Betting Strategy

Suppose Michelle currently has \$2 and is allowed to play a game of chance three times. If she bets b dollars on a play of the game then with probability 0.4 she wins b dollars while with probability 0.6 she loses b dollars. (Each bet must be in a whole number of dollars. She can choose to bet \$0 on a game.) Suppose Michelle wants to maximise her probability of having at least \$5 after the three games. What strategy of bets should she use to achieve this?

Data p probability of winning.

Stages Games $j \in \{0, 1, 2\}$

State Money s_j at start of game j.

Actions Money b_j to bet on game j.

Value Function

$V_j(s_j)$ = maximum probability of at least \$5 after three games if we start game j with \$$s_j$.

We want $V_0(2)$

$$V_3(s) = \begin{cases} 1, & \text{if } s \geq 5 \\ 0, & \text{if } s < 5 \end{cases}$$

$$V_j(s_j) = \max_{0 \leq b_j \leq s_j} \{ p\, V_{j+1}(s_j + b_j) + (1-p)\, V_{j+1}(s_j - b_j) \}$$