MATH3202/7232 Operations Research & Mathematical Planning 2020

Week 10 – Parking and Web Building

Optimal Stopping

Jenny drives along a straight road towards a particular shop, looking for a vacant parking space. Vacancies occur at random, on average once in every 10 places. Once past the shop, Jenny will take the next available space. But at what point in approaching the shop should she accept a vacant space?

Web Building

Female orb-weaving spiders need to gain energy to grow in weight from 35 mg, at their sexual maturity, to 80 mg, where they can lay a first batch of eggs. These spiders are sit-and-wait predators, building a web to catch prey and gain energy. They replace their web every day and so can change the size of web they use. Larger webs are more likely to catch prey but require more energy to build and more time in the open, making them more vulnerable to their own predators.

Suppose the spiders have a choice of building webs where the total length of the sticky spiral is 4 m, 8 m or 12 m. Building a web of size w requires an energy cost of

$$\alpha_w(s) = -0.125\, w + 0.005\, w\, s,$$

where s is the current weight of the spider in mg. Each day a spider also has a basal metabolic expenditure, α_m, of 0.4 mg.

The table below shows the daily probability, λ_w, of catching a prey and the daily probability, β_w, that the spider will be eaten for the different web sizes.

Web size, w	4	8	12
λ_w	0.66	0.77	0.82
β_w	0.01	0.02	0.03

The energy value of a prey item is 6 mg. We assume that at most one prey can be caught each day. If the weight of a spider drops below 25 mg then it dies from starvation.

At the end of each day let s be the weight of the spider in mg with $p = s - \lfloor s \rfloor$. Then the weight of the spider at the start of the next day will be $\lfloor s \rfloor + 1$ with probability p and $\lfloor s \rfloor$ with probability $1 - p$. (This keeps the possible states more manageable.)

Suppose a spider starts a day with weight s. What is the maximum probability that she will reach the egg-laying weight of 80 mg within the next 10 days? What strategy should she pursue to achieve this?

Reference

This model is a finite-time version of one presented in Venner, S., Chadès, I., Bel-Venner, M., & Pasquet, A. (2006). Dynamic optimization over infinite-time horizon: Web-building strategy in an orb-weaving spider as a case study. *Journal of Theoretical Biology* doi:10.1016/j.jtbi.2006.01.008.