Sets: $I$ regions (1a)
$J$ settings
$K$ blends
$T$ months

Data:

$ash_{ij}$ ash content from region $i \in I$ washed at setting $j \in J$

$yield_{ij}$ yield for coal from region $i \in I$ washed at setting $j \in J$

$rmax_{it}$ max coal that can be mined in region $i \in I$ in month $t \in T$

$bmin_{kt}$ / $bmax_{kt}$ min/max blend $k \in K$ sold in month $t \in T$

$amax_k$ max ash in blend $k \in K$

$sell_k$ selling price of blend $k \in K$

$pmax_t$ wash plant max throughput in month $t \in T$

Var: $x_{ijkt}$ as per Qn text.

$$\max \sum_{\substack{i \in I, j \in J \\ k \in K, t \in T}} sell_k \, yield_{ij} \, x_{ijkt}$$

Constraints:

Max coal / region/month

$$\sum_{\substack{j \in J \\ k \in K}} x_{ijkt} \leq smax_{it} \quad \forall\, i \in I,\ t \in T$$

Min/max blend sold/month

$$bmin_{kt} \leq \sum_{\substack{i \in I \\ j \in J}} yield_{ij}\, x_{ijkt} \leq bmax_{kt} \quad \forall\, k \in K,\ t \in T$$

Washplant throughput/month

$$\sum_{\substack{i \in I \\ j \in J,\ k \in K}} x_{ijkt} \leq pmax_t \quad \forall\, t \in T$$

Ash maximum

$$\sum_{\substack{i \in I \\ j \in J}} ash_{ij}\, yield_{ij}\, x_{ijkt} \leq amax \cancel{\sum_{\substack{i \in I \\ j \in J}} yield_{ij}\, x_{ijkt}} \quad \forall\, k \in K,\ t \in T$$

$$x_{ijkt} \geq 0 \quad \forall\, i \in I,\ j \in J,\ k \in K,\ t \in T$$

(1b) Var

$y_{ijkt} \in \{0, 1\}$

1 if $ijk$ combo used in month $t$.

Constraints

Max 10 combinations/month

$$\sum_{\substack{i \in I \\ j \in J,\ k \in K}} y_{ijkt} \leq 10 \quad \forall\, t \in T$$

Link $x + y$

$$x_{ijkt} \le y_{ijkt}\, rmax_{it}$$

If used, used at least 1000.

$$x_{ijkt} \ge 1000\, y_{ijkt} \quad \rightarrow \forall i \in I, j \in J, k \in K, t \in T$$

var, data

$$\boxed{\sum_{t' \le t} spend_{t'}} \le \sum_{t' \in t} budget_{t'} \qquad \forall t \in T$$

$\forall t' \in T$
$t' \le t$

$$\cancel{unspent_0}\ s_0 = 0$$

data

$$spend_t + s_t = budget_t + s_{t-1}$$

sum of variables

3a

<u>Data.</u> $p_{ij}$ Prob of winning election $i$ if we spend \$$j$ million

<u>Stage.</u> $t$, election about to be held.

<u>State.</u> $m_t$ money on hand in millions

$w_t$ elections already won

<u>Action</u> $a_t$ amount to spend on election $t$ in millions.

<u>Transition</u>. $m_{t+1} = m_t - a_t$

$$w_{t+1} = \begin{cases} w_t & \text{if lose election } (1-p_{ta_t}) \\ w_t + 1 & \text{if win election } (p_{ta_t}) \end{cases}$$

<u>Value Fn</u>: $V_t(m_t, w_t)$ is the max prob win 3 or more elections, given we start election $t$ with \$$m_t$ million and $w_t$ previous wins.

Solution of problem $V_1(10, 0)$

<u>End Cases</u>

$$V_t(m_t, w_t) = \begin{cases} 1 & \text{if } w_t \geq 3 \\ 0 & \text{if } t \geq 5 \ \& \ w_t < 3 \end{cases}$$

<u>General Cases</u>

$$V_t(m_t, w_t) = \max_{0 \leq a_t \leq \min(5, m_t)} \begin{pmatrix} p_{ta_t} \cdot V_{t+1}(m_t - a_t, w_t + 1) \\ + (1 - p_{ta_t}) \cdot V_{t+1}(m_t - a_t, w_t) \end{pmatrix}$$

3b

$$
V_3(4,2) = \begin{array}{cll}
a_t & \text{Win} & \text{Lose} \\
0 & 0.3\, V_4(4,3) & +\ 0.7\, V_4(4,2) \\
1 & 0.5\, V_4(3,3) & +\ 0.5\, V_4(3,2) \\
2 & 0.65\, V_4(2,3) & +\ 0.35\, V_4(2,2) \\
3 & 0.7\, V_4(1,3) & +\ 0.3\, V_4(1,2) \\
4 & 0.75\, V_4(0,3) & +\ 0.25\, V_4(0,2)
\end{array}
$$

Substitute:

$V_4(m,3) = 1$

$\boxed{V_4(m,2) = p_{4m}}$ $\quad m \leq 5$

$$
\begin{array}{cl}
0 & 0.3 + 0.7 * 0.9 = 0.93 \\
1 & 0.5 + 0.5 * 0.88 = 0.94 \\
2 & 0.65 + 0.35 * 0.85 = 0.9475 \\
3 & 0.7 + 0.3 * 0.8 = 0.94 \\
4 & 0.75 + 0.25 * 0.7 = 0.925
\end{array}
$$

Spend \$2 million on Election 3, for a 0.9475 prob. of winning 3 or more elections.