

Please make sure to follow the hand-in instructions. Also, please present your answers in order, showing the working for each answer. Answering yes/no is not enough. You should rather present an argument or derivation of your answer.

Consider the vectors $v_1 = [-1 \ 0 \ -1]^T$, $v_2 = [0 \ -1 \ 1]^T$, and $v_3 = [0 \ 1 \ 1]^T$. Let the 3×3 matrix A have columns v_1 , v_2 , and v_3 . That is $A = [v_1 \ v_2 \ v_3]$.

1. Do the vectors v_1 , v_2 , and v_3 constitute a set of **orthogonal** vectors?
2. Are the vectors v_2 and v_3 **orthogonal**?
3. Execute the Gram-Schmidt procedure on the vectors v_1 , v_2 , and v_3 in this order. Present the output as columns of an orthogonal matrix Q .
4. Determine Q^{-1} .
5. What is the nullspace of the matrix A ?
6. What is the rank of the matrix A ?
7. Are the vectors v_1 , v_2 , and v_3 a basis for \mathbb{R}^3 ?
8. Now execute the Gram-Schmidt procedure on the vectors v_3 , v_2 , and v_1 in the order $(3 \rightarrow 2 \rightarrow 1)$. Present the output as the columns of an orthogonal matrix M .
9. Define the matrix $B = QM$. Is B an orthogonal matrix?
10. Consider now the 9×9 block matrix,

$$G = \begin{bmatrix} Q & 0 & 0 \\ 0 & M & 0 \\ 0 & 0 & Q \end{bmatrix},$$

where each 0 is a 3×3 matrix of zero values. What is the inverse matrix of G ?