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New Linguistic Z-Number Petri Nets for Knowledge Acquisition and Representation Under Large Group Environment

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Abstract In this paper, we develop a new model named linguistic Z-number Petri nets for knowledge acquisition and representation in the large group environment. First, the linguistic Z-number production rules are introduced for knowledge representation, where truth degrees, threshold values, and certainty factors are described by linguistic Z-numbers. Subsequently, a knowledge acquisition approach is put forward to obtain the knowledge parameters of linguistic Z-number Petri nets based on a large group of experts. To reduce the complexity of knowledge reasoning, a simplification method is proposed to simplify the structure of linguistic Z-number Petri nets. Finally, a real case of chemical security risk assessment is provided to demonstrate the practicability and effectiveness of the proposed linguistic Z-number Petri net model. The results show that risk level of the given chemical plant is high and efficient actions should be taken to identify threat drivers and reduce security risk. Moreover, through a sensitivity analysis and a comparative analysis, it is concluded that the proposed linguistic Z-number Petri nets can represent experts' complex and uncertain cognitive information comprehensively and acquire more precise and reasonable knowledge from domain experts.

Keywords Expert system · Linguistic Petri net · Linguistic Z-number · Knowledge representation · Knowledge acquisition

1 Introduction

With the development of artificial intelligence, applying expert systems to simulate human thinking has become a hot research topic nowadays. Expert system is an intellectual programming system that uses the knowledge captured from experts to solve specific problems reaching the level of experts [1]. Two crucial issues in developing an expert system are the acquisition of experts' professional knowledge and the representation of the obtained knowledge rules. So far, various knowledge representation methods have been introduced in the literature, which include multilevel flow modelling [2], ontology [3], hierarchical censored production rules [4], dynamic uncertain causality graph [5], and domain knowledge graph [6]. Combining fuzzy logic with Petri nets, the fuzzy Petri nets (FPNs) [7] are a promising mathematical modeling tool for knowledge representation and reasoning of expert systems. The FPN is a marked graph in which places represent propositions, transitions represent inference rules, and directed arcs represent the relationships between places and transitions [8]. It provides a unified form to deal with imprecise, vague or uncertain knowledge information [9]. Recently, the FPNs have received considerable attention from researchers and practitioners and been extensively applied to many fields, such as fault diagnosis method of distribution network [10], security risk assessment of civil aviation airport [11], quantitative modelling of biological systems [12], aluminium electrolysis cell condition identification [13], and failure system cause analysis [14].

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In traditional FPNs, fuzzy set theory was used for handling the vagueness of real-world knowledge. Nevertheless, due to the increasingly complexity of expert systems, the fuzzy sets used in previous studies cannot reflect the reliability of knowledge information [15]. Moreover, domain experts often express their judgements using natural language because of the inherent vagueness of human cognition and the lack of data and experience [13, 16]. To depict the fuzziness and randomness of linguistic decision-making information, the concept of linguistic Z-numbers was suggested by Wang et al. [17] based on Z-numbers [18] and linguistic term sets. It is an ordered pair of fuzzy numbers (A, B), in which A represents the fuzzy constraint of information and B represents the degree of reliability of A [19]. For a linguistic Z number, the two fuzzy numbers A and B are described with linguistic expressions [20]. As a result, the linguistic Z-numbers can represent uncertain linguistic information and consider the reliability of experts, and its description is more consistent with human cognition [21, 22]. Because of these advantages, the linguistic Z-numbers have been employed to model the uncertainty in various decision-making problems, e.g., green supplier selection [23], concept evaluation of information axiom [24], failure mode risk analysis [25], railway passenger train causes analysis [26], and quality function deployment [27].

On the other hand, knowledge acquisition is the first and one critical step in developing an expert system. However, few researches have been conducted on how to determine knowledge parameters of FPNs in the past. In this point, Liu et al. [8] suggested the use of fuzzy evidential reasoning method to describe the experience and knowledge of experts. Li et al. [28] put forward an interval-valued intuitionistic FPN model for acquiring and sharing tacit knowledge in knowledge intensive organizations. Xu et al. [29] introduced a picture FPN method to represent imprecise knowledge considering experts' conflicting opinions, and Xu et al. [30] proposed a bipolar FPN model for extracting knowledge from domain experts considering their non-cooperative behaviors. With the increasing complexity of expert systems, it is more reasonable to acquire knowledge based on a large number of experts [31, 32], which leads to the large group knowledge acquisition problem. Moreover, it is necessary to apply a consensus reaching method during the knowledge acquisition process [33, 34], since experts usually differ in backgrounds, knowledge structure, and professional experience in the large group.

Considering the above analyses, this article aims to put forward a new type of FPNs, called linguistic Z-number Petri nets (LZPNs), to enhance the performance of traditional FPNs in knowledge representation and reasoning. In summary, the primary contributions of this study are highlighted as follows:

(1) The linguistic Z-number production rules are defined based on linguistic Z-numbers to express experts' knowledge information, which can depict both fuzziness and randomness of uncertain knowledge and capture the reliability of experts' judgments.
(2) A new large group knowledge acquisition approach is proposed to acquire the knowledge parameters of LZPNs with a consensus reaching process. Based on a large number of experts with different backgrounds, the knowledge rules obtained will be acceptable and reliable and the discrepancies among experts can be minimized.
(3) The existing knowledge reasoning algorithms of FPNs are facing a challenge for large and complex expert systems. Thus, a simplification method is introduced to simplify the structure of LZPNs and reduce the complexity of knowledge reasoning.
(4) A practical example concerning security risk assessment in a chemical plant is given to illustrate the proposed LZPN model. Furthermore, a sensitivity analysis and a comparison analysis with previous methods are performed to reveal its effectiveness and superiority.

The rest part of this paper is organized as follows: Section 2 presents the definitions of traditional FPNs and reviews the related works on FPNs. Section 3 presents the basic concepts of linguistic Z-numbers. The new LZPN model and a large group knowledge acquisition method are provided in Sect. 4. A security risk assessment case is implemented in Sect. 5 to exemplify the developed LZPNs. Finally, conclusions of this study and possible future works are provided in Sect. 6.

2 Related work

2.1 Definition of FPNs

The FPNs were first proposed in [35] by integrating the graphical power of Petri nets and competencies of fuzzy sets to represent fuzzy production rules (FPRs) of an expert system [36]. Later, Chen et al. [7] redefined the FPN model in a structural way so as to describe knowledge precisely. The formal definition of an FPN is an 8-tuple shown as follows [7]:

$$\mathrm{FPN} = (P, T, D, I, O, f, \alpha, \beta), \tag{1}$$

where

(1) $P = \{p_1, p_2, \ldots, p_m\}$ is a finite set of places;

(2) $T = \{t_1, t_2, \ldots, t_n\}$ is a finite set of transitions;
(3) $D = \{d_1, d_2, \ldots, d_m\}$ denotes a finite set of propositions with $P \cap T \cap D = \emptyset$ and $|P| = |D|$;
(4) $I : T \to P^{\infty}$ denotes an input function, a mapping from transitions to bags of places. If an arc can be found between transition t_j and input places p_i, then $I_{ij} = 1$, otherwise $I_{ij} = 0$, $i = 1, 2, \ldots, m, j = 1, 2, \ldots, n$;
(5) $O : T \to P^{\infty}$ denotes an output function, a mapping from transitions to bags of places. If an arc can be found between transition t_j and output place p_i, then $O_{ij} = 1$, otherwise $O_{ij} = 0$, $i = 1, 2, \ldots, m, j = 1, 2, \ldots, n$;
(6) $f : T \to [0, 1]$ denotes an association function, a mapping from transitions to certainty values between 0 and 1;
(7) $\alpha : P \to [0, 1]$ denotes an association function, a mapping from places to certainty values between 0 and 1;
(8) $\beta : P \to D$ denotes an association function, a bijective mapping from places to propositions.

The FPRs are expressed in the form of IF–THEN rules to capture imprecise and vague knowledge in expert systems [1]. To strengthen the representation and reasoning capabilities of FPRs, Yeung and Tsang [37] further introduced weighted FPRs, which can be divided into five types as follows:

Type 1 A simple weighted FPR
R: IF d_j THEN d_k $(\mu; \lambda; w)$;
Type 2 A composite weighted fuzzy conjunctive rule in the antecedent
R: IF d_{j1} AND d_{j2} AND…AND d_{jm} THEN d_k $(\mu; \lambda_1, \lambda_2, \ldots, \lambda_m; w_1, w_2, \ldots, w_m)$;
Type 3 A composite weighted fuzzy conjunctive rule in the consequent
R: IF d_j THEN d_{k1} AND d_{k2} AND…AND d_{km} $(\mu; \lambda; w)$;
Type 4 A composite weighted fuzzy disjunctive rule in the antecedent
R: IF d_{j1} OR d_{j2} OR…OR d_{jm} THEN d_k $(\mu; \lambda_1, \lambda_2, \ldots, \lambda_m; w_1, w_2, \ldots, w_m)$;
Type 5 A composite weighted fuzzy disjunctive rule in the consequent:
R: IF d_j THEN d_{k1} OR d_{k2} OR…OR d_{km} $(\mu; \lambda; w)$.

Note that d_j and d_k denote the antecedent proposition and the consequent proposition with fuzzy variables, respectively. The parameter $\mu \in [0, 1]$ is the certainty factor of the rule;λ and w are threshold value and weight specified for the antecedent proposition, respectively. In addition, rules of Types 4 and 5 will not be discussed in the following sections because they can be converted into several rules of Type 1.

2.2 FPN Improvement

In the literature, many alternative FPN methods were proposed to improve the performance of traditional FPNs [38]. On the one hand, various uncertainty methods have been integrated into FPNs for depicting vague and imprecise knowledge information. For example, fuzzy numbers were used in the weighted FPN model [39] for risk assessment considering veto factors. The intuitionistic fuzzy sets were used in [40] to deal with uncertain and imprecise knowledge, and the bipolar fuzzy sets were adopted in [30] for knowledge representation and acquisition. A knowledge representation scheme based on FPNs and genetic-particle swarm optimization was designed by Wang et al. [41] for the dynamic representation of fuzzy knowledge. A generalized weighted intuitionistic FPN model was proposed by Suraj [42], in which the truth values and the weights of places were represented by intuitionistic fuzzy numbers. Chiachío et al. [43] presented a model named plausible Petri nets to represent plausible (uncertain) information in pervasive computing environments. Combining interval-valued intuitionistic sets with cloud model, an extended Petri net model was constructed in [44] to cope with the subway fire risk assessment problem. Yue et al. [45] utilized the interval type-2 fuzzy sets to capture fuzzy knowledge information, and Yue et al. [9] employed the interval-valued intuitionistic fuzzy sets to describe the uncertainty of experts' experiential cognition. In addition, the cloud model theory [36], the linguistic 2-tuples [16], the interval type-2 fuzzy sets [46], and the fuzzy evidential reasoning method [47] have been combined with FPNs for uncertain knowledge representation.

On the other hand, some scholars focused on knowledge reasoning and proposed different advanced FPN methods. For instance, Yue et al. [13] suggested an extended FPN model with an interval type-2 fuzzy ordered weighted averaging operator to improve the knowledge reasoning capability of FPNs. Considering the changeable arc weights in knowledge reasoning, Amin and Shebl [48] proposed an adaptive fuzzy higher order Petri net model to dynamically adjust the weights of an expert system. To analyze coupling relationship evidently, a layered FPN method was suggested by Li et al. [49] to deal with equipment failure risk assessment. An improved FPN method based on attribute theory and attribute granular computing was put forward in [50] for knowledge representation and reasoning under the variable fuzzy criterion. An intelligent Petri net model was proposed in [51] by incorporating fuzzy rules to Petri nets to describe the behavior of adaptive systems. Besides, the equivalent transformation algorithm [52] and the decomposition algorithm [53] were suggested to address the state space explosion issue of FPNs in knowledge inference.

Zhang et al. [54] introduced a fuzzy inference Petri net method for the control of operator functional state in human–machine hybrid control systems. Liu et al. [12] developed a class of fuzzy continuous Petri nets for modeling biological systems with uncertain kinetic data. Gupta et al. [55] provided a layered FPN model to conduct process equipment failure risk analysis and assessment. Sun et al. [56] presented a fused FPN method for the shared control of brain-computer interface systems. The hierarchical fuzzy colored Petri nets were used by Majma and Babamir [57] for the monitoring and adaptation of pacemaker behavior, the colored fuzzy Petri nets were utilized by Assaf et al. [58] for modelling and analyzing membrane systems, and the time sequence hierarchical FPNs were applied in [10] for the fault diagnosis of distribution networks. Other modified FPN models include the dynamic timed FPNs [59], the 2-dimensional uncertain linguistic Petri nets [60], the Pythagorean FPNs [31], the R-numbers Petri nets [61], and the grey reasoning Petri nets [32].

From the literature review, it can be found that a variety of uncertainty theories have made progress in the expression of experts' experiential knowledge, but they are inefficient at handling the reliability of relevant judgement information. In addition, while various FPN models have been used for knowledge representation and reasoning, the determination of knowledge parameters for FPNs has received little attention from researchers, especially in the setting of large expert group. Moreover, majority of the reasoning algorithms used in current FPNs are facing an enormous challenge called state explosion issue where computation complexity would increase greatly with the growth of FPN scale. To fill these gaps, we establish a new FPN model called LZPNs in this paper for knowledge acquisition and representation in the large group environment. The main novelties of this study are summarized as follows: (1) The linguistic Z-numbers are first employed to represent expert knowledge in a flexible and comprehensive way; (2) An innovative large group knowledge acquisition approach is proposed based on a consensus reaching process; (3) A simplification method is introduced to simplify the structure of linguistic Z-number Petri nets for knowledge reasoning.

3 Preliminary

3.1 Linguistic Scale Functions

Let $S = \{s_0, s_1, ..., s_{2g}\}$ be a finite and totally ordered linguistic term set where s_i is a possible value for a linguistic variable and g is a nonnegative integer. Especially, s_0 and s_{2g} denote the lower and the upper limits of linguistic terms, respectively. Then, the linguistic term set S satisfies that $s_i > s_j$ if and only if $i > j$ [16].

To preserve all given linguistic information, the discrete linguistic term set S was extended by Xu [62] to a continuous form $\overline{S} = \{s_i | i \in [0, 2\xi]\}$, in which $\xi (\xi > g)$ is a sufficiently large positive integer. If $s_i \in S$, then s_i is called an original linguistic term; otherwise, it is a virtual linguistic term.

Definition 1 [63] Let $S = \{s_0, s_1, ..., s_{2g}\}$ be a linguistic term set and θ_i is a numerical value ranging from 0 to 1. Then the linguistic scale function is a mapping from s_i to θ_i defined by.

$$\dot{f}(s_i) = \theta_i, \quad i = 0, 1, \ldots, 2g, \tag{2}$$

where $0 \leq \theta_0 \leq \theta_1 \leq ... \leq \theta_{2g} \leq 1$ and θ_i reflects the preference of decision makers when they choose s_i.

A linguistic scale function denotes the semantic value of a linguistic term, and the following two different forms were proposed in [63]:

(1) The first function based on subscript function is represented as

$$\dot{f}_1(s_i) = \theta_i = \frac{i}{2g}, \quad i = 0, 1, ..., 2g. \tag{3}$$

(2) The second function based on exponential scale is expressed as

$$\dot{f}_2(s_i) = \theta_i = \left(\frac{i}{2g}\right)^g, \quad i = 0, 1, ..., 2g. \tag{4}$$

3.2 Linguistic Z-numbers

The concept of linguistic Z-numbers was introduced by Wang et al. [17] to express uncertain linguistic decision information flexibly.

Definition 2 [17] Let X be a universe of discourse,$S = \{s_0, s_1, ..., s_{2g}\}$ and $S' = \{s'_0, s'_1, ..., s'_{2g'}\}$ be two linguistic term sets. Furthermore, let $A_{\varphi(x)} \in S$ and $B_{\varphi(x)} \in S'$. Then the linguistic Z-number set $\tilde{Z}$ in X is defined as follows:

$$\tilde{Z} = \{(x, A_{\varphi(x)}, B_{\varphi(x)}) | x \in X\}, \tag{5}$$

where $A_{\varphi(x)}$ is a fuzzy restriction on the values that the uncertain variable x is allowed to take, and $B_{\varphi(x)}$ is a measure of reliability for the first component.

The linguistic Z-number set is turned to $(A_{\phi(x)}, B_{\varphi(x)})$ when X has only one element. For simplicity, $\tilde{z}_i = \left(A_{\phi(i)}, B_{\varphi(i)}\right)$ is called a linguistic Z-number, in which $A_{\phi(i)} \in S$ and $B_{\varphi(i)} \in S'$ are two linguistic terms.

Definition 3 [17] Suppose that $\tilde{z}_i = \left(A_{\phi(i)}, B_{\varphi(i)}\right)$ and $\tilde{z}_j = \left(A_{\phi(j)}, B_{\varphi(j)}\right)$ are two linguistic Z-numbers, $\dot{f}$ and $\dot{f}'$ are two linguistic scale functions with their inverse functions $\dot{f}^{-1}$ and $\dot{f}'^{-1}$, respectively. Then, the operational laws of linguistic Z-numbers are defined below:

(1) $\tilde{z}_i \oplus \tilde{z}_j = \left(\dot{f}^{-1}\left(\dot{f}\left(A_{\phi(i)}\right) + \dot{f}\left(A_{\phi(j)}\right)\right), \dot{f}'^{-1}\left(\frac{\dot{f}\left(A_{\phi(i)}\right)\times\dot{f}'\left(B_{\phi(i)}\right)+\dot{f}\left(A_{\phi(j)}\right)\times\dot{f}'\left(B_{\phi(j)}\right)}{\dot{f}\left(A_{\phi(i)}\right)+\dot{f}\left(A_{\phi(j)}\right)}\right)\right)$;

(2) $\lambda\tilde{z}_i = \left(\dot{f}^{-1}\left(\lambda\dot{f}\left(A_{\phi(i)}\right)\right), B_{\phi(i)}\right), \ \lambda \geq 0$

(3) $\tilde{z}_i \otimes \tilde{z}_j = \left(\dot{f}^{-1}\left(\dot{f}\left(A_{\phi(i)}\right)\dot{f}\left(A_{\phi(j)}\right)\right), \dot{f}'^{-1}\left(\dot{f}'\left(B_{\phi(i)}\right)\dot{f}'\left(B_{\phi(j)}\right)\right)\right)$;

(4) $\tilde{z}_i^{\lambda} = \left(\dot{f}^{-1}\left(\dot{f}\left(A_{\phi(i)}\right)^{\lambda}\right), \dot{f}'^{-1}\left(\dot{f}'\left(B_{\phi(i)}\right)^{\lambda}\right)\right), \ \lambda \geq 0.$

Definition 4 [23] Let $\tilde{z}_i = \left(A_{\phi(i)}, B_{\phi(i)}\right)(i = 1, 2, 3, ..., n)$ be a set of linguistic Z-numbers. Then, the linguistic Z-number weighted average (LZWA) operator is defined as:

$$LZWA(\tilde{z}_1, \tilde{z}_2, ..., \tilde{z}_n) = \left(\dot{f}^{-1}\left(\sum_{i=1}^{n} w_i\dot{f}\left(A_{\phi(i)}\right)\right), \dot{f}'^{-1}\left(\frac{\sum_{i=1}^{n} w_i\dot{f}\left(A_{\phi(i)}\right)\dot{f}'\left(B_{\phi(i)}\right)}{\sum_{i=1}^{n} w_i\dot{f}\left(A_{\phi(i)}\right)}\right)\right), \tag{6}$$

where w_i is the weight of $\tilde{z}_i$ for i = 1,2,…,n with $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$.

Definition 5 [17] Let $\tilde{z}_i = \left(A_{\phi(i)}, B_{\varphi(i)}\right)$ and $\tilde{z}_j = \left(A_{\phi(j)}, B_{\varphi(j)}\right)$ be two linguistic Z-numbers. Then, the distance between them is determined by

$$\dot{d}(\tilde{z}_i, \tilde{z}_j) = \frac{1}{2}\left(\dot{f}\left(A_{\phi(i)}\right) \times \dot{f}'\left(B_{\phi(i)}\right) - \dot{f}\left(A_{\phi(j)}\right) \times \dot{f}'\left(B_{\phi(j)}\right) + \max\left\{\left|\dot{f}\left(A_{\phi(i)}\right) - \dot{f}\left(A_{\phi(j)}\right)\right|, \left|\dot{f}'\left(B_{\phi(i)}\right) - \dot{f}'\left(B_{\phi(j)}\right)\right|\right\}\right) \tag{7}$$

Definition 6 [25] Let $\tilde{z}_i = \left(A_{\phi(i)}, B_{\varphi(i)}\right)$ be a linguistic Z-number based on the linguistic term sets $S = \left\{s_0, s_1, ..., s_{2g}\right\}$ and $S' = \left\{s'_0, s'_1, ..., s'_{2g'}\right\}$. Then, it can be defuzzied into a crisp value by

$$\psi(\tilde{z}_i) = \frac{\dot{d}(\tilde{z}_i, \tilde{z}_{\min})}{\dot{d}(\tilde{z}_{\max}, \tilde{z}_{\min})}, \tag{8}$$

where $\tilde{z}_{\min} = \left(s_0, s'_0\right)$ and $\tilde{z}_{\max} = \left(s_{2g}, s'_{2g'}\right)$.

4 Framework for Large Group Knowledge Acquisition and Representation

In this section, we present a new LZPN model for knowledge acquisition and representation under large group environment. The specific process is divided into three parts as shown in Fig. 1 and explained in the following subsections.

4.1 The Proposed LZPN Model

4.1.1 Definition of LZPNs

In this section, a new FPN model called LZPNs is proposed for uncertain knowledge representation and reasoning.

Definition 7 Let $\tilde{\Omega}$ be the set of all linguistic Z-numbers based on the linguistic term sets $S = \left\{s_0, s_1, ..., s_{2g}\right\}$ and $S' = \left\{s'_0, s'_1, ..., s'_{2g'}\right\}$. Then, the structure of an LZPN is defined as:

$$\text{LZPN} = \left(P, T, I, O, \tilde{M}, \widetilde{Th}, \tilde{U}, LW, GW\right) \tag{9}$$

where

(1) P, T, I and O are the same as those in Eq. (1);

(2) $\tilde{M} : P \to \tilde{\Omega}$ represents a linguistic Z-number marking vector $\tilde{M} = (\tilde{\alpha}_1, \tilde{\alpha}_2, ..., \tilde{\alpha}_m)^T$. The element $\tilde{\alpha}_i$ is the truth degree of p_i for i = 1,2,…,m, and the initial marking vector is denoted by $\tilde{M}_0$;

(3) $\widetilde{Th} : P \to \tilde{\Omega}$ represents a linguistic Z-number threshold vector $\widetilde{Th} = \left(\tilde{\lambda}_1, \tilde{\lambda}_2, ..., \tilde{\lambda}_m\right)^T$ which assigns threshold values to the m places;

(4) $\tilde{U} : T \to \tilde{\Omega}$ is a linguistic Z-number certainty factor vector $\tilde{U} = (\tilde{\mu}_1, \tilde{\mu}_2, ...\tilde{\mu}_n)$, which assigns certainty degrees to the n transitions;

(5) $LW : P \to [0, 1]$ is a local weight vector $LW = (lw_1, lw_2, ..., lw_m)^T$ of places, reflecting relative importance of a place contributing to its following transition;

(6) $GW : T \to [0, 1]$ is a global weight vector $GW = (gw_1, gw_2, ..., gw_n)$ of transitions, signifying how much a transition impacts its output places.

4.1.2 Linguistic Z-number Production Rules

Based on the concept of weighted FPRs [1], linguistic Z-number production rules (LZPRs) are introduced in this study. The LZPRs are classified into three types as described below:

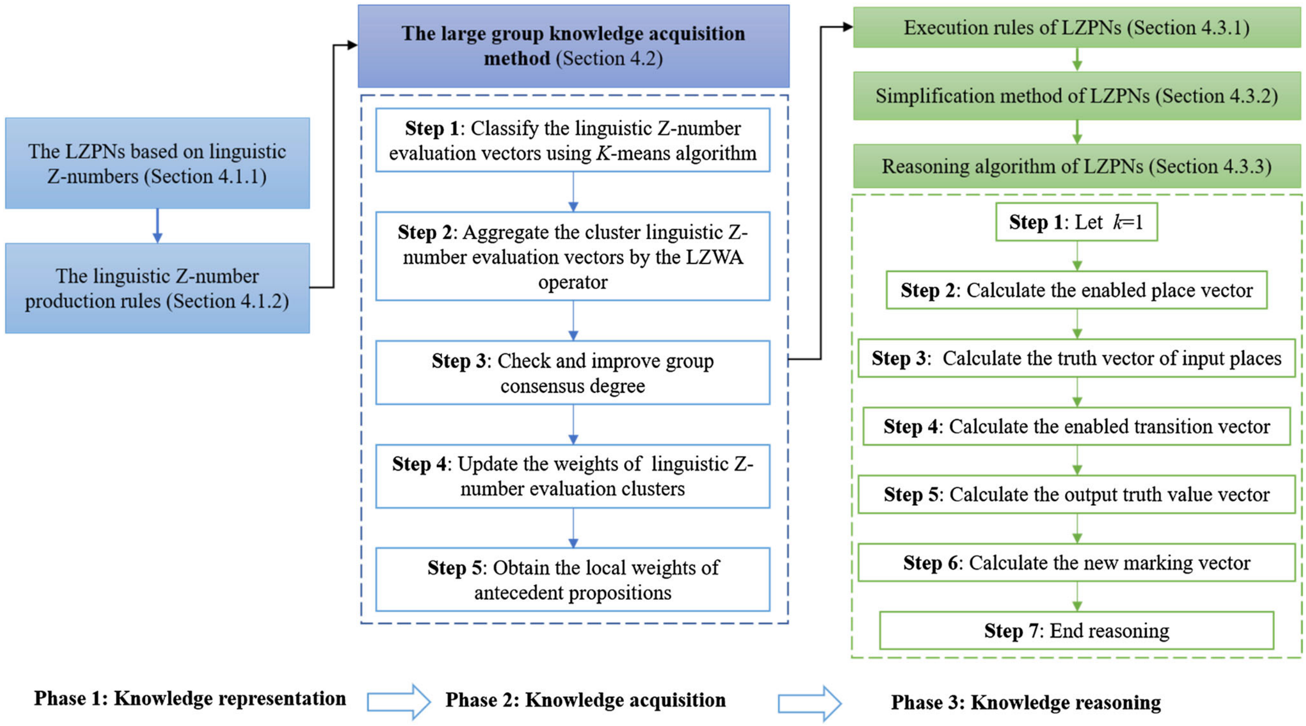

Fig. 1 Flowchart of the proposed knowledge acquisition and representation framework

Type 1: A simple LZPR

R_1: IF d_j THEN d_k $\left(\tilde{\lambda}; lw; \tilde{\mu}; gw\right)$;

Type 2: A composite conjunctive rule in the antecedent R_2: IF d_{j1} AND d_{j2} AND…AND d_{jm} THEN d_k $\left(\tilde{\lambda}_1, \tilde{\lambda}_2, \ldots \tilde{\lambda}_m; lw_1, lw_2, \ldots, lw_m; \tilde{\mu}; gw\right)$;

Type 3: A composite conjunctive rule in the consequent R_3: IF d_j THEN d_{k1} AND d_{k2} AND…AND d_{km} $\left(\tilde{\lambda}; lw; \tilde{\mu}; gw\right)$.

The above three types of LZPRs can be mapped into LZPNs as shown in Fig. 2.

4.2 The Large Group Knowledge Acquisition Method

The section develops a knowledge acquisition method to obtain the knowledge parameters of LZPRs based on large numbers of experts. In what follows, the local weight (*LW*) is used as an example to explain the proposed knowledge acquisition method.

Suppose that n LZPRs with respect to L antecedent propositions $d_l(l = 1, 2, \ldots, L)$ are modelled by a LZPN. A set of experts $E_h(h = 1, 2, \ldots, H; H \geq 20)$ from different backgrounds are invited to assess local weights of the LZPRs. Let $\tilde{Z}_h = \left(\tilde{z}_1^h, \tilde{z}_2^h, \ldots, \tilde{z}_L^h\right)^T$ be a linguistic Z-number weight evaluation vector provided by the expert E_h, where $\tilde{z}_l^h = \left(A^h_{\varphi(l)}, B^h_{\varphi(l)}\right)$ is the linguistic Z-number local weight determined by the hth expert for the antecedent proposition d_l based on the linguistic term sets $S = \{s_0, s_1, \ldots, s_{2g}\}$ and $S' = \{s'_0, s'_1, \ldots, s'_{2g'}\}$. Next, the steps of the proposed knowledge acquisition method for determining the *LW* are introduced.

Step 1 Classify the linguistic Z-number evaluation vectors.

Various clustering algorithms have been proposed in the literature [21]. Among them the *K*-means algorithm [64], because of its simplicity and robustness, is the most popular partitioning-based clustering algorithm. Compared with other clustering algorithms, the *K*-means algorithm is a fast iteration algorithm and has been applied to many fields. Hence, in this step, the *K*-means algorithm is adopted to classify the linguistic Z-number evaluation vectors $\tilde{Z}_h(h = 1, 2, \ldots, H)$ into comparatively a small number of clusters.

Step 1.1 First, $\dot{H}$ linguistic Z-number evaluation vectors denoted as $\tilde{Z}'_{\dot{h}}(\dot{h} = 1, 2, \ldots, \dot{H})$ are randomly selected from the linguistic Z-number evaluation vectors $\tilde{Z}_h(h = 1, 2, \ldots, H)$ as cluster centers.

Step 1.2 The linguistic Z-number evaluation vectors $\tilde{Z}_h$ should be assigned to the $\dot{h}$th cluster, if $\dot{d}\left(\tilde{Z}_h, \tilde{Z}'_{\dot{h}}\right) \leq \dot{d}\left(\tilde{Z}_h, \tilde{Z}'_{\dot{h}'}\right)$ for $h = 1, 2, \ldots, H$, $\dot{h}' = 1, 2, \ldots, \dot{H}$,

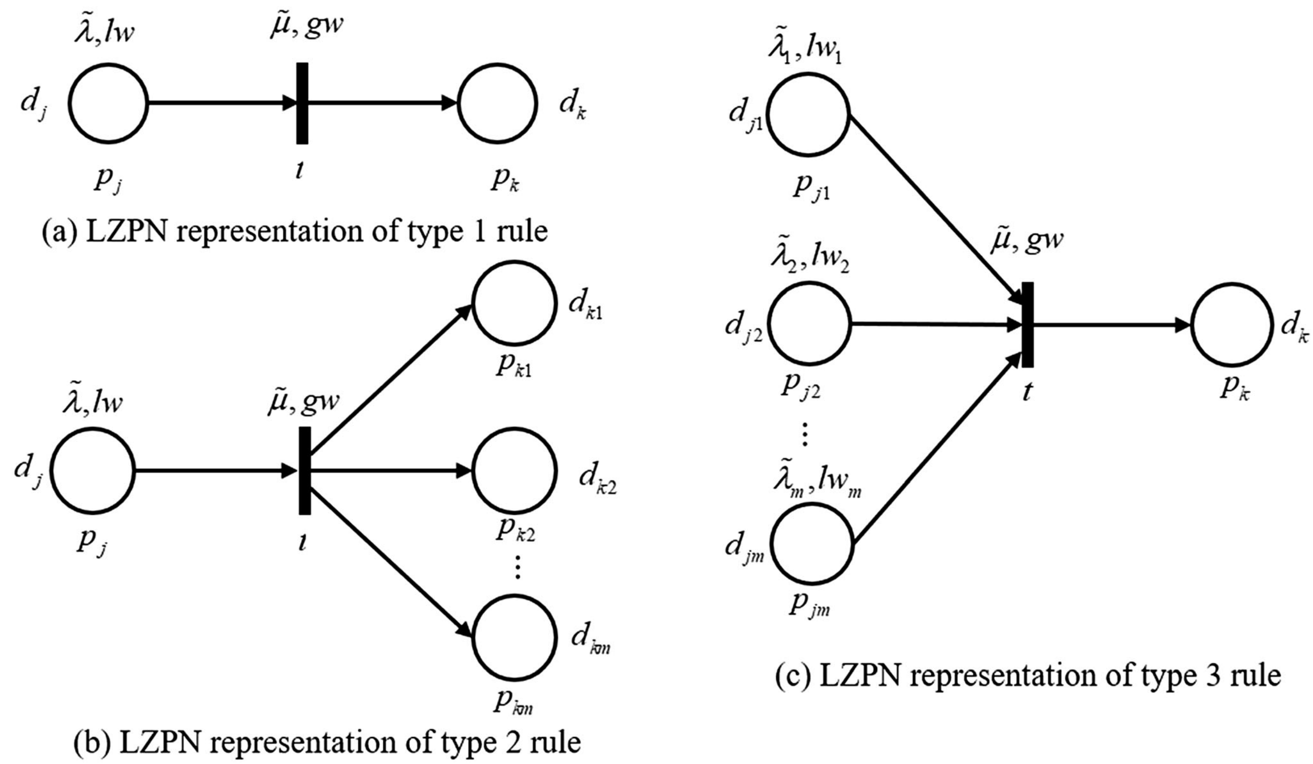

Fig. 2 LZPN representation of LZPRs

and $h' \neq h$. In this way, the H linguistic Z-number evaluation vectors $\tilde{Z}_h(h = 1, 2, ..., H)$ can be grouped into $\dot{H}$ clusters. The number of vectors in the $\dot{h}$th cluster is defined as $v_{\dot{h}}$ and $\sum_{\dot{h}=1}^{\dot{H}} v_{\dot{h}} = H$, and their linguistic Z-number evaluation vectors are denoted as $\tilde{Z}^c_u = \left(\tilde{z}^c_{u1}, \tilde{z}^c_{u2}, ..., \tilde{z}^c_{uL}\right)^T (u = 1, 2, ..., v_{\dot{h}})$.

Step 1.3 The centers of $\dot{H}$ clusters are updated to obtain $\tilde{Z}''_{\dot{h}} = \left(\tilde{z}''_{\dot{h}1}, \tilde{z}''_{\dot{h}2}, ..., \tilde{z}''_{\dot{h}L}\right)^T (\dot{h} = 1, 2, ..., \dot{H})$, in which

$$\tilde{z}''_{\dot{h}l} = LZWA\left(\tilde{z}^c_{1l}, \tilde{z}^c_{2l}, ..., \tilde{z}^c_{v_{\dot{h}}l}\right) = \sum_{u=1}^{v_{\dot{h}}} \frac{1}{v_{\dot{h}}} \tilde{z}^c_{ul}, \quad l = 1, 2, ..., L. \tag{10}$$

Step 1.4 When the new cluster centers $\tilde{Z}''_{\dot{h}}(\dot{h} = 1, 2, ..., \dot{H})$ are the same as the cluster centers obtained in the previous iteration, go to Step 2; Otherwise, let $\widetilde{Z'}_{\dot{h}} = \widetilde{Z''}_{\dot{h}}(\dot{h} = 1, 2, ..., \dot{H})$ and go back to Step 1.2.

Step 2 Aggregate the cluster linguistic Z-number evaluation vectors.

Let $r = 1$. The parameter r means the number of iterations. Suppose the experts are allocated the same weight before consensus reaching process [34].

Step 2.1 Based on the cluster result obtained in the last step, the cluster linguistic Z-number evaluation vector $\tilde{Z}^{*(r)}_{\dot{h}} = \left(\tilde{z}^{*(r)}_{\dot{h}1}, \tilde{z}^{*(r)}_{\dot{h}2}, ..., \tilde{z}^{*(r)}_{\dot{h}L}\right)^T$ of the $\dot{h}$th cluster can be determined as $\tilde{Z}^{*(r)}_{\dot{h}} = \tilde{Z}''_{\dot{h}}$.

Step 2.2 By aggregating the cluster linguistic Z-number evaluation vectors $\tilde{Z}^{*(r)}_{\dot{h}}(\dot{h} = 1, 2, ..., \dot{H})$, a group linguistic Z-number evaluation vector $\tilde{Z}^{*(r)} = \left(\tilde{z}^{*(r)}_1, \tilde{z}^{*(r)}_2, ..., \tilde{z}^{*(r)}_L\right)^T$ is obtained by

$$\tilde{z}^{*(r)}_l = \mathrm{LZWA}\left(\tilde{z}^{*(r)}_{1l}, \tilde{z}^{*(r)}_{2l}, ..., \tilde{z}^{*(r)}_{\dot{H}l}\right) = \sum_{\dot{h}=1}^{\dot{H}} w^{*(r)}_{\dot{h}} \tilde{z}^{*(r)}_{\dot{h}l}, \quad l = 1, 2, ..., L, \tag{11}$$

where $w^{*(r)}_{\dot{h}}$ is the weight of the $\dot{h}$th cluster in the rth iteration. It is assumed as $w^{*(1)}_{\dot{h}} = \frac{v_{\dot{h}}}{H}$ in the first iteration.

Step 3 Check and improve group consensus degree.

Once the group linguistic Z-number evaluation vector $\tilde{Z}^{*(r)}$ is obtained, the consensus degree between $\tilde{Z}^{*(r)}_{\dot{h}}$ and $\tilde{Z}^{*(r)}$ is calculated by

$$\begin{aligned} CD\left(\tilde{Z}^{*(r)}_{\dot{h}}, \tilde{Z}^{*(r)}\right) &= 1 - \dot{d}\left(\tilde{Z}^{*(r)}_{\dot{h}}, \tilde{Z}^{*(r)}\right) \\ &= 1 - \frac{1}{L}\sum_{l=1}^{L} d\left(\tilde{z}^{*(r)}_{\dot{h}l}, \tilde{z}^{*(r)}_l\right). \end{aligned} \tag{12}$$

Then, the group consensus degree is obtained by

$$GCD_{(r)} = \sum_{\dot{h}=1}^{\dot{H}} w_{\dot{h}}^{*(r)} CD\left(\tilde{Z}_{\dot{h}}^{*(r)}, \tilde{Z}^{*(r)}\right). \tag{13}$$

If $GCD_{(r)}$ is bigger than the consensus threshold ε, then the group consensus is acceptable and go to Step 5. Otherwise, the cluster linguistic Z-number evaluation vectors should be changed to improve $GCD_{(r)}$. Note that the consensus threshold ε ranges from 0 to 1; the larger the value of ε, the larger the consensus level of the group will be.

Step 4 Let $r = r + 1$, update the weight of each cluster by Eq. (14) and return to Step 2.2.

$$w_{\dot{h}}^{*(r+1)} = \frac{w_{\dot{h}}^{*(r)} CD\left(\tilde{Z}_{\dot{h}}^{*(r)}, \tilde{Z}^{*(r)}\right)}{GCD_{(r)}}, \quad \dot{h} = 1, 2, ..., \dot{H}. \tag{14}$$

Step 5 Obtain the local weights of antecedent propositions.

When the acceptable group consensus degree is reached, the final group linguistic Z-number evaluation vector $\tilde{Z}^* = \left(\tilde{z}_1^*, \tilde{z}_2^*, ..., \tilde{z}_L^*\right)^T$ can be obtained. Then, the local weights of the L antecedent propositions $LW = (lw_1, lw_2, ..., lw_L)^T$ are defuzzified and normalized by

$$lw_l = \frac{\psi\left(\tilde{z}_l^*\right)}{\sum_{l=1}^{L} \psi\left(\tilde{z}_l^*\right)}, \quad l = 1, 2, ..., L. \tag{15}$$

Note that the knowledge parameters GW, $\widetilde{Th}$ and $\tilde{U}$ can be determined in the same way. The final group linguistic Z-number evaluation vector of GW needs be defuzzified and normalized, while the $\widetilde{Th}$ and $\tilde{U}$ can be obtained from their final group linguistic Z-number evaluation vectors directly.

4.3 Knowledge Reasoning of LZPNs

4.3.1 Execution Rules of LZPNs

Let p_O be an output place of the transition t, and $I(t) = \{p_{I1}, p_{I2}, ..., p_{Im'}\}$ be the input places of t with threshold values $\left\{\tilde{\lambda}_{I1}, \tilde{\lambda}_{I2}, ..., \tilde{\lambda}_{Im'}\right\}$ and local weights $\{lw_{I1}, lw_{I2}, ..., lw_{Im'}\}$. Next, the enabling and firing rules of LZPNs are explained.

For the transition t, it is enabled if

$$\tilde{\alpha}(p_{Ij}) \geq \tilde{\lambda}_{Ij}, j = 1, 2, ..., m', \tag{16}$$

where $\tilde{\alpha}(p_{Ij})$ is the truth degree of input place p_{Ij}.

The transition t fires instantly once it is enabled. After t is fired, the tokens in its input places are copied, and a token is put into each of its output places. For the output place p_O, its truth degree from transition t is computed by

$$\tilde{\alpha}(p_O) = LZWA(\tilde{\alpha}(p_{I1}), \tilde{\alpha}(p_{I2}), ..., \tilde{\alpha}(p_{Im'})) \otimes \tilde{\mu}, \tag{17}$$

where the local weights $\{lw_{I1}, lw_{I2}, ..., lw_{Im'}\}$ are used in the LZWA operator.

If more than one input transitions $t_i (i = 1, 2..., n', n' \geq 2)$ are enabled for p_O, then its truth degree is calculated by

$$\tilde{\alpha}(p_O) = LZWA(\tilde{\alpha}(p_{O1}), \tilde{\alpha}(p_{O2}), ..., \tilde{\alpha}(p_{On'})), \tag{18}$$

where $\tilde{\alpha}(p_{Oi})$ is the truth degree of p_O determined by the ith input transition, and the global local weights $\{gw_1, gw_2, ..., gw_{n'}\}$ are used in the LZWA operator.

4.3.2 Simplification Method of LZPNs

As with FPNs, the implementation of inference using the LZPN model will be difficult with the increasing scale of LZPNs. Thus, based on the threshold values of places, an LZPN can be simplified before the knowledge reasoning process.

Suppose that N LZPRs $\{R_1, R_2, ..., R_N\}$ are mapped into an LZPN model with $\{t_1, t_2, ..., t_N\}$. The input places of t_i are denoted as $\{p_{I1}, p_{I2}, ..., p_{Iu}\}$ with their truth degrees $\{\tilde{\alpha}(p_{I1}), \tilde{\alpha}(p_{I2}), ..., \tilde{\alpha}(p_{Iu})\}$ and threshold values $\left\{\tilde{\lambda}_{I1}, \tilde{\lambda}_{I2}, ..., \tilde{\lambda}_{Iu}\right\}$. Inspired by [52], the method to simplify the LZPN are introduced below.

Step 1 For the transition t_i, if $\tilde{\alpha}\left(p_{Ij}\right) < \tilde{\lambda}_{Ij}$, which means that the input place p_{Ij} is unenabled, then remove the transition t_i.

Step 2 For the removed transition t_i, its consequent places are denoted as $\{p_{O1}, p_{O2}, ..., p_{Ou}\}$. For the place p_{Oq},

(1) if p_{Oq} is the consequent place of other transitions, then reserve the place;
(2) if p_{Oq} is the antecedent place of other transitions, then delete those transitions.

Step 3 Following the above steps, a simplified LZPN can be obtained by checking all the N transitions.

4.3.3 Reasoning Algorithm of LZPNs

Once a simplified LZPN is determined, a reasoning algorithm should be implemented to determine the final reasoning result of a modeled expert system. To explain the reasoning process of LZPNs clearly, the following basic matrix operators are introduced first [40]:

(1) Operator ⊕:

$$\mathbf{A} \oplus \mathbf{B} = \mathbf{D}, \tag{19}$$

where $\mathbf{A}$,$\mathbf{B}$, and $\mathbf{D}$ are all $m \times n$-dimensional matrices with a_{ij}, b_{ij}, and d_{ij} being their elements, respectively, and $d_{ij} = \max\{a_{ij}, b_{ij}\},\ i = 1, 2, ..., m; j = 1, 2, ..., n$.

(2) Operator $\circ$:

$$\mathbf{A} \circ \mathbf{B} = \mathbf{D}, \tag{20}$$

where $\mathbf{A}$, $\mathbf{B}$, and $\mathbf{D}$ are all $m \times n$ -dimensional matrices with a_{ij}, b_{ij}, and d_{ij} being their elements, respectively, and $d_{ij}{=}a_{ij} \times b_{ij}$, $i = 1, 2, ..., m; j = 1, 2, ..., n$.

(3) Operator $\triangleright$:

$$\mathbf{A} \triangleright \mathbf{B} = \mathbf{D}, \tag{21}$$

where $\mathbf{A}$, $\mathbf{B}$, and $\mathbf{D}$ are all $m \times n$ -dimensional matrices with a_{ij}, b_{ij}, and d_{ij} being their elements respectively, and $d_{ij}{=}\ 1$ if $a_{ij} \geq b_{ij}$, $d_{ij}{=}\ 0$ if $a_{ij} < b_{ij}$, $i = 1, 2, ..., m, j = 1, 2, ..., n$.

Based on the simplified LZPN being obtained, the proposed concurrent inference algorithm is described as follows:

Input: I and O are $m \times n$ dimensional matrices, $\tilde{U}$ and GW are n dimensional vectors, LW, $\widetilde{Th}$, and $\tilde{M}_0$ are m dimensional vectors.

Output: $\tilde{M}_k$ is an m dimensional vector.

Step 1: Let $k = 1$. The parameter k denotes the time of iterations.

Step 2: Calculate the enabled place vector $D^{(k)}$ that represents the enable input places of transitions by

$$D^{(k)} = \tilde{M}_{k-1} \triangleright \widetilde{Th}. \tag{22}$$

Step 3: If $D^{(k)}$ is a nonzero vector, then calculate the truth vector of input places $\tilde{\Gamma}^{(k)}$ by (23); Otherwise, go to Step 7.

$$\tilde{\Gamma}^{(k)} = LZWA\left((I \circ W_L)^T, \tilde{M}_{k-1}\right), \tag{23}$$

where $W_L = [LW, LW, ..., LW]_{m\times n}$.

Step 4: Calculate the enabled transition vector $F^{(k)}$ by

$$F^{(k)} = (E \times I) \triangleright \left(\left(D^{(k)}\right)^T \times I\right), \tag{24}$$

where $E = (1)_{1\times m}{=}\ [1, 1, ..., 1]$.

Step 5: If $F^{(k)}$ is a nonzero vector, then calculate the output truth value vector $\tilde{\Psi}^{(k)}$ by (25); Otherwise, go to Step 7.

$$\tilde{\Psi}^{(k)} = \left(F^{(k)} \circ \tilde{\Gamma}^{(k)}\right) \circ \tilde{U}. \tag{25}$$

Step 6: Calculate the new marking vector $\tilde{M}_k$ by

$$\tilde{M}_k = \tilde{M}_{k-1} \oplus LZWA\left((O \circ W_G), \tilde{\Psi}^{(k)}\right), \tag{26}$$

where $W_G = [GW, GW, ..., GW]^T_{m\times n}$.

If $\tilde{M}_k = \tilde{M}_{k-1}$, then go to Step 7; Otherwise, let $k = k + 1$ and go back to Step 2.

Step 7: End reasoning.

5 Illustrative Example

In this section, a security risk assessment case in the chemical industry [39] is provided to demonstrate the proposed LZPNs.

5.1 Background

In the petrochemical industry, many hazardous chemicals are handled in the production and storage activities. Chemical plants have a strong appeal to terrorists as severe consequences and important social impact can be caused. Security risk assessment is important to assess the risk level of a plant in order to take efficient actions to reduce its security risk. The risks originating from terrorists' attacks should be examined to determine if the existing security measures are adequate or need enhancement. Normally, four elements are considered for the security risk assessment of chemical plant, i.e., threat analysis, vulnerability analysis, security countermeasure, and mitigation and emergency response. To reflect the importance and relationships of these risk factors, the developed LZPN model is adopted to assess the security risk assessment of a refinery plant in this case study.

In security risk assessment, the logical relationships among risk factors can be transformed into the relationships of transitions and places of an LZPN. Let $d_i (i = 1, 2, ..., 17)$ be the propositions in the security risk assessment. Then, LZPRs of the security risk assessment are denoted as follows:

R_1: IF d_1 THEN d_{15} $\left(\tilde{\lambda}_1; lw_1;\ \tilde{u}_1; gw_1\right)$

R_2: IF d_2 THEN d_{15} $\left(\tilde{\lambda}_2; lw_2;\ \tilde{u}_2; gw_2\right)$

R_3: IF d_3 THEN d_{15} $\left(\tilde{\lambda}_3; lw_3;\ \tilde{u}_3; gw_3\right)$

R_4: IF d_4 THEN d_{15} $\left(\tilde{\lambda}_4; lw_4;\ \tilde{u}_4; gw_4\right)$

R_5: IF d_5 THEN d_{15} $\left(\tilde{\lambda}_5; lw_5;\ \tilde{u}_5; gw_5\right)$

R_6: IF d_6 and d_7 and d_8 and d_9 and d_{10} and d_{11} and d_{12} and d_{13} and d_{14} and d_{15} and d_{16} THEN d_{17} $\big(\tilde{\lambda}_6, \tilde{\lambda}_7, \tilde{\lambda}_8, \tilde{\lambda}_9, \tilde{\lambda}_{10}, \tilde{\lambda}_{11}, \tilde{\lambda}_{12}, \tilde{\lambda}_{13}, \tilde{\lambda}_{14}, \tilde{\lambda}_{15}, \tilde{\lambda}_{16};\ lw_6, lw_7, lw_8, lw_9, lw_{10}, lw_{11}, lw_{12}, lw_{13}, lw_{14}, lw_{15}, lw_{16};\ \tilde{u}_6; gw_6\big)$.

Based on the above LZPRs, an LZPN model for the security risk assessment is established as shown in Fig. 3. The places in the LZPN related to their propositions are listed in Table 1. As a result, the security risk assessment problem can be converted to evaluating the truth value of the terminating place p_{17}.

5.2 Implementation

In this section, the proposed large group knowledge acquisition and representation framework is implemented to address the refinery security risk assessment problem.

First, to determine the knowledge parameters of the obtained LZPRs, 25 experts $E_h(h = 1, 2, ..., 25)$ were involved to give their judgements by utilizing the linguistic term sets: $S = \{s_0 = Very\ Low,\ s_1 = Low, s_2 = Moderate, s_3 = High, s_4 = Very\ High\}$ and $S' = \{s'_0 = Seldom, s'_1 = Often, s'_2 = Usually\}$.

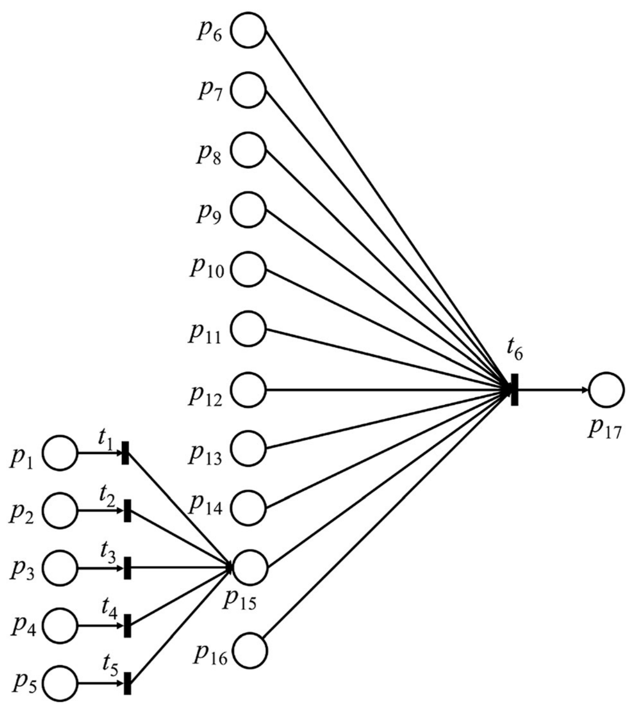

Fig. 3 LZPN for the security risk assessment

Based on the experts' evaluations, knowledge parameters of the six LZPRs can be obtained according to the proposed knowledge acquisition method. For the LZPR R_6, the linguistic Z-number evaluation vectors of local weights $\tilde{Z}_h = \left(\tilde{z}_6^h, \tilde{z}_7^h, ..., \tilde{z}_{16}^h\right)^T (h = 1, 2, ..., 25)$ are displayed in Table 2. Note that lw_1, lw_2, lw_3, lw_4, and lw_5 are one in this example. Next, the knowledge acquisition process for the local weight (LW) is explained.

Step 1 This step is to cluster the linguistic Z-number evaluation vectors $\tilde{Z}_h(h = 1, 2, ..., 25)$ using the K-means clustering algorithm. Without loss of generality, the number of clusters is set as four (i.e., H = 4) in the case.

Step 1.1 Four cluster centers are randomly selected and denoted as:

$$\tilde{Z}'_1 = \begin{pmatrix} (s_{0.1}, s'_{0.1}) (s_{0.1}, s'_{0.1}) (s_{0.1}, s'_{0.1}) (s_{0.1}, s'_{0.1}) \\ (s_{0.1}, s'_{0.1}) (s_{0.1}, s'_{0.1}) (s_{0.1}, s'_{0.1}) (s_{0.1}, s'_{0.1}) \\ (s_{0.1}, s'_{0.1}) (s_{0.1}, s'_{0.1}) (s_{0.1}, s'_{0.1}) \end{pmatrix}^T,$$

$$\tilde{Z}'_2 = \begin{pmatrix} (s_{4.0}, s'_{1.8}) (s_{3.8}, s'_{2.0}) (s_{4.0}, s'_{2.0}) (s_{4.0}, s'_{2.0}) \\ (s_{4.0}, s'_{2.0}) (s_{4.0}, s'_{1.8}) (s_{3.8}, s'_{2.0}) (s_{4.0}, s'_{2.0}) \\ (s_{4.0}, s'_{2.0}) (s_{4.0}, s'_{2.0}) (s_{4.0}, s'_{2.0}) \end{pmatrix}^T,$$

$$\tilde{Z}'_3 = \begin{pmatrix} (s_{0.5}, s'_{0.1}) (s_{0.5}, s'_{0.1}) (s_{0.5}, s'_{0.1}) (s_{0.5}, s'_{0.1}) \\ (s_{0.5}, s'_{0.1}) (s_{0.5}, s'_{0.1}) (s_{0.5}, s'_{0.1}) (s_{0.5}, s'_{0.1}) \\ (s_{0.5}, s'_{0.1}) (s_{0.5}, s'_{0.1}) (s_{0.5}, s'_{0.1}) \end{pmatrix}^T,$$

$$\tilde{Z}'_4 = \begin{pmatrix} (s_{1.5}, s'_{2.0}) (s_{1.5}, s'_{2.0}) (s_{1.5}, s'_{1.8}) (s_{1.5}, s'_{2.0}) \\ (s_{1.5}, s'_{2.0}) (s_{1.5}, s'_{2.0}) (s_{1.5}, s'_{2.0}) (s_{1.5}, s'_{2.0}) \\ (s_{1.5}, s'_{2.0}) (s_{1.5}, s'_{2.0}) (s_{1.5}, s'_{1.8}) \end{pmatrix}^T.$$

Step 1.2 The linguistic Z-number evaluation vectors $\tilde{Z}_h(h = 1, 2, ..., 25)$ are divided into the following four clusters:

Cluster 1: $\tilde{Z}_3, \tilde{Z}_7, \tilde{Z}_{11}, \tilde{Z}_{15}, \tilde{Z}_{19}, \tilde{Z}_{23}$; Cluster 2: $\tilde{Z}_1, \tilde{Z}_5, \tilde{Z}_9, \tilde{Z}_{13}, \tilde{Z}_{17}, \tilde{Z}_{21}$;

Cluster 3: $\tilde{Z}_2, \tilde{Z}_6, \tilde{Z}_{10}, \tilde{Z}_{14}, \tilde{Z}_{18}, \tilde{Z}_{22}$; Cluster 4: $\tilde{Z}_4, \tilde{Z}_8, \tilde{Z}_{12}, \tilde{Z}_{16}, \tilde{Z}_{20}, \tilde{Z}_{24}, \tilde{Z}_{25}$.

Table 1 Places of the LZPN and their propositions

Places	Propositions	Places	Propositions
p_1	Access control is poor	p_{10}	Chemicals that can be utilized as precursors for WMD exist
p_2	Perimeter protection is poor	p_{11}	Worst case impact on-site is severe
p_3	Mitigation potential is poor	p_{12}	Worst case impact off-site is severe
p_4	Proper lighting is poor	p_{13}	History of security incidents is frequent
p_5	Application of metal detector/X-ray/CCTV is poor	p_{14}	Many terrorist groups exist in region
p_6	Location has a high density of population	p_{15}	Existing security measures are poor
p_7	The processing area is visible	p_{16}	Personal preparedness and training is poor
p_8	Inventory is very large	p_{17}	Security risk is very high
p_9	Ownership is not private		

Table 2 Linguistic Z-number evaluation vectors of local weights

Local weights	$\tilde{Z}_1$	$\tilde{Z}_2$	$\tilde{Z}_3$	$\tilde{Z}_{23}$	$\tilde{Z}_{24}$	$\tilde{Z}_{25}$
lw_6	$(s_4, s'_{1.8})$	$(s_{0.5}, s'_{0.1})$	$(s_{0.1}, s'_{0.1})$	$(s_{0.1}, s'_{0.2})$	$(s_{0.1}, s'_2)$	$(s_{0.2}, s'_2)$
lw_7	$(s_{3.8}, s'_2)$	$(s_{0.5}, s'_{0.1})$	$(s_{0.1}, s'_{0.1})$	$(s_{0.1}, s'_{0.1})$	$(s_{0.1}, s'_{1.8})$	$(s_{0.1}, s'_2)$
lw_8	(s_4, s'_2)	$(s_{0.5}, s'_{0.1})$	$(s_{0.1}, s'_{0.1})$	$(s_{0.2}, s'_{0.1})$	$(s_{0.1}, s'_2)$	$(s_{0.2}, s'_2)$
lw_9	(s_4, s'_2)	$(s_{0.5}, s'_{0.1})$	$(s_{0.1}, s'_{0.1})$	$(s_{0.1}, s'_{0.1})$	$(s_{0.1}, s'_2)$	$(s_{0.1}, s'_2)$
lw_{10}	(s_4, s'_2)	$(s_{0.5}, s'_{0.1})$	$(s_{0.1}, s'_{0.1})$	$(s_{0.1}, s'_{0.1})$	$(s_{0.1}, s'_2)$	$(s_{0.1}, s'_{1.8})$
lw_{11}	$(s_4, s'_{1.8})$	$(s_{0.5}, s'_{0.1})$	$(s_{0.1}, s'_{0.1})$	$(s_{0.1}, s'_{0.1})$	$(s_{0.2}, s'_2)$	$(s_{0.1}, s'_{1.8})$
lw_{12}	$(s_{3.8}, s'_2)$	$(s_{0.5}, s'_{0.1})$	$(s_{0.1}, s'_{0.1})$	$(s_{0.1}, s'_{0.1})$	$(s_{0.1}, s'_{1.8})$	$(s_{0.1}, s'_{1.8})$
lw_{13}	(s_4, s'_2)	$(s_{0.5}, s'_{0.1})$	$(s_{0.1}, s'_{0.1})$	$(s_{0.1}, s'_{0.2})$	$(s_{0.1}, s'_2)$	$(s_{0.1}, s'_2)$
lw_{14}	(s_4, s'_2)	$(s_{0.5}, s'_{0.1})$	$(s_{0.1}, s'_{0.1})$	$(s_{0.1}, s'_{0.1})$	$(s_{0.1}, s'_2)$	$(s_{0.1}, s'_2)$
lw_{15}	(s_4, s'_2)	$(s_{0.5}, s'_{0.1})$	$(s_{0.1}, s'_{0.1})$	$(s_{0.2}, s'_{0.1})$	$(s_{0.2}, s'_2)$	$(s_{0.1}, s'_2)$
lw_{16}	(s_4, s'_2)	$(s_{0.5}, s'_{0.1})$	$(s_{0.1}, s'_{0.1})$	$(s_{0.1}, s'_{0.1})$	$(s_{0.1}, s'_2)$	$(s_{0.1}, s'_2)$

Step 1.3 Based on Eq. (10), the centers of the four clusters are updated as:

$$\tilde{Z}''_1 = \begin{pmatrix} (s_{0.100}, s'_{0.117})\,(s_{0.100}, s'_{0.1})\,(s_{0.133}, s'_{0.113})\,(s_{0.117}, s'_{0.114}) \\ (s_{0.117}, s'_{0.1})\,(s_{0.100}, s'_{0.1})\,(s_{0.100}, s'_{0.1})\,(s_{0.117}, s'_{0.143}) \\ (s_{0.100}, s'_{0.117})\,(s_{0.117}, s'_{0.1})\,(s_{0.100}, s'_{0.1}) \end{pmatrix}^T,$$

$$\tilde{Z}''_2 = \begin{pmatrix} (s_{4.000}, s'_{1.867})\,(s_{3.933}, s'_{1.967})\,(s_{3.933}, s'_{1.968})\,(s_{4.000}, s'_{2.0}) \\ (s_{3.967}, s'_{1.967})\,(s_{3.933}, s'_{1.900})\,(s_{3.967}, s'_{1.967})\,(s_{4.000}, s'_{1.933}) \\ (s_{3.967}, s'_{1.967})\,(s_{4.000}, s'_{2.0})\,(s_{4.000}, s'_{1.967}) \end{pmatrix}^T,$$

$$\tilde{Z}''_3 = \begin{pmatrix} (s_{3.417}, s'_{0.1})\,(s_{3.417}, s'_{0.159})\,(s_{3.417}, s'_{0.1})\,(s_{3.350}, s'_{0.120}) \\ (s_{3.383}, s'_{0.1})\,(s_{3.383}, s'_{0.1})\,(s_{3.383}, s'_{0.1})\,(s_{3.417}, s'_{0.120}) \\ (s_{3.417}, s'_{0.1})\,(s_{3.417}, s'_{0.1})\,(s_{3.417}, s'_{0.120}) \end{pmatrix}^T,$$

$$\tilde{Z}''_4 = \begin{pmatrix} (s_{0.314}, s'_{1.981})\,(s_{0.3}, s'_{1.981})\,(s_{0.314}, s'_{1.845})\,(s_{0.3}, s'_{1.990}) \\ (s_{0.314}, s'_{1.981})\,(s_{0.329}, s'_{1.991})\,(s_{0.314}, s'_{1.981})\,(s_{0.3}, s'_{1.990}) \\ (s_{0.3}, s'_{1.990})\,(s_{0.314}, s'_{1.991})\,(s_{0.3}, s'_{1.857}) \end{pmatrix}^T.$$

Step 1.4 Since $\tilde{Z}'_1 \neq \tilde{Z}''_h (h = 1, 2, 3, 4)$, Steps 1.2 and 1.3 are repeated. After three iterations, the cluster is convergent and the final four clusters are determined as:

Cluster 1: $\tilde{Z}_2, \tilde{Z}_3, \tilde{Z}_7, \tilde{Z}_{11}, \tilde{Z}_{15}, \tilde{Z}_{19}, \tilde{Z}_{23}$; Cluster 2: $\tilde{Z}_1, \tilde{Z}_5, \tilde{Z}_9, \tilde{Z}_{13}, \tilde{Z}_{17}, \tilde{Z}_{21}$;

Cluster 3: $\tilde{Z}_6, \tilde{Z}_{10}, \tilde{Z}_{14}, \tilde{Z}_{18}, \tilde{Z}_{22}$; Cluster 4: $\tilde{Z}_4, \tilde{Z}_8, \tilde{Z}_{12}, \tilde{Z}_{16}, \tilde{Z}_{20}, \tilde{Z}_{24}, \tilde{Z}_{25}$.

Step 2 This step is to aggregate the cluster linguistic Z-number evaluation vectors $\tilde{Z}_h (h = 1, 2, \ldots, 25)$.

Step 2.1 Let $r = 1$. The cluster linguistic Z-number evaluation vector of each cluster is obtained as:

$$\tilde{Z}_1^{*(1)} = \begin{pmatrix} (s_{0.157}, s'_{0.109})\,(s_{0.157}, s'_{0.100})\,(s_{0.186}, s'_{0.108})\,(s_{0.171}, s'_{0.108}) \\ (s_{0.171}, s'_{0.100})\,(s_{0.157}, s'_{0.100})\,(s_{0.157}, s'_{0.100})\,(s_{0.171}, s'_{0.125}) \\ (s_{0.157}, s'_{0.109})\,(s_{0.171}, s'_{0.100})\,(s_{0.157}, s'_{0.100}) \end{pmatrix}^T,$$

$$\tilde{Z}_2^{*(1)} = \begin{pmatrix} (s_{4.000}, s'_{1.867})\,(s_{3.933}, s'_{1.966})\,(s_{3.933}, s'_{1.968})\,(s_{4.000}, s'_{2.000}) \\ (s_{3.967}, s'_{1.966})\,(s_{3.933}, s'_{1.900})\,(s_{3.967}, s'_{1.966})\,(s_{4.000}, s'_{1.933}) \\ (s_{3.967}, s'_{1.966})\,(s_{4.000}, s'_{2.0})\,(s_{4.000}, s'_{1.967}) \end{pmatrix}^T,$$

$$\tilde{Z}_3^{*(1)} = \begin{pmatrix} (s_{4.0}, s'_{0.100})\,(s_{4.0}, s'_{0.160})\,(s_{4.0}, s'_{0.100})\,(s_{3.92}, s'_{0.120}) \\ (s_{3.96}, s'_{0.100})\,(s_{3.96}, s'_{0.100})\,(s_{3.96}, s'_{0.100})\,(s_{4.0}, s'_{0.120}) \\ (s_{4.0}, s'_{0.100})\,(s_{4.0}, s'_{0.100})\,(s_{4.0}, s'_{0.120}) \end{pmatrix}^T,$$

$$\tilde{Z}_4^{*(1)} = \begin{pmatrix} (s_{0.314}, s'_{1.982})\,(s_{0.3}, s'_{1.981})\,(s_{0.314}, s'_{1.845})\,(s_{0.3}, s'_{1.990}) \\ (s_{0.314}, s'_{1.982})\,(s_{0.329}, s'_{1.991})\,(s_{0.314}, s'_{1.982})\,(s_{0.3}, s'_{1.990}) \\ (s_{0.3}, s'_{1.990})\,(s_{0.314}, s'_{1.990})\,(s_{0.3}, s'_{1.857}) \end{pmatrix}^T.$$

Step 2.2 By Eq. (11), the group linguistic Z-number evaluation vector $\tilde{Z}^{*(1)}$ is calculated as:

$$\tilde{Z}^{*(1)} = \begin{pmatrix} (s_{1.892}, s'_{1.084})\,(s_{1.872}, s'_{1.151})\,(s_{1.884}, s'_{1.118})\,(s_{1.876}, s'_{1.166}) \\ (s_{1.880}, s'_{1.133})\,(s_{1.872}, s'_{1.101})\,(s_{1.876}, s'_{1.135})\,(s_{1.892}, s'_{1.123}) \\ (s_{1.88}, s'_{1.130})\,(s_{1.896}, s'_{1.150})\,(s_{1.888}, s'_{1.136}) \end{pmatrix}^T.$$

Note that the weights of the four clusters are determined as $w_1^{*(1)} = 0.28, w_2^{*(1)} = 0.24, w_3^{*(1)} = 0.2, w_4^{*(1)} = 0.28$.

Step 3 Using Eqs. (12) and (13), the group consensus degree is computed as $GCD_{(1)} = 0.585$. In this example, the consensus threshold value ε is set as 0.6. As $GCD_{(1)}$- $< \varepsilon$, the cluster linguistic Z-number evaluation vectors are modified by the experts.

Step 4 With Eq. (14), the four clusters' weights are updated as:$w_1^{*(2)} = 0.29, w_2^{*(2)} = 0.16, w_2^{*(2)} = 0.22, w_3^{*(2)} = 0.33$. Then let $r = 2$ and return to Step 2.2.

At last, the group consensus degree is derived as $GCD_2 = 0.887$ and the consensus is acceptable. The final group linguistic Z-number evaluation vector is determined as:

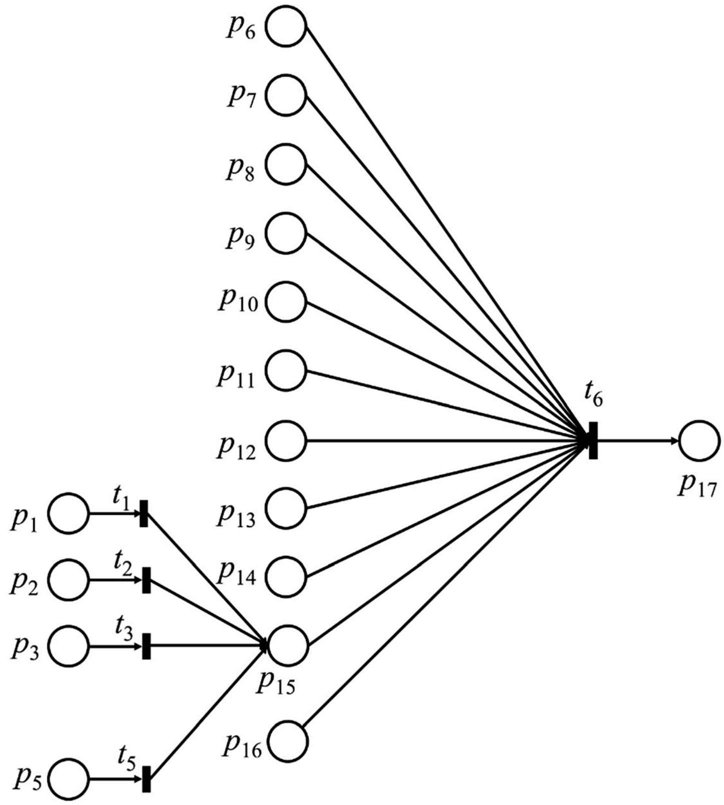

Fig. 4 Simplified LZPN model for the security risk assessment

$\tilde{Z}^* = \big((s_{1.5}, s'_{0.875}), (s_{1.5}, s'_{0.875}), (s_{2.125}, s'_{0.853}), (s_1, s'_{0.1}), (s_{2.125}, s'_{0.853}), (s_{2.125}, s'_{0.853}), (s_{2.125}, s'_{0.853}), (s_{1.5}, s'_{0.792}) (s_{2.0}, s'_{1.375}), (s_{3.875}, s'_{1.871}), (s_{1.875}, s'_{1.367})\big)^T$.

Step 5 Based on Eq. (15), the local weights of the 11 antecedent propositions are determined as:

$LW = [1 \;\; 1 \;\; 1 \;\; 1 \;\; 0.067\; 0.067\; 0.085\; 0.029\; 0.085\; 0.085\; 0.085\; 0.0610.115\; 0.209\; 0.112\; 0]^T$.

Similarity, the knowledge parameters of GW, $\tilde{U}$ and $\widetilde{Th}$ are acquired as shown here:

$GW = [0.262\; 0.262\; 0.132\; 0.082\; 0.262\; 1]$,

$$\widetilde{Th} = \big[(s_{0.275}, s'_{0.218})(s_{0.275}, s'_{0.218})(s_{0.275}, s'_{0.218})(s_3, s'_{1.333}) (s_{0.275}, s'_{0.218})(s_{0.275}, s'_{0.218}) (s_{0.275}, s'_{0.218})(s_{0.275}, s'_{0.218})(s_{0.275}, s'_{0.218}) (s_{0.275}, s'_{0.218})(s_{0.275}, s'_{0.218})(s_{0.275}, s'_{0.218}) (s_{0.275}, s'_{0.218})(s_{0.275}, s'_{0.218})(s_{0.275}, s'_{0.218})(s_{0.275}, s'_{0.218})\big]^T,$$

$$\tilde{U} = \big[(s_{3.8}, s'_{1.855})(s_{3.8}, s'_{1.855})(s_{3.8}, s'_{1.855})(s_{3.8}, s'_{1.855}) (s_{3.8}, s'_{1.855})(s_{3.8}, s'_{1.855})\big]..$$

For the security risk assessment, the truth degrees of the starting places are obtained as follows:

$$\tilde{\alpha}(p_1)=(s_{2.5}, s'_{0.5}), \tilde{\alpha}(p_2)=(s_{2.5}, s'_{0.5}), \tilde{\alpha}(p_3)=(s_{2.5}, s'_{0.5}), \tilde{\alpha}(p_4)=(s_{2.5}, s'_{1.5}), \tilde{\alpha}(p_5)=(s_{2.5}, s'_{1.5}), \tilde{\alpha}(p_6)=(s_{1.5}, s'_{0.2}), \tilde{\alpha}(p_7)=(s_{1.5}, s'_{0.2}), \tilde{\alpha}(p_8)=(s_4,s'_2), \tilde{\alpha}(p_9)=(s_4, s'_2), \tilde{\alpha}(p_{10})=(s_{1.5}, s'_{0.2}), \tilde{\alpha}(p_{11})=(s_4, s'_2), \tilde{\alpha}(p_{12})=(s_{2.5}, s'_{1.5}), \tilde{\alpha}(p_{13})=(s_{2.5}, s'_{1.5}), \tilde{\alpha}(p_{14})=(s_{2.5}, s'_{1.5}), \tilde{\alpha}(p_{16})=(s_{2.5}, s'_{1.5}).$$

According to the introduced simplification method of LZPNs, the truth degree of p_4 is smaller than its threshold value $(s_3, s'_{1.333})$. Thus, a simplified LZPN can be obtained as shown in Fig. 4.

As a result, the initial marking vector of the simplified LZPN is expressed as:

$$\tilde{M}_0 = \big[(s_{2.5}, s'_{0.5})(s_{2.5}, s'_{0.5})(s_{2.5}, s'_{0.5})(s_{2.5}, s'_{1.5}) (s_{1.5}, s'_{0.2}) (s_{1.5}, s'_{0.2})(s_4,s'_2) (s_4,s'_2)(s_{1.5}, s'_{0.2}) (s_4, s'_2)(s_{2.5}, s'_{1.5}) (s_{2.5}, s'_{1.5})(s_{2.5}, s'_{1.5}) (s_0, s'_0)(s_{2.5}, s'_{1.5})(s_0, s'_0)\big]^T.$$

Further, from Fig. 4, we can obtain

$$I = \begin{bmatrix} 10000\\ 01000\\ 00100\\ 00010\\ 00001\\ 00001\\ 00001\\ 00001\\ 00001\\ 00001\\ 00001\\ 00001\\ 00001\\ 00001\\ 00001\\ 00000 \end{bmatrix}_{16\times 5}, O = \begin{bmatrix} 00000\\ 00000\\ 00000\\ 00000\\ 00000\\ 00000\\ 00000\\ 00000\\ 00000\\ 00000\\ 00000\\ 00000\\ 00000\\ 11110\\ 00000\\ 00001 \end{bmatrix}_{16\times 5}.$$

Next, the reasoning algorithm of LZPNs is implemented to determine of the given chemical plant' security risk value.

(1) Let $k = 1$ and the enabled place vector $D^{(1)}$ is computed as:
$D^{(1)} = \tilde{M}_0 \triangleright \widetilde{Th} = [1$
$1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 0\ 1\ 0]^T$.

(2) The truth vector of input places $\tilde{\Gamma}^{(1)}$ is determined as:
$$\tilde{\Gamma}^{(1)} = LZWA\left((I \circ W_L)^T, \tilde{M}_0\right) = \left[(s_{2.5}, s'_{0.5})(s_{2.5}, s'_{0.5})(s_{2.5}, s'_{0.5})(s_{2.5}, s'_{1.5})(s_{2.056}, s'_{1.486})\right].$$

(3) The enabled transition vector $F^{(1)}$ is computed as:
$$F^{(1)} = (E \times I) \triangleright \left(\left(D^{(1)}\right)^T \times I\right) = [1\ \ 1\ 1\ 1\ \ 0].$$

(4) The output truth value vector $\Psi^{(1)}$ is calculated as:
$$\tilde{\Psi}^{(1)} = \left(F^{(1)} \circ \tilde{\Gamma}^{(1)}\right) \circ \tilde{U} = \left[(s_{2.375}, s'_{0.464})(s_{2.375}, s'_{0.464})(s_{2.375}, s'_{0.464})(s_{2.375}, s'_{1.391})(s_0, s'_0)\right].$$

(5) The new marking vector $\tilde{M}_1$ is acquired as:
$$\tilde{M}_1 = \tilde{M}_0 \oplus LZWA\left((O \circ W_G), \tilde{\Psi}^{(1)}\right) = \left[(s_{2.5}, s'_{0.5})(s_{2.5}, s'_{0.5})(s_{2.5}, s'_{0.5})(s_{2.5}, s'_{1.5})(s_{1.5}, s'_{0.2})(s_{1.5}, s'_{0.2})(s_4, s'_2)(s_4, s'_2)(s_{1.5}, s'_{0.2})(s_4, s'_2)(s_{2.5}, s'_{1.5})(s_{2.5}, s'_{1.5})(s_{2.5}, s'_{1.5})(s_{2.178}, s'_{0.728})(s_{2.5}, s'_{1.5})(s_0, s'_0)\right]^T.$$

(6) Since $\tilde{M}_1 \neq \tilde{M}_0$, let $k = 2$ and we continue to next iteration and obtain
$$\tilde{M}_2 = \left[(s_{2.5}, s'_{0.5})(s_{2.5}, s'_{0.5})(s_{2.5}, s'_{0.5})(s_{2.5}, s'_{1.5})(s_{1.5}, s'_{0.2})(s_{1.5}, s'_{0.2})(s_4, s'_2)(s_4, s'_2)(s_{1.5}, s'_{0.2})(s_4, s'_2)(s_{2.5}, s'_{1.5})(s_{2.5}, s'_{1.5})(s_{2.5}, s'_{1.5})(s_{2.178}, s'_{0.728})(s_{2.5}, s'_{1.5})(s_{2.386}, s'_{1.250})\right]^T.$$

(7) Since $\tilde{M}_2 \neq \tilde{M}_1$, let $k = 3$ and we get
$$\tilde{M}_3 = \left[(s_{2.5}, s'_{0.5})(s_{2.5}, s'_{0.5})(s_{2.5}, s'_{0.5})(s_{2.5}, s'_{1.5})(s_{1.5}, s'_{0.2})(s_{1.5}, s'_{0.2})(s_4, s'_2)(s_4, s'_2)(s_{1.5}, s'_{0.2})(s_4, s'_2)(s_{2.5}, s'_{1.5})(s_{2.5}, s'_{1.5})(s_{2.5}, s'_{1.5})(s_{2.178}, s'_{0.728})(s_{2.5}, s'_{1.5})(s_{2.386}, s'_{1.250})\right]^T.$$

Since $\tilde{M}_3 = \tilde{M}_2$, the knowledge reasoning is ended and the final truth values of all the places can be acquired from $\tilde{M}_3$. Thus, the truth degree of the terminating place p_{17} is $(s_{2.386}, s'_{1.250})$, which means that the risk level of terrorist attacks for the considered refinery is high. The threat of terrorists striking is considered real as this plant handles large amounts of flammable, explosive or toxic hazardous materials that can cause great loss and social impact. Therefore, the plant should identify threat-drivers and take security countermeasures to reduce the target attractiveness and consequences in case of a successful attack.

5.3 Sensitivity Analysis

In the above application, the number of clusters is set as four (H = 4) for determining the knowledge parameters of LZPRs. In this section, a sensitivity analysis is conducted by choosing different number of clusters in the large group knowledge acquisition process. According to the proposed knowledge acquisition approach, we can obtain the local weights of the 11 antecedent places as presented in Fig. 5, where H is set to [1, 25]. From Fig. 5, it can be observed that the values of local weights vary with the change of H. For most of the cases, the value of lw_{14} is the largest, followed by values of lw_{13} and lw_{15}. And the value of lw_8 is the smallest in the 11 antecedent places. The fourteenth local weight (lw_{14}) seems to be the most importance place

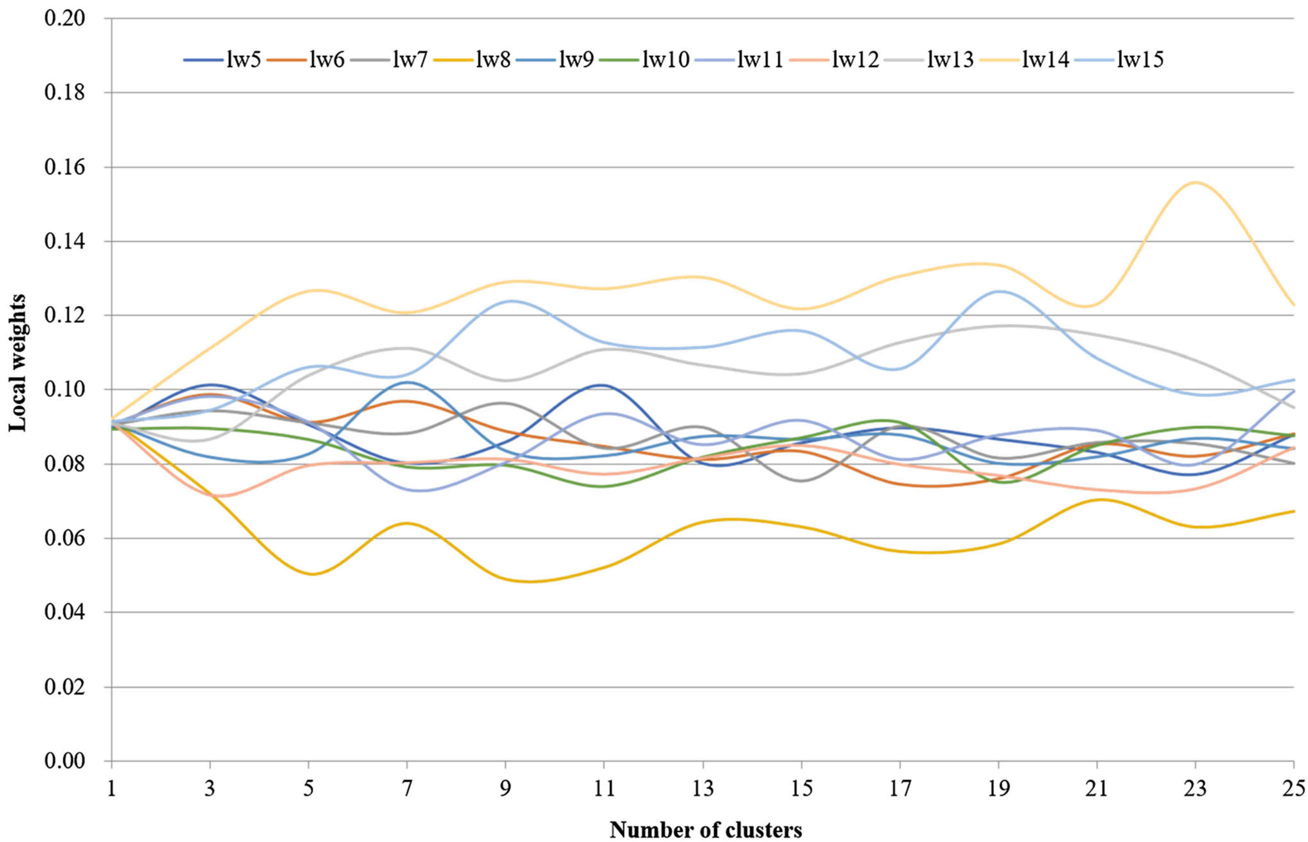

Fig. 5 Local weight results of sensitivity analysis

contributing to the transition t_6. And, the value of lw_8 is the place contributing the lowest to the transition t_6.

The sensitivity analysis shows that the number of clusters has an impact on the final values of local weights and there are differences in the results obtained by selecting different number of clusters. Therefore, considering the number of clusters based on actual situation is of vital importance. In general, the number of clusters is a user-specified parameter. Since the variation of H may lead to different values of knowledge parameters, experts may have difficulty in identifying the most suitable number of clusters. The suitable number of clusters is beneficial for obtaining the knowledge parameters more precisely. Thus, experts are supposed to choose an appropriate number of clusters for the reliability of the final results according to the distribution characteristics of data and the characteristics of clustering requirements in practical applications.

5.4 Comparison Analysis

In this section, a comparison analysis with relevant existing FPN models is conducted to show the effectiveness of the proposed LZPNs. The same security risk analysis is solved by the linguistic reasoning Petri nets (LRPNs) [16], the weighted FPNs (WFPNs) [39], and the traditional FPNs [7]. According to these FPN models, inference results of the terminating place p_{17} are determined and listed in Table 3. Because the WFPNs and the FPNs are fuzzy logic-based methods, their certainty factor vector U and initial marking vector M_0 are given as follows:

Table 3 Reasoning results by different FPN methods

Models	p_{17}	Security risk status
LZPNs	$(s_{2.386}, s'_{1.250})$	High
LRPNs	$\Delta(0.564)$	High
WFPNs	0.57	High
FPNs	0.46	Moderate

$$U = [1\ \ 1\ \ 1\ \ 1\ \ 1\ \ 1],$$

$$M_0 = [0.4\ \ 0.4\ \ 0.4\ \ 0.6\ \ 0.6\ \ 0.2\ \ 0.2\ \ 1.0\ \ 1.0\ \ 0.0\ \ 1.0\ \ 0.6\ \ 0.6\ \ 0.6\ \ 0.0\ \ 0.6\ 0.0]^T.$$

From Table 3, it can be seen that the security risk status derived by the traditional FPNs is different from that determined by the other three models. This inconsistence can be explained by the shortcomings of the traditional

FPN method. First, min/max operators are used in the FPNs for knowledge reasoning. As a result, extreme truth values of places will have a great impact on the final reasoning result. Second, local and global weights are not considered in the traditional FPNs. This may be unreasonable when propositions and rules have different importance in an expert system. Third, the reliability of experts' knowledge information cannot be reflected in the normal FPN model. Thus, domain experts cannot flexibly express their assessments of knowledge parameters. These imply a lack of accuracy of the reference results with the traditional FPNs.

In addition, we can see that the security risk status of the refinery plant is high according to the WFPNs, the LRPNs and the proposed LZPNs. This verifies the effectiveness of the proposed LZPN model for knowledge representation and reasoning. However, fuzzy numbers and linguistic 2-tuples were adopted in the WFPNs and the LRPNs, respectively, for depicting expert knowledge. They ignore the reliability of cognitive information, and are not able to describe the uncertainty of knowledge information sufficiently. Besides, the WFPNs don't consider global weights and threshold values in the knowledge reasoning, which may cause biased reasoning results. The reasoning of the WFPNs and the LRPNs is also time consuming due to the increasing of scale for big and complex expert systems. Furthermore, only a small group of experts is involved in determining knowledge parameters of the WFPNs and the LRPNs, which cannot reduce the influence of unfair and biased information. By contrast, based on a large group of experts, the knowledge parameters acquired by the proposed large group knowledge acquisition approach are more reliable.

As to reasoning complexity, the fuzzy reasoning algorithm of the traditional FPNs is implemented using a reachability tree-based method, which require the enumeration of all possible paths from the starting places to the terminating ones, leading to a less efficient reasoning algorithm. In contrast, the WFPNs, the LRPNs, and the proposed LZPNs algorithm adopt a matrix equation format and can execute knowledge reasoning in a parallel way. It will stop at a finite number of iterations $k \leq h^{*} + 1$, where h^{*} is the number of transitions in the longest place-transition direct path. Furthermore, the presented reasoning algorithm is more efficient than the WFPNs and the LRPNs since it is based on a simplified FPN with less places and transitions in the knowledge inference process (as shown in Fig. 4).

According to the above comparative analysis, it can be concluded that the proposed LZPN model is more feasible and effective in addressing practical knowledge acquisition and representation problems. In contrast to the traditional FPN model and its improved versions, the proposed LZPN model has the following advantages: (1) Using linguistic Z-numbers, the proposed LZPNs are more efficient to deal with experts' knowledge information and capture the reliability of their cognitive information in knowledge representation. (2) Based on the proposed knowledge acquisition approach, more precise and reliable knowledge parameters can be determined on the basis of a large number of experts in knowledge acquisition. (3) With the simplification method and the matrix operations-based parallel reasoning algorithm, the structure of LZPNs can be simplified and the knowledge reasoning process can be more efficient. Therefore, the proposed model is suitable for modeling complex rule-based systems and is more effective for knowledge representation and reasoning under the large group environment.

6 Conclusions

In this study, we proposed a new kind of modified FPN model, named LZPNs, for knowledge acquisition and representation in the large group environment. The linguistic Z-numbers are adopted to handle human cognitive information and capture the reliability of expert evaluations in knowledge representation. A new knowledge acquisition approach was introduced to determine the knowledge parameters of LZPNs based on a large number of experts. Further, an application example regarding chemical security risk assessment is utilized to demonstrate the proposed LZPN model. Via sensitivity analysis and comparison analysis, it is concluded that the proposed LZPN model can overcome the defects of FPNs and is a preferable approach for knowledge management. To sum up, the proposed model can capture experts' complex and uncertain cognitive information comprehensively in knowledge representation; it is able to acquire more precise and reasonable knowledge parameters of LZPNs based on a large group of domain experts; it can simplify the structure of LZPNs and reduce the cost and complexity of knowledge reasoning especially for large-scale expert systems.

There exist several limitations of the proposed LZPN model, which can be addressed in the future research. First, the input places of transitions are assumed to be independent in this study. In real applications, however, various types of relationships may exist among input places. Thus, further extension of the proposed model can be explored to solve the knowledge representation problems with interrelated input places. Second, only one format of assessments is allowed to describe experts' knowledge in the proposed LZPN method. In the future, the proposed method can be extended for the cases in which the knowledge data have different forms such as interval values, fuzzy numbers, and linguistic terms. In addition, the LZPNs can be implemented to solve other practical expert systems to further demonstrate its effectiveness and practicability.

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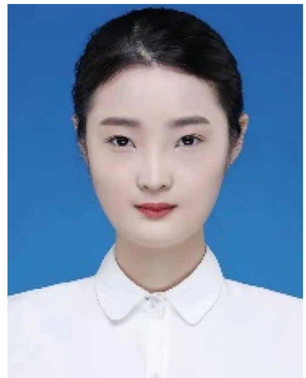

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