Concept Reduction Methods

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Abstract <To be prepared>

1 The Motivating Problem

The title of Adrienn Buruzs's PhD thesis [1] is "Evaluation of Sustainable Regional Waste Management Systems with Fuzzy Cognitive Map". As the title suggests, she analyzed the internal driving forces, dynamic behavior and sustainability of Integrated Waste Management Systems (IWMSs), which are very complex systems including many aspects (environmental, economic, social, institutional, legal and technical) and stakeholders. Even at an early stage of her investigations became apparent that Fuzzy Cognitive Map (FCM) is an appropriate tool to describe the large number of interacting and coupled entities and it copes with the inherent uncertainties of the system.

At first, a new FCM model [3] was created, which contained six concepts. These concepts were identified on the basis of the literature. The strength of relationships among concepts were defined by the results of a survey filled out by 75 stakeholders. The simulation results provided by FCM were validated later in [5]. Time series data were collected based on the relevant literature and it served as the input of a Bacterial Evolutionary Algorithm to learn the connection weights among the already specified concepts and parameter λ of the threshold function. The goal of optimization was to find an FCM that generates as similar time series as possible. Unfortunately, a strong contradiction was explored between the models created by experts and machine learning.

In order to resolve the experienced problem the concepts of the original model were decomposed to further 4-7 sub-concepts according to the System-of-Systems approach, which led to a very detailed, completely new model of IWMS [4]. A

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workshop was organized with the help of 12 stakeholders who decided the sub-concepts and their interconnections. The result of their work is a FCM containing 33 concepts in total (Fig. 1). Unfortunately such an extremely complex model is often confusing for the experts (Fig. 2), and to work with them may be very laborious. Note that the number of connections is a quadratic function of the number of concepts.

That is why, in general, the following approach is suggested to follow in practice: start with an obviously oversized, fine-grained model. Experts are often uncertain about the importance of system components thus it worth include most or all of them in the preliminary model. Then start reducing the model automatically, in an algorithmic way until the balance of model size and required accuracy is found. The numerical accuracy of reduced models are always lower by their nature, but it does not cause a problem in practice until the decisions suggested by them are the same. On the other hand, simpler models are easier to understand, their visual representation is clearer, and it is often more important for experts and managers. In the following sections several possible ways of model reduction is presented.

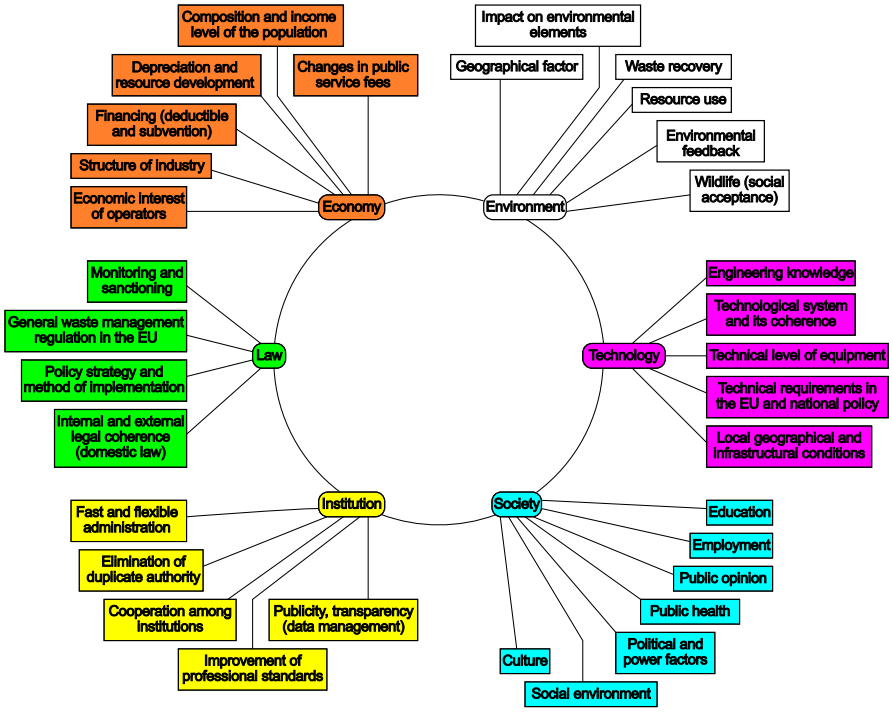

Fig. 1 The main concepts and their sub-concepts of regional IWMS.

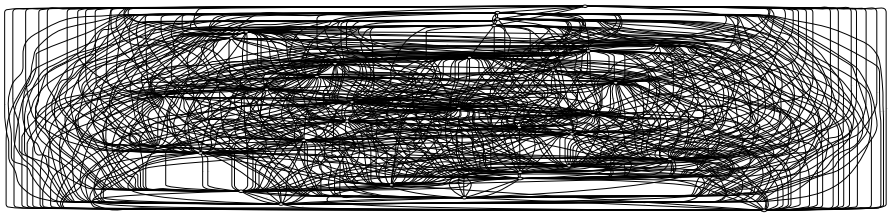

Fig. 2 The 33 concept model of regional IWMS and the relationships among its concepts. It is hard to illustrate complex FCM models appropriately and graphical model visualizations often confuse experts.

2 Early model reduction methods

Several methods had been suggested to solve the problem of oversized models before the complex model of IWMS saw the light of the day. These approaches are based on different perspectives.

In [2] an FCM is learned using historical data and its concepts are grouped into clusters in a unique way. The clustering is based on the DEMATEL [6] method. The concepts are arranged on a two dimensional plot. The vertical axis classifies concepts to cause and effect groups, the position of concepts along the horizontal axis expresses the importance of them. Based on this rearrangement of concepts two clustering methods are suggested. The first one uses K-Means clustering to create clusters according to the cause-effect behavior of concepts. The cluster centers replace the original concepts in the reduced model. The second method contains two consecutive steps, and takes also the importance of concepts into consideration. Regardless of the methods applied, experts have to define the number of clusters and they must be disjoint.

FCM was used to predict and discover knowledge about the HIV-1 drug resistance in [12]. The protease protein was modeled by FCM and causalities among sequence positions were estimated by Particle Swarm Optimization. Furthermore, Ant Colony Optimization was also applied in order to find the strongest sequence positions related to the resistance target. After that, some concepts and their connections were removed to decrease the complexity of the model until the quality of inference remained acceptable.

Another reduction method was introduced in [10]. Weak concepts and their connections were removed, then the inference capabilities of the simplified model were tested with time series data collected from real-world applications. The reduced model replaced the original if its estimation errors were acceptable.

3 Fuzzy Tolerance Relations-based reduction methods

The first results on a novel state reduction method family were presented in [8, 9]. The members of this family differ only in the applied metric, which is used to measure the "distance" of concepts from each other. The methods may be considered as generalizations of the state reduction of finite state machines and sequential systems with partially defined states. They are widely applied in digital design where the complexity of the problem makes it possible [11]. The common idea is to create clusters of identical or similar concepts, and use these clusters instead of their members in the reduced model. The methods are based on Fuzzy Tolerance Relations (FTR), which is an extension of compatibility relations among crisp concepts. The methods examine the connection weights among concepts, because they define the effect on other concepts.

3.1 Description of the main functions used for reduction

At the very beginning the new model contains exactly as many clusters as the number of concepts in the original model. These clusters are all disjoint single element sets containing one of the original concepts. In the following, the i^{th} concept is denoted by C_i, and similarly, the i^{th} cluster is denoted by K_i. The number of concepts is n. In the next steps further concepts are added to the clusters if they are "close enough" to each member of the cluster. The "distance" of concepts can be measured by various metrics. These metrics can be selected according to the specialities of the problem, and they differentiate the members of the algorithm family. Finally, some of these expanded clusters may be identical, containing exactly the same concepts. In order to keep clusters unique, only one of these clusters is retained.

One of the main functions is called $buildCluster$ (Algorithm 1). Its goal is to create a cluster that initially contains only its $initialConcept$. Later this cluster will be expanded by merger of other concepts. The second parameter, ϵ specifies the maximum allowed distance between current cluster members and the concept under examination. The value of ϵ must be in the [0, 1] interval. This parameter plays an important role in model reduction, and have to be chosen properly by experts. Low values hardly reduce the size of the model, but high values may cause oversimplification. Its appropriate value is completely problem dependent.

Function $isNearA$ is called several times in $buildCluster$ to decide whether the current concept C_i can become a member of cluster K or not. The number of function calls depends on how many member concepts do the cluster already have. This function (Algorithm 2) implements one of the possible metrics, but can be repleced by any of the metrics presented here. The last one was suggested in [7].

Metric "A" calculates the absolute difference of two connection weights, $w_{i,k}$ and $w_{j,k}$ from concept C_i, a cluster member candidate and concept C_j, a concept of cluster K to a third concept C_k, where $i \neq j \neq k$, and $i, j, k = 1, ..., n$ and n is the number of concepts. If the half of both this distance and the distance of weights

Algorithm 1 The *buildCluster* function

```
1: function BUILDCLUSTER(initialConcept, ε)
2:     K ← {initialConcept}
3:     for i ← 0; i < n; i++ do
4:         if i ≠ initialConcept then
5:             member ← true
6:             while member and HASNEXTELEMENT(K) do
7:                 j ← NEXTELEMENT(K)
8:                 member ← ISNEARA(j, i, ε)
9:             end while
10:            if member then
11:                K ← K + {i}
12:            end if
13:        end if
14:    end for
15:    return c
16: end function
```

Algorithm 2 Function *isNearA* implementing *Metric "A"*

```
1: function ISNEARA(i, j, ε)
2:     near ← true
3:     for k ← 0; k < n and near = true; k++ do
4:         if k ≠ i and k ≠ j then
5:             if |w_{i,k} − w_{j,k}| / 2 ≥ ε or |w_{k,i} − w_{k,j}| / 2 ≥ ε then
6:                 near ← false
7:             end if
8:         end if
9:     end for
10:    return near
11: end function
```

in the opposite direction are below the design variable ϵ for all C_k, C_i is added to cluster K.

Metric "B" (Algorithm 3) is a slightly modified version of Metric "A". The latter works well in most cases, but sometimes a small proportion of weight differences exceed the allowed value, and prevent the merger of concepts. The second metric provides a simple solution to this problem with the usage of parameter p. The metric allows the merger even if the distances are greater than ϵ in a small, less than p proportion of the investigated cases.

Theoretically the value of p can be any in the $[0,1]$ interval, but it should be rather low, however. High p values makes possible to practically merge any concepts regardless the value of ϵ, which propably leads to an unusable model.

Metric "B" allowed to lower the value of parameter ϵ with a simple trick, but the third approach (Metric "C", Algorithm 4) allows the omittance of the second parameter p by using the normalized, squared Euclidean distance. It is much easier to tune only one parameter instead of two in practice.

Finally, Metric "D" (Algorithm 5) works with the Manhattan-distance of concepts.

Algorithm 3 Function *isNearB* implementing *Metric "B"*

```
1: function isNearB(i, j, ϵ, p)
2:     near ← 0
3:     far ← 0
4:     for k ← 0; k < n; k++ do
5:         if k ≠ i and k ≠ j then
6:             if |w_{i,k} − w_{j,k}| / 2 < ϵ then
7:                 near ← near + 1
8:             else
9:                 far ← far + 1
10:            end if
11:            if |w_{k,i} − w_{k,j}| / 2 < ϵ then
12:                near ← near + 1
13:            else
14:                far ← far + 1
15:            end if
16:        end if
17:    end for
18:    if near = 0 or far/near ≥ p then
19:        return false
20:    else
21:        return true
22:    end if
23: end function
```

Algorithm 4 Function *isNearC* implementing *Metric "C"*

```
1: function isNearC(i, j, ϵ)
2:     sum ← 0
3:     for k ← 0; k < n; k++ do
4:         if k ≠ i and k ≠ j then
5:             sum ← sum + (w_{i,k} − w_{j,k})^2
6:             sum ← sum + (w_{k,i} − w_{k,j})^2
7:         end if
8:     end for
9:     if sum / ((n−2)*8) < ϵ then
10:        return true
11:    else
12:        return false
13:    end if
14: end function
```

The applied metrics, its parameters (ϵ, p) and also the details of implementation not specified here may affect the result of reduction. For example, if the concepts provided by *nextElement* are in various orders, the content of clusters may be different, even is the size of the reduced model remains the same. The results should be revised by experts.

The *buildCluster* function creates one of the clusters of the reduced model. Another function, *buildAllClusters* (Algorithm 6) calls *buildCluster* subsequently with different *initialConcept* values. According to the nature of the method, multi-

Algorithm 5 Function *isNearD* implementing *Metric "D"*

```
1: function isNearD(i, j, ϵ)
2:     sum ← 0
3:     for k ← 0; k < n; k++ do
4:         if k ≠ i and k ≠ j then
5:             sum ← sum + |w_{i,k} − w_{j,k}|
6:             sum ← sum + |w_{k,i} − w_{k,j}|
7:         end if
8:     end for
9:     if sum/((n−2)*4) < ϵ then
10:        return true
11:    else
12:        return false
13:    end if
14: end function
```

ple clusters may contain the same concepts. Only one the same clusters will be kept by the algorithm.

Algorithm 6 The *buildAllClusters* function

```
1: function buildAllClusters(ϵ)
2:     clusters ← {}
3:     for i ← 0; i < n; i++ do
4:         K ← buildCluster(i, ϵ)
5:         if !isElementOf(K, clusters) then
6:             clusters ← clusters + {K}
7:         end if
8:     end for
9:     return clusters
10: end function
```

Function *buildAllClusters* returns the clusters, the concepts of the reduced model, but the weights among clusters have to be defined also. Function *getWeight* (Algorithm 7) investigates the members of its parameter clusters, a and b, and calculates the average (arithmetic mean) weight of relationships between concepts included in cluster a to the concepts of cluster b. This function must be called to all possible a, b pairs to completely define the reduced model.

3.2 Theoretical background of FTR-based methods

Acknowledgements If you want to include acknowledgments of assistance and the like at the end of an individual chapter please use the `acknowledgement` environment – it will automatically be rendered in line with the preferred layout.

Algorithm 7 The *getWeight* function

```
1: function getWeight(a, b)
2:     count ← 0
3:     sum ← 0
4:     while hasNextElement(a) do
5:         i = nextElement(a)
6:         while hasNextElement(b) do
7:             j = nextElement(b)
8:             if i ≠ j then
9:                 count ← count + 1
10:                sum ← sum + w_{i,j}
11:            end if
12:        end while
13:    end while
14:    if count = 0 then
15:        return 0
16:    else
17:        return sum/count
18:    end if
19: end function
```

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