THE UNIVERSITY OF QUEENSLAND
AUSTRALIA

This exam paper must not be removed from the venue

Venue _____

Seat Number _____

Student Number |__|__|__|__|__|__|__|__|

Family Name _____

First Name _____

School of Mathematics & Physics

EXAMINATION

Semester One Final Examinations, 2016

MATH3202 Operations Research and Mathematical Planning

This paper is for St Lucia Campus students.

Examination Duration: 120 minutes

Reading Time: 10 minutes

Exam Conditions:

This is a Central Examination

This is an Open Book Examination

During reading time - write only on the rough paper provided

This examination paper will be released to the Library

Materials Permitted In The Exam Venue:

(No electronic aids are permitted e.g. laptops, phones)

Calculators - Casio FX82 series or UQ approved (labelled)

Materials To Be Supplied To Students:

1 x 14 Page Answer Booklet

Instructions To Students:

Additional exam materials (eg. answer booklets, rough paper) will be provided upon request.

There are **40 marks** available on this exam from **3** questions.

Provide your answers in the booklet provided.

For Examiner Use Only

Question	Mark

Total _____

Question 1 – Revised Simplex Algorithm

10 marks

Suppose we are solving the following linear programming problem:

$$maximise\ z = 2x_1 + x_2 + 2x_3$$

Subject to:

$$\begin{aligned} 4x_1 + 3x_2 + 8x_3 + x_4 &= 12 \\ 4x_1 + x_2 + 12x_3 + x_5 &= 8 \\ 4x_1 - x_2 + 3x_3 + x_6 &= 8 \end{aligned}$$

$$x_1, x_2, x_3, x_4, x_5, x_6 \geq 0$$

Assume we have a current basis of x_1, x_4, x_6. Demonstrate your understanding of the steps of the Revised Simplex Algorithm by answering the following:

a) What is the basic feasible solution at this stage? What is the value of the objective?

b) What is the entering variable for the next step of the Revised Simplex Algorithm?

c) What is the leaving variable?

d) What is the new value of the objective? Verify that the new solution is optimal.

e) If the right hand side of the second constraint is changed to $8 + \delta$ for some value of $\delta > 0$ will the value of z increase or decrease? By how much?

f) Assuming no other data changes, what value does the objective function coefficient of x_3 have to exceed so that x_3 is non-zero in the optimal solution?

Hint The following information may be useful:

$$\begin{bmatrix} 4 & 1 & 0 \\ 4 & 0 & 0 \\ 4 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1/4 & 0 \\ 1 & -1 & 0 \\ 0 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 3 & 0 \\ 4 & 1 & 0 \\ 4 & -1 & 1 \end{bmatrix} \begin{bmatrix} -1/8 & 3/8 & 0 \\ 1/2 & -1/2 & 0 \\ 1 & -2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Question 2 – Linear and Integer Programming

12 marks total

The state of Arcadia needs to plan its weekly waste management strategy. The basic strategy consists of collecting garbage in towns, moving it to incinerators and burning it, then moving the burnt garbage (known as debris) to landfill locations. They know the following information:

- The amount of garbage produced in each town every week, in tonnes.
- The reduction factor for each incinerator. This is a number between 0 and 1 that specifies the conversion from garbage to debris. For example, a conversion factor of 0.2 would mean that burning 10 tonnes of garbage produces 2 tonnes of debris.
- The operating cost per tonne of garbage for each incinerator.
- The capacity in tonnes per week of garbage for each incinerator.
- The haulage cost per tonne from all towns to all incinerators and from all incinerators to all landfill locations.
- The capacity in tonnes per week of debris for each landfill location.

The garbage from a town may be split across more than one incinerator and the debris from an incinerator may be split across more than one landfill.

a) Develop a linear programming model to determine the optimal allocation of garbage from towns to incinerators and debris from incinerators to landfills. Clearly define all sets, data, variables, objective function and constraints.
[9 marks]

b) Arcadia are considering introducing the restriction that each incinerator is assigned to exactly one landfill (i.e. the debris from an incinerator cannot be split over multiple landfill sites). Modify your model to handle this by the addition of binary variables and appropriate constraints. You need only specify the new variables and new constraints.
[3 marks]

Question 3 – Dynamic Programming

18 marks total

a) Consider an acyclic network defined by a set of nodes N and a set of arcs A. We know the travel time for each arc and the value for visiting each node. We wish to construct a maximum value path from a specified origin to a specified destination, subject to the constraint that the total travel time of the path is no more than a maximum duration.

Provide a dynamic programming formulation to solve this problem. You should use Bellman's equations and define data, stages, states, actions and the transition and value functions.

[9 marks]

b) At the beginning of the day a machine can be in one of two states – Good or Bad. Each morning it is possible to undertake maintenance on the machine, at a cost of $400. If the machine is Bad at the end of any day, we incur a cost of $300.

If the machine is Good at the beginning of the day, it will be Good at the end of the day with probability 0.95 if it is maintained and it will be Good with probability 0.8 if it is not maintained.

If the machine is Bad at the beginning of the day, it will be Good with probability 0.9 if it is maintained and it will be Good with probability 0.1 if it is not maintained.

Use stochastic dynamic programming to model this problem for a three day period, assuming the machine starts in the Good state. What strategy of maintenance is optimal and what is the expected cost of this strategy?

[9 marks]

END OF EXAMINATION