THE UNIVERSITY OF QUEENSLAND
AUSTRALIA

This exam paper must not be removed from the venue

Venue _____

Seat Number _____

Student Number |__|__|__|__|__|__|__|__|

Family Name _____

First Name _____

School of Mathematics & Physics

EXAMINATION

Semester One Final Examinations, 2018

MATH3202 Operations Research and Mathematical Planning (Theory)

This paper is for St Lucia Campus students.

Examination Duration: 120 minutes

Reading Time: 10 minutes

Exam Conditions:

This is a Central Examination

This is an Open Book Examination

During reading time - write only on the rough paper provided

This examination paper will be released to the Library

Materials Permitted In The Exam Venue:

(No electronic aids are permitted e.g. laptops, phones)

Calculators - Casio FX82 series or UQ approved (labelled)

Materials To Be Supplied To Students:

1 x 14 Page Answer Booklet

Instructions To Students:

There are **40** marks available on this exam from **3** questions.

Provide your answers in the booklet provided.

Additional exam materials (eg. answer booklets, rough paper) will be provided upon request.

For Examiner Use Only

Question	Mark

Total _____

There was an error in question 1 of the 2018 exam. An extra simplex iteration was required to get the optimal solution.

Question 1 – Revised Simplex Algorithm

10 marks

Suppose we are solving the following linear programming problem:

$$maximise\ z = 35x_1 + 60x_2$$

Subject to:

$$\begin{aligned} 8x_1 + 12x_2 + x_3 &= 120 \\ 15x_2 + x_4 &= 60 \\ 3x_1 + 6x_2 + x_5 &= 48 \end{aligned}$$

$$x_1, x_2, x_3, x_4, x_5 \geq 0.$$

Assume we have a current basis of x_2, x_3, x_5. Demonstrate your understanding of the steps of the Revised Simplex Algorithm by answering the following:

a) What is the basic feasible solution at this stage? What is the value of the objective? *[2 marks]*

b) What is the entering variable for the next step of the Revised Simplex Algorithm? *[1 mark]*

c) What is the leaving variable? *[1 mark]*

d) What is the new value of the objective? Verify that the new solution is optimal. *[2 marks]*

e) If the right-hand side of the third constraint is changed to $48 + \delta$ for some value of $\delta > 0$, what will happen to the value of z? *[2 marks]*

f) Assuming no other data changes, what value does the objective function coefficient of x_1 have to reduce to so that x_1 is zero in the optimal solution? *[2 marks]*

Hint The following information may be useful:

$$\begin{bmatrix} 8 & 12 & 1 \\ 0 & 15 & 0 \\ 3 & 6 & 0 \end{bmatrix} \begin{bmatrix} 0 & -\frac{2}{15} & \frac{1}{3} \\ 0 & \frac{1}{15} & 0 \\ 1 & \frac{4}{15} & -\frac{8}{3} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 12 & 1 & 0 \\ 15 & 0 & 0 \\ 6 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & \frac{1}{15} & 0 \\ 1 & -\frac{4}{5} & 0 \\ 0 & -\frac{2}{5} & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Question 2 – Linear and Integer Programming

16 marks total

A hospital seeks your advice to schedule operations for a day. Each day is divided into four time slots for operations. You are also given the following information:

- the set of operating theatres;
- the set of surgeons; and
- the set of operations.

For each operation you are given:

- the operating rooms to which it can be assigned;
- the surgeons to which it can be assigned;
- the duration of the operation as either 1, 2, 3 or 4 time slots; and
- a measure of the cost of not doing the operation today.

Once an operation starts, it must continue uninterrupted for its duration with the same surgeon in the same operating theatre.

The hospital wants to allocate operations to time slots, operating theatres and surgeons so as to minimise the cost of the unallocated operations. A surgeon can be assigned to at most one operation in each time slot. An operating theatre can be assigned to at most one operation in each time slot.

a) Assuming that each operation has a duration of just one time slot, develop an integer programming model to determine the optimal allocation of operations to surgeons, operating theatres and time slots. Clearly define all sets, data, variables, objective function and constraints. You may wish to use binary variables x_{ijkt} to indicate that operation i is done in operating theatre j by surgeon k in time slot t, with other variables as required. *[10 marks]*

b) Extend your model to allow for operations with a duration of several time slots. You need only specify the constraints that have been altered. *[4 marks]*

c) The hospital wishes to impose a further rule that surgeons can only operate for three time slots, unless they are doing an operation which is four time slots long. Modify your model to include this constraint. *[2 marks]*

Question 3 – Dynamic Programming

14 marks total

Jenny likes to listen to music when she works out at the gym, as it helps to motivate her. She uses the streaming music service Spotify on her phone but sadly does not have a premium account, so Spotify will randomly suggest songs for her to listen to that are not on her playlist. She can skip those songs but has a limited number of skips to use.

Each time she goes to the gym, her workout lasts for 12 songs. Songs in her **playlist** motivate her the most. Of the suggested songs, they can be **good**, **neutral** or **bad**. Good songs are ones that still motivate her, though not as much as the workout songs in her playlist. Neutral songs don't hurt her routine but don't provide much motivation either. Bad songs actively sap her motivation and should be avoided. The probability of each song type and associated motivational value are given in the following table:

Song Type j	**Probability** p_j	**Motivational Value** m_j
Playlist	0.5	10
Good	0.2	5
Neutral	0.1	2
Bad	0.2	-2

Spotify will only allow Jenny to skip five songs during her workout.

a) Provide a general dynamic programming formulation to solve Jenny's problem. You should use Bellman's equation and identify the data, stages, states, actions and the transition and value functions. *[8 marks]*

b) Suppose that Jenny has three songs left to play and still has one skip remaining. A **good** song starts – should she skip it? Use your formulation to calculate the relevant function values to support your decision. *[6 marks]*

END OF EXAMINATION