THE UNIVERSITY OF QUEENSLAND
AUSTRALIA

This exam paper must not be removed from the venue

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Seat Number ________

Student Number |__|__|__|__|__|__|__|__|

Family Name ______________________

First Name ______________________

School of Mathematics & Physics

EXAMINATION

Semester One Final Examinations, 2013

MATH3202 Operations Research and Mathematical Planning

This paper is for St Lucia Campus students.

Examination Duration: 120 minutes

Reading Time: 10 minutes

Exam Conditions:

This is a Central Examination

This is an Open Book Examination

During perusal - writing is not permitted at all

This examination paper will be released to the Library

Materials Permitted In The Exam Venue:

(No electronic aids are permitted e.g. laptops, phones)

Calculators - Casio FX82 series or UQ approved (labelled)

Materials To Be Supplied To Students:

1 x 14 Page Answer Booklet

Instructions To Students:

There are **30** marks available on this exam from **3** questions.

Provide your answers in the booklet provided.

For Examiner Use Only

Question	Mark

Total ________

Question 1

8 marks

Suppose we are solving the following linear programming problem:

$$maximise\ z = x_1 + 9x_2 + x_3$$

Subject to:

$$\begin{aligned} x_1 + \ 2x_2 + 3x_3 &\leq 9 \\ 3x_1 + 2x_2 + 2x_3 &\leq 15 \\ 2x_1 + 3x_2 + x_3 &\leq 14 \end{aligned}$$

$$x_1, x_2, x_3 \geq 0$$

Assume we have a starting basis of all slack variables. Demonstrate your understanding of the steps of the Revised Simplex Algorithm by answering the following:

a) What is the basic feasible solution at this stage? What is the value of the objective?

b) What is the entering variable for the next step of the Revised Simplex Algorithm?

c) What is the leaving variable?

d) What is the new value of the objective? Verify that the new solution is optimal.

e) If the right hand side of the 2nd constraint is changed to $15 + \delta$ for some value of $\delta > 0$ will the value of z increase or decrease? By how much?

f) Assuming no other data changes, what value does the objective function coefficient of x_3 have to exceed so that x_3 is non-zero in the optimal solution?

Hint The following information may be useful:

$$\begin{bmatrix} 2 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1/2 & 0 & 0 \\ -1 & 1 & 0 \\ -3/2 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Question 2

12 marks total

A fertiliser company imports its main raw material, Ammonium Phosphate (AP), into a number of ports in Australia. At each port they know the following information:

- Maximum amount of AP that can be stored there
- Initial amount of AP in stock
- The demand for AP in every month in the planning period
- The cost per tonne per month of storing AP

There are a number of different options for ships that the company can use to import AP. For each different type of ship they know:

- The maximum tonnage the ship can carry
- The cost of ordering a ship of that type
- The cost of stopping at a port for a ship of that type

(For example, if a ship of type 1 is used in a particular month, it costs $100,000 plus $10,000 per port visited. So if in May the company orders a ship of this type to stop at 4 ports, and in September it orders a ship of this size to stop at 2 ports, their cost is $140,000 in May and $120,000 in September.)

All amounts of AP and capacities are given in tonnes.

The company needs to come up with a shipping plan for the planning period. At most one ship in total can be ordered in any month, though it can visit as many ports as are required. The shipping plan will show what size ship (if any) is ordered in a month, which ports it visits and how much AP is unloaded at each port.

a) Formulate the AP shipping problem as a mixed integer programming problem. You should explain your notation for sets, parameters, variables, objective function and constraints so that they can easily be linked back to the problem description. *[8 marks]*

b) In practice there are multiple raw material types to be imported, with demand by month and storage capacity information for each combination of raw material and port. Also, each ship type has a small number of holds of known capacity (adding up to the capacity of the ship) and only one type of raw material can be stored in each hold. Describe in words or with a formulation how to extend your model to cover these extra details. *[4 marks]*

Question 3

10 marks total

a) A company must meet the following demands on time:

Month	*July*	*August*	*September*	*October*
Demand	1	1	2	2

It costs \$4 to place an order (regardless of the amount) and a \$2 per-unit holding cost is assessed against each month's ending inventory. At the beginning of July there are 0 units in stock.

Use dynamic programming to determine an ordering policy that minimises total cost. *[6 marks]*

b) Suppose now that demands in (a) need not be met on time. Assume that all lost demand is backlogged and that a \$1 per-unit shortage cost is assessed against the number of shortages incurred during each month. All demand must be met by the end of October.

Use dynamic programming to determine a revised ordering policy that minimises total cost. *[4 marks]*

END OF EXAMINATION