THE UNIVERSITY OF QUEENSLAND
AUSTRALIA

This exam paper must not be removed from the venue

Venue ____________________

Seat Number ________

Student Number |__|__|__|__|__|__|__|__|

Family Name _____________________

First Name _____________________

School of Mathematics & Physics

EXAMINATION

Semester One Final Examinations, 2014

MATH3202 Operations Research and Mathematical Planning

This paper is for St Lucia Campus students.

Examination Duration: 120 minutes

Reading Time: 10 minutes

Exam Conditions:

This is a Central Examination

This is an Open Book Examination

During reading time - writing is not permitted at all

This examination paper will be released to the Library

Materials Permitted In The Exam Venue:

(No electronic aids are permitted e.g. laptops, phones)

Calculators - Casio FX82 series or UQ approved (labelled)

Materials To Be Supplied To Students:

1 x 14 Page Answer Booklet

Instructions To Students:

There are **30** marks available on this exam from **3** questions.

Provide your answers in the booklet provided.

For Examiner Use Only

Question	Mark

Total ________

Question 1

8 marks

Suppose we are solving the following linear programming problem:

$$maximise\ z = 5x_1 + 4x_2 + 3x_3$$

Subject to:

$$\begin{aligned} 2x_1 + 3x_2 + x_3 + x_4 &= 5 \\ 4x_1 + x_2 + 2x_3 + x_5 &= 11 \\ 3x_1 + 4x_2 + 2x_3 + x_6 &= 8 \end{aligned}$$

$$x_1, x_2, x_3, x_4, x_5, x_6 \geq 0$$

Assume we have a current basis of x_1, x_5, x_6. Demonstrate your understanding of the steps of the Revised Simplex Algorithm by answering the following:

a) What is the basic feasible solution at this stage? What is the value of the objective?

b) What is the entering variable for the next step of the Revised Simplex Algorithm?

c) What is the leaving variable?

d) What is the new value of the objective? Verify that the new solution is optimal.

e) If the right hand side of the first constraint is changed to $5 + \delta$ for some value of $\delta > 0$ will the value of z increase or decrease? By how much?

f) Assuming no other data changes, what value does the objective function coefficient of x_2 have to exceed so that x_2 is non-zero in the optimal solution?

Hint The following information may be useful:

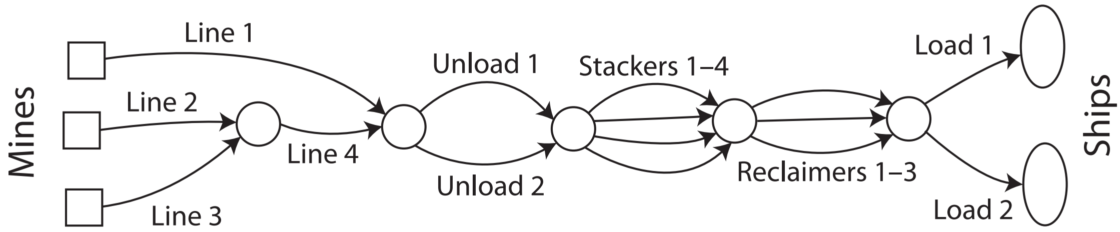
$$\begin{bmatrix} 2 & 0 & 0 \\ 4 & 1 & 0 \\ 3 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1/2 & 0 & 0 \\ -2 & 1 & 0 \\ -3/2 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 & 0 \\ 4 & 2 & 1 \\ 3 & 2 & 0 \end{bmatrix} \begin{bmatrix} 2 & 0 & -1 \\ -3 & 0 & 2 \\ -2 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Question 2

12 marks total

Throughput of coal from mine to port to ship is a critical issue for Australia's largest coal systems. Press reports of huge queues of ships waiting to be loaded are common and these queues are expensive for the industry. A typical but simple diagram for a coal system is shown below.



More generally, we can describe a coal system using a collection of nodes and arcs. Some of the nodes are source nodes (mines) and some are sink nodes (ships). For each arc we know the origin and destination nodes and the maximum weekly throughput of the arc.

a) Develop a linear programming model of a general coal system that determines how much coal to move on each arc so as to maximise the total throughput. This throughput is the total amount of coal moved out of the source nodes, which will be equal to the total amount of coal moved into the sink nodes. For all other nodes the total amount of coal moved into the node will be same as the total amount of coal moved out of the node.

   Clearly define all sets, data, variables, objective function and constraints.
   [6 marks]

b) In order to keep the system running smoothly, it needs to be maintained. Assume we are given a set of maintenance tasks applying to the arcs, with at most one task for each arc. For each arc we know whether or not it has a maintenance task and the effort (in man days) for the maintenance task.

   We wish to schedule all the known maintenance tasks over the next *T* weeks. For each week we know the maximum man days available for maintenance, which may vary from week to week.

   Assume that each maintenance task must be started and finished in the same week, and that when an arc is being maintained its throughput goes down to 0 for the whole week.

   Develop a mixed integer programming model to produce a maintenance schedule for the next *T* weeks so as to maximise the total throughput. Once again clearly define all sets, data, variables, object function and constraints.
   [6 marks]

Question 3

10 marks total

a) Suppose stamps can be bought in denominations of \$0.10, \$0.70 and \$1.20. Design a dynamic programming formulation that will find the minimum number of stamps necessary for a postage value of \$x (in any multiple of \$0.10). Show how your formulation works to calculate the minimum number of stamps necessary for a postage value of \$1.40. *[5 marks]*

b) Suppose Michelle currently has \$2 and is allowed to play a game of chance three times. If she bets *b* dollars on a play of the game then with probability 0.4 she wins *b* dollars while with probability 0.6 she loses *b* dollars. (Each bet must be in a whole number of dollars. She can choose to bet \$0 on a game.) Use stochastic dynamic programming to find her maximum probability of having at least \$5 after the three games. What strategy of bets should she use to achieve this? *[5 marks]*

END OF EXAMINATION