MATH3202/7232 Operations Research & Mathematical Planning 2020

Tutorial 12 – Optimal Keyboard Layout

A standard English keyboard has three rows, with 10, 9 and 7 keys on each. We will label these keys 0, …, 25, with keys 0,…,9 on the first row, 10,…,18 on the second row, and 19,…,25 on the third row.

Suppose we know the average time, t_{ij}, to press key j immediately after pressing key i. For example, if $t_{0,1} = 66$ then it takes 66 time units to press the second key ('W' on a regular keyboard) after pressing the first key ('Q' on a regular keyboard).

Furthermore, suppose we have data, f_{ab}, which gives the frequency of each pair of letters ab in 100,000 pairs of letters in a corpus of text. For example, $f_{CE} = 643$ indicates that the pair 'CE' occurred 643 times. Thus, if we had assigned 'C' to the first key of the keyboard and 'E' to the second key, it would take $643 \times 66 = 42438$ time units to type the 643 'CE' pairs. From this, the total time taken to type the 100,000 pairs would be

$$T = \sum_{i,j=0}^{25} t_{ij}\ f_{p(i)p(j)}$$

where $p(k)$ is the letter assigned to key k.

A Python stub on Blackboard gives values for t_{ij} and f_{ab}, along with the function for calculating the total cost for a particular permutation $p(k)$.

Use simulated annealing to determine an optimal layout for the 26 English letters on the keyboard. (For comparison, the regular QWERTY keyboard has a total cost of 5,863,759 time units from this data.)

Reference

This problem was originally presented by Burkard, R.E. & Offermann, J. (1977) Entwurf von Schreibmaschinentastaturen mittels quadratischer Zuordnungsprobleme. *Zeitschrift für Operations Research* 21: B121. doi:10.1007/BF01918175

The data was obtained from the Quadratic Assignment Problem Library (QAPLIB) at http://anjos.mgi.polymtl.ca/qaplib/inst.html