Numerical Methods

Course Title: Numerical Methods
Course No: BIT203
Nature of the Course: Theory + Lab
Semester: III

Full Marks: 60 + 20 + 20
Pass Marks: 24 + 8 + 8
Credit Hrs: 3

Course Description:
This course covers different concepts of numerical techniques of solving non-linear equations, system of linear equations, integration and differentiation, and ordinary and partial differential equations.

Course Objective:
The main objective of this course is to provide concepts of numerical techniques for solving different types of equations and developing algorithms for solving scientific problems.

Detailed Syllabus

Unit	Teaching Methodology	Teaching Hours
Unit 1: Solution of Nonlinear Equations (7 Hrs.)		
1.1 Errors in Numerical Calculations (True Error, Relative Error, Approximate Error, Relative Approximate Error), Sources of Errors (Round-off Error, Truncation Error), Propagation of Errors, Review of Taylor's Theorem	Lecture + Laboratory Work	1
1.2 Concept of Linear and Non-linear Equations, Solving Non-linear Equations: Trial and Error Method, Bisection Method, Newton Raphson Method, Secant Method, Fixed Point Method, False Position Method, Newton's Method for Calculating Multiple Roots, Evaluating Polynomials with Horner's Method, Derivation, Algorithm and Example of each method		6
Unit 2: Interpolation and Regression (8 Hrs.)		
2.1 Concept of Interpolation and Extrapolation, Lagrange's Interpolation, Newton's Interpolation using divided differences, forward differences and backward differences, Derivation, Algorithm and Example of each method	Lecture + Laboratory Work	5
2.2 Concept of Regression, Regression vs. Interpolation, Least Squares Methods, Linear Regression, Non-linear Regression: Exponential regression by linearization, and Polynomial,		3

Derivation, Algorithm and Example of each method		
Unit 3: Numerical Differentiation and Integration (9 Hrs.)		
3.1 Concept of Differentiation, Differentiating Continuous Functions (Two-Point Forward and Backward Difference Formulae, Three-Point Formula), Differentiating Tabulated Functions by using Newton’s Differences (Divided, Forward and Backward Difference Formulae), Maxima and minima of Tabulated Functions, Derivation, Algorithm and Example of each method 3.2 Concept of Integration, Newton-Cote's Quadrature Formulae: Trapezoidal rule, Multi-Segment Trapezoidal rule, Simpson's 1/3 rule, Multi-Segment Simpson's 1/3 rule, Simpson's 3/8 rule, Multi-Segment Simpson's 3/8 rule, Derivation, Algorithm and Example of each method	Lecture + Laboratory Work	5 4
Unit 4: Solving System of Linear Equations (8 Hrs.)		
4.1 Existence of Solutions, Properties of Matrices, Matrix Representation, Gaussian Elimination Method, Partial and Complete Pivoting with Gaussian Elimination Method, Gauss-Jordan method, Inverse of matrix using Gauss-Jordan method, Derivation, Algorithm and Example of each method 4.2 Concept of LU Decomposition, Matrix factorization and Solving System of Linear Equations by using Doolittle and Cholesky Algorithm, Derivation, Algorithm and Example of each method 4.3 Iterative Solutions of System of Linear Equations, Jacobi Iteration Method, Gauss-Seidel Method, Derivation, Algorithm and Example of each method 4.4 Eigen Values and Eigen Vectors Problems, Power Method, Algorithm and Example of the method	Lecture + Laboratory Work	3 2 2 1
Unit 5: Solution of Ordinary Differential Equations (8 Hrs.)		
5.1 Concept of Differential Equations, Initial Value Problem, Taylor Series Method, Euler's Method, Heun's Method, Runge-Kutta Methods, Derivation, Algorithm and Example of each method	Lecture + Laboratory Work	5 3

5.2 Solving System of Ordinary Differential Equations, Solution of the Higher Order Equations, Boundary Value Problems, Shooting Method, Derivation, Algorithm and Example of each method.		
Unit 6: Solution of Partial Differential Equations (5 Hrs.)		
6.1 Concept of Partial Differential Equations, Classification of PDE, Deriving Difference Equations, Solving Laplacian Equation and Poisson's Equation, Derivation, Algorithm and Example of each method	Lecture + Laboratory Work	5

Text Books

1. W. Chency and D. Kincaid, "Numerical Mathematics and Computing", 7th Edition,
2. Brooks Cole Publisher.
3. C.F. Gerald and P.O. Wheatley, "Applied Numerical Analysis", 9th Edition, Addison
4. Wesley Publisher.

Reference Books

1. W.H. Press, B.P. Flannery et al., "Numerical Recipes: Art of Scientific Computing", 3rd Edition, Cambridge Press.
2. J. M. Mathews and K. Fink, “Numerical Methods using MATLAB “, 4th Edition, Prentice
3. Hall Publication.
4. S. S. Sastry, “Introductory Methods of Numerical Analysis”, 5th Edition, PHI Publication.
5. E Balagurusamy, Numerical Methods, 1st Edition, MC Graw Hill India.
6. A. S. Saud, Numerical Methods with Practical Approach, Numerical Methods, KEC Publication and Distributors.

Model Question

Attempt ANY TWO {10 ×2=20}

1. How Secant methods differs from Newton Raphson method? Derive the formula for Secant Method. Solve the equation $cosx+2sinx-x^2=0$ using Secant method. Assume error precision is 0.01. {2+4+4}
2. How interpolation differs from regression? Write down algorithm and program for Lagrange interpolation. {2+4+4}
3. Explain the working of Jacobi Iteration method? Solve the following system of equations using the method. Assume error precision is 0.01. Compare Jacobi Iteration method with Gauss-Seidel method. {4+4+2}

$5x-2y+3z=-1$

$-3x+9y+z=2$

$2x-y-7z=-3$

Group B

Attempt ANY EIGHT {5 ×8=40}

4. Define the terms true error and relative error? Write down algorithm for Horner' method to evaluate polynomial and use the method to evaluate the polynomial $2x^3-3x^2+5x-2$ at $x=3$. {1+2+2}
5. Construct Newton's backward difference table for the given data points and approximate the value of $f(x)$ at $x=45$.

X	10	20	30	40	50
$f(x)$	0.173	0.342	0.5	0.643	0.766

6. Fit the quadratic curve through the following data points and estimate the value of $f(x)$ at $x=2$.

x	1	3	4	5	6
y	2	7	8	7	5

7. Derive formula for the Doolittle LU decomposition matrix factorization method.
8. How can we calculate derivatives of continuous functions? Write down algorithm and program for differentiating continuous function using two point forward difference formula.
9. Find following integral using composite trapezoidal rule using 2 segments (k=2) and 4 segments (k=4).

$$\int_2^8 (x^3+2)dx$$

10. Approximate the solution of $y' = 2x + y$, $y(0) = 1$ using Euler's method with step size of 0.1. Approximate the value of $y(0.4)$.
11. Solve the Poisson's equation $\nabla^2 f = xy$ with $f = 2$ on boundary by assuming square domain $0 \le x \le 3,\ 0 \le y \le 3$ and $h = 1$.
12. How boundary value problems differs from initial value problems? Discuss shooting method for solving boundary value problem.

Lab Manual

Lab Sheet#	Chapter
Lab Sheet#1 1. Write programs to implement Bisection method, Newton Raphson method, Secant method, False Position method, and Fixed Point method for solving non-linear equations. 2. Write a program to implement Horner’s method of polynomial evaluation.	Chapter 1
Lab Sheet#2 1. Write programs for Lagrange interpolation, Newton’s divided/forward/backward difference interpolation. 2. Write programs to implement linear and non-linear regression.	Chapter 2
Lab Sheet#3 1. Write programs to implement Two-Point Forward Difference, Two-Point Backward Difference and Three-Point algorithm to compute derivatives of continuous functions. 2. Write programs to implement Newton’s Divided, Forward and Backward Difference algorithms to compute derivatives of discrete functions. 3. Write a program to compute maxima and minima of tabulated functions. 4. Write program to compute integral value of functions using trapezoidal rule, multi-segment trapezoidal rule, Simpson’s rule 1/3 rule, multi-segment Simpson’s 1/3 rule, Simpson’s rule 3/8 rule and multi-segment Simpson’s 3/8 rule.	Chapter 3
Lab Sheet#4 1. Write programs to solve system of non-linear equations using Gauss-Elimination and Gauss Jordan method. 2. Write programs to factorize matrix using Doolittle and Cholesky method 3. Write programs to solve system of non-linear equations using Jacobi Iteration and Gauss-Seidel method. 4. Write a program to find eigenvalue and eigenvector using power method.	Chapter 4
Lab Sheet#5 1. Write programs to implement Taylor’s method, Euler’s method, Heun’s method and RK method to solve ordinary differential equations. 2. Write a program to solve second order ODE. 3. Write a program to solve boundary value problem using shooting method.	Chapter 5
Lab Sheet#6 1. Write programs to solve Laplacian Equation and Poisson's Equation.	Chapter 6