

Please make sure to follow the hand-in instructions. Also, please present your answers in order, showing the working for each answer. Answering yes/no is not enough. You should rather present an argument or derivation of your answer. **Tip:** Do NOT waste time on excessive computations because the quiz can be solved without requiring big computations.

1. Consider the vector $u = [1 \ 2]^T$ and the vector $v = [1 \ 10]^T$. Let the matrix A be the outer product $A = u v^T$. Determine the eigenvalues of A .
2. Now set $u = [1 \ 2 \ 3 \ 4 \ 5]^T$ and $v = [1 \ 10 \ 10^2 \ 10^3 \ 10^4]^T$. With these values determine the eigenvalues of $A = u v^T$. Hint: Remember that the sum of the eigenvalues is the trace.
3. Let B be a 3×2 matrix with $\text{rank}(B) = 2$, and set the vector $c = [0 \ 1 \ 1]^T$. Assume that,

$$B(B^T B)^{-1} B^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/2 & 1/2 \\ 0 & 1/2 & 1/2 \end{bmatrix}.$$

Let x^* be the value of x that minimizes $\|Bx - c\|$. Determine the value of Bx^* and the value of $\|Bx^* - c\|$.

4. Continuing with the same B , prove that $B^T B$ is a positive definite matrix.
5. What are the 3 eigenvalues of $B(B^T B)^{-1} B^T$? Why?
6. Consider the sequence $x(0), x(1), x(2), \dots$ of vectors in \mathbb{R}^3 with

$$x(k+1) = B(B^T B)^{-1} B^T x(k) - \frac{1}{2}x(k),$$

for $k = 1, 2, \dots$. Argue about the value of the limit,

$$\lim_{k \rightarrow \infty} \|x(k)\|.$$

Does it depend on $x(0)$? Or is it the same limit for all $x(0)$? What is the limit?