FPT Algorithms – Incremental Dominating Set Solutions

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2.5 DOMINATING SET

Dominating set is one of natural properties of graphs, while DOMINATING SET problem is one of complex problems studied in complexity theory. DOMINATING SET problem is categorized in $NP - complete$ **class [GJ79].**

DOMINATING SET

Instance **A graph** $G = (V, E)$ **and** $k \in \mathbb{N}$.

Question **Is there a dominating set** $D \subseteq V$ **for** G **such that** $|D| \leqslant k$**? [GJ79]**

DOMINATING SET has been proved to be a $W[2] - complete$ **problem by Downey and Fellows in 1995 [DF95]. In another words, this problem is not a** FPT **problem and does not have kernel. Nevertheless, the incremental edition of this problem,** $INCREEMENTAL \quad DOMINATING \quad SET$ **problem, can be classified as a** FPT **problem [RGD14].**

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3.4.1 Hamming Distance

Before talking about the INCREEMENTAL DOMINATING SET problem, we need clarify some concepts to help us understand the increment

problem.

Assuming there are two vectors with the same length, we can check if the symbols in the corresponding positions of the two vectors are same or different. We call the number of positions where the symbols are different as *Hamming Distance* [Ham50]. Obviously, this technique can be applied in graphs. Firstly, given two graphs $G = (V, E)$ and $G' = (V, E')$, both have the same set of vertices but different set of edges. We say that set E' is obtained by a series of *edge edit operations* from E, which refers to edge deletion and edge addition. Secondly, we can establish two 0/1 vectors to indicate E and E'. Thirdly, we can find the Hamming distance between E and E', which is denoted by $d_e(G, G')$. We call $d_e(G, G')$ as *edge edit distance*. In the fourth step, if there exists a solution of vertex set $S \subset V$ for graph G and there may or may not exist another solution $S' \subset V$ for G' with respect to a certain graph problem, we can also establish two 0/1 vectors to indicate S and S'. Finally, we can define the Hamming distance $d_H(S, S')$ as the *vertex solution set distance* [RGD14].

3.4.2 INCREEMENTAL DOMINATING SET

With the assistance of the Hamming distance of $d_e(G, G')$ and $d_H(S, S')$, we can define INCREEMENTAL PROBLEM.

INCREEMENTAL PROBLEM (INC-PROBLEM)

Instance A graph $G = (V, E)$ and a set $S \subseteq V$ where S has a certain

property P for G,

A graph $G' = (V, E')$ with $d_e(G, G') \leqslant k$,

$k, r \in \mathbb{N}$

Parameter (k, r)

Question **Is there a set $S' \subseteq V$ such that S' has property P for G' and $d_H(S, S') \leqslant r$**

[RGD14]

With regards to the property of dominating set of graph, the definition of INCREEMENTAL DOMINATING SET problem can be presented in the following form:

INCREEMENTAL DOMINATING SET (INC-DS)

Instance **A graph $G = (V, E)$ and a dominating set $S \subseteq V$ for G,**

A graph $G' = (V, E')$ with $d_e(G, G') \leqslant k$,

$k, r \in \mathbb{N}$

Parameter (k, r)

Question **Is there a dominating set $S' \subseteq V$ such that $d_H(S, S') \leqslant r$**

[RGD14]

There is another problem relevant to INC-DS, we also present it in the following form:

DOMINATING A VERTEX COVER (DOM-A-VC)

Instance **A graph $G = (V, E)$ and a vertex cover $C \subseteq V$ for G,**

$r \leqslant k, r, k \in \mathbb{N}$

Parameter (k,r)

Question Is there a set $D \subseteq V$ such that $|D| \leqslant r$ and D dominates the vertex cover C?

[RGD14]

3.4.3 Harmness in INC-DS

We have already known graph G' is obtained from G by a series of edge edit operations. If a certain edge edit operation results in a non-dominated vertex in G', we say that such edge edit operation does $harm$ the vertex solution set S for G [RGD14]. Conversely, we can say some edge edit operations do not harm S if no non-dominated vertex is introduced. Edge addition, for instance, will not harm S. In another words, edge deletion is likely to harm S. We will discuss which kind of edge deletion can be $harmful$.

Definition 3.4.1 *H-edit [RGD14] Given an instance* (G, G', S, k, r) *of INC-DS,* v *is a non-dominated vertex in* G'*. Let* $U = N_G(v) \cap S$*. There is a linear order* $>_L$ *on* U *and* u_m *is the greatest vertex in* U *under* $>_L$*. The deletion of edge* (u_m, v) *in the transition from* G *to* G' *is called* $H - edit$*.*

Since v is a non-dominated vertex in G', $v \notin S$ in G. In another words, at least one of v's neighbours is in S, which can be expressed as $U = N_G(v) \cap S = \{u_1, u_2, \cdots, u_m\}$. Obviously, as long as one edge between v and one of its neighbours exists, v will not be a non-dominated vertex in G'. Under the linear order $>_L$, the greatest element u_m ensures that any deletion of edge (u_i,v) will not harm S when $i \neq m$. It proves that an edge edit operation harms S if and only if it is an H-edit [RGD14].

3.4.4 Reduction Rules for INC-DS

Let $h \leqslant k$ be the number of harmful edge edit operations in the transition from G to G', the remaining $k - h$ edge edit operations will not harm S. In another words, we can construct an instance $I^* = (G^*, G', h, r)$ from $I = (G, G', k, r)$ via $k - h$ harmless edge edit operations. Obviously, if I is a $Yes - Instance$ if and only if I^* is a $Yes - Instance$ [RGD14].

Lemma 3.4.2 *For a given instance $(I, (k, r)$ of the problem INC-DS$(k, r))$, there is a parameterized reduction to an instance $(I', (k, r))$ of DOM-A-VC , such that $|I'| \leqslant |I|$. [RGD14]*

Let $I = (G, G', S, k, r)$ be an instance of largeINC-DS(k, r) . Let $C = V(G')\mathcal{N}_{G'}[S]$ and $B = N_{G'}(S)$. The sets C, B and S form a partition of the vertices of G'. Since there are $h \leqslant k$ harmful edge edit operations when transforming G to G', $|C| = h \leqslant k$, which also implies that the vertices in

C are only dominated by vertices in B or C. We can apply some reduction rules to graph G'.

- **Reduction Rule 1: If $v \in S$, remove v and its incident edges;**
- **Reduction Rule 2: If $v \in B$ and $N(v) \cap C = \emptyset$, remove v and its incident edges;**
- **Reduction Rule 3: If (u, v) is an edge and $u, v \subseteq B$, remove the edge (u, v);**

[RGD14]

After applying the reduction rules, we can find the remains of B is an independent set in the new obtained graph H. In another words, C is a vertex cover for H. An instance $(I, (k, r))$ of the problem INC-DS(k, r) is reduced to an instance $(I', (k, r))$ of DOM-A-VC . If I' is a $Yes - Instance$, there exists a set D dominating C, whose size $\leqslant r$. Thus,there exists a dominating set $S' = S \cup D$ with $d_H(S, S') \leqslant r$ in G', or we can say I is a $Yes - Instance$. On the contrary, if I is a $Yes - Instance$, which suggests that there is a dominating set S' with $d_H(S, S') \leqslant r$ in G', we can find a set of vertices $D = S'\ S$ to dominate the vertices in C. In another words, I' is a $Yes - Instance$. In summary, I is a $Yes - Instance$ if and only if I' is a $Yes - Instance$.

4

Conclusion

Appendix A

Source Code

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