\end{overview}
```

Background Information

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```
\begin{backgroundinformation}{Background Information}
...
\end{backgroundinformation}
```

Legal Text

If you want to emphasize complete paragraphs of texts in an `Legal Text` we recommend to use

```
\begin{legaltext}{Legal Text}
...
\end{legaltext}
```

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Appendix

When placed at the end of a chapter or contribution (as opposed to at the end of the book), the numbering of tables, figures, and equations in the appendix section continues on from that in the main text. Hence please *do not* use the `appendix` command when writing an appendix at the end of your chapter or contribution. If there is only one the appendix is designated “Appendix”, or “Appendix 1”, or “Appendix 2”, etc. if there is more than one.

$$a \times b = c \tag{6}$$

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