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Unit-3

NON PARAMETRIC TEST

⊛. Advantages of non parametric test:

→ Can be applied to qualitative data as well as quantative data.

→ For relatively small sample it is the only possible test.

→ It is simple to understand, quicker and eaiser to apply.

→ It is less time consuming.

→ It does not require complicated sampling theory.

→ It needs no assumption about population from which sample is selected.

⊛. Disadvantages of non parametric test:

→ It can not be used to estimate the parameters of population.

→ These tests are less reliable and less powerful than parametric tests.

→ These tests are less efficient than parametric tests.

→ In these tests many times lot of information are discarded or unused.

→ Lot of tables are needed for tests.

⊛. Measurement Scale:

The measurement consisting of counting the number of units or parts of units displayed by objects and phenomena is called measurement scale. Following are the different types of measurement scale.

i) Nominal scale → It is simplest and lowest level of measurement scale. It is simply a system of assining a number or the symbols to objects or events to distinguish one from another in order to label them. The symbols or numbers have no numeral meaning. The arithmetic operations can not be used for these numerals.

ii) Ordinal scale → The second and the lowest level of ordered scale is the ordinal scale. It is the quantification of items by ranking.

In this scale, the numerals are arranged in some order but the gaps between the positions of numerals are not made equal. It represents qualitative values in ascending or descending order.

iii) Interval scale → In addition to ordering the data, this scale uses equidistant units to measure the difference between the scores. It assumes data has equal intervals. This scale does not have absolute zero but only arbitrary zero. Interval scale is the developed form of ordinary scale.

iv) Ratio scale → Ratio scale is the ideal scale and an extended form of interval scale. It is most powerful scale of measurement. It possesses the characteristics of nominal, ordinal and interval scale. Ratio scale has an absolute zero or true zero or natural zero of measurement.

2. One Sample test:

ⓐ Run test: It is non parametric test used to determine the randomness of the selected samples. So, for Run test we have following steps;

i) Problem to test

Null Hypothesis: (H_0): Sample observations are in random order.

Alternate Hypothesis (H_1): Sample observations are not in random order.

ii) Test statistics,

No. of Runs = R

Sample size = n

no. of one type of sequence = n_1.

no. of another type of sequence = n_2.

iii) Critical value: At 5% level of significance for value n_1 and n_2 we look in table and find lower limit and upper limit.

iv) Decision:- If value of R lies between lower and upper limit then H_0 is accepted otherwise rejected.

v) Conclusion:- We make conclusion on the basis of decision.

Q1. Following is a sequence of head (H) and tail (T) in a tossing of a coin 14 times is; HTTHHHTHTTHHTH.
Test whether heads and tails occur in random order.

random word आए question मा जहिले run test गर्ने

Solution:

Given,

$$\underbrace{H}_{1}\ \underbrace{TT}_{2}\ \underbrace{HHH}_{3}\ \underbrace{T}_{4}\ \underbrace{H}_{5}\ \underbrace{TT}_{6}\ \underbrace{HH}_{7}\ \underbrace{T}_{8}\ \underbrace{H}_{9}$$

symbols दिएको बेला यसरी गर्ने अनि value दिएको बेला +ve र -ve गरेर Q.N.2 जस्तै

No. of Runs $(R) = \boxed{9}$

Sample size $(n) = 14$

no. of heads $(n_1) = 8$

no. of tails $(n_2) = 6$.

Problem to test:

Null Hypothesis (H_0): Head and tail occur in random order.

Alternate Hypothesis (H_1): Head and tail do not occur in random order.

Critical value:

At 5% level of significance for value of $n_1 = 8$ and $n_2 = 6$;

$$\left.\begin{aligned} \text{lower limit} &= 3 \\ \text{Upper limit} &= 12 \end{aligned}\right\} \text{table values}$$

Decision: Since value of $R = 9$ lies between 3 and 12. So, H_0 is accepted, i.e, H_1 is rejected.

Conclusion: Hence, Head and tail occur in random order.

Q2. From the given data from an experiment are;
64, 82, 58, 61, 78, 72, 69, 65, 79, 75, 80.
Test the randomness of data.

Solution: Given, 64, 82, 58, 61, 78, 72, 69, 65, 79, 75, 80.

Problem to test:

Null Hypothesis (H_0):- The data are in random order.

Alternate Hypothesis (H_1):- The data are not in random order.

Now,

$$\text{Median} = \text{value of } \left(\frac{n+1}{2}\right)^{th} \text{ item}$$

$$= \text{value of } 6^{th} \text{ item}$$

$$= 72.$$

Now, the values which are less than median (i.e, 72) are symbolized by negative sign and the values which are greater than median are symbolised by positive sign and equal to median is symbolized by zero, and it is not counted as run.

Now, For finding value of R

$$\underset{1}{\underset{-}{64}},\ \underset{2}{\underset{+}{82}},\ \underbrace{\underset{-}{58},\ \underset{-}{61}}_{3},\ \underset{4}{\underset{+}{78}},\ \underset{0}{72},\ \underbrace{\underset{-}{69},\ \underset{-}{65}}_{5},\ \underbrace{\underset{+}{79},\ \underset{+}{75},\ \underset{+}{80}}_{6}$$

Here, the number of runs $(R) = 6$.
no. of positive signs $(n_1) = 5$.
no. of negative signs $(n_2) = 5$.

Critical value: At 5% level of significance for value of $n_1 = 5$ and $n_2 = 5$ the lower and upper limit are;
lower limit = 2
Upper limit = 10.

Decision: Since $R=6$ lies between 2 and 10. So, H_0 is accepted. i.e, H_1 is rejected.

Conclusion: Hence the data are in random order.

Note: If the number of observation n_1 or n_2 is greater than 20 then test statistics $Z = \frac{r - \mu_r}{\sigma_r}$ which follows normal distribution mean 0 and deviation 1 $[\text{i.e} \sim N(0,1)]$.

where,

$$\mu_r = \frac{2n_1n_2}{n_1+n_2} + 1$$

$$\& \ \sigma_r^2 = \frac{2n_1n_2(2n_1n_2 - n_1 - n_2)}{(n_1+n_2)^2(n_1+n_2-1)}.$$

Example: The following is the arrangement of defective (d) and non-defective (n) pieces of keyboard produced in the given order by a certain machine: nnnnndddd nnnnnnnnnnddnndddddnnnddnndnnn.

Test the randomness at the 0.01 level of significance.

Solution:
Given, nnnnndddd nnnnnnnnnnddnnddd nnnddnndnnn.

Here, Number of non-defective $(n_1) = 25$
Number of defective $(n_2) = 13$
Number of runs $(r) = 11$.

Problem to test:

Null Hypothesis (H_0):- The defective and non-defective pieces produced by machine are in random order.

Alternate Hypothesis (H_1): The defective and non-defective pieces produced by machine are not in random order.

Test Statistics:

Since $n_1 > 20$, so test statistics,

$$Z = \frac{r - \mu_r}{\sigma_r}$$

where, $\mu_r = \frac{2n_1n_2}{n_1+n_2} + 1 = \frac{2\times25\times13}{25+13} + 1 = 18.105$

& $\sigma_r^2 = \frac{2n_1n_2(2n_1n_2 - n_1 - n_2)}{(n_1+n_2)^2(n_1+n_2-1)} = \frac{2\times25\times13(2\times25\times13-25-13)}{(25+13)^2(25+13-1)}$

$= 7.44$

So, $\sigma_r = \sqrt{7.44} = 2.72$

$\therefore Z = \frac{r-\mu_r}{\sigma_r} = \frac{11-18.105}{2.72} = -2.612$

$|Z| = 2.612$

Critical Value:
At 0.01 level of significance the value of Z_{tab} is $Z_{0.01} = 2.58$

Decision: Since $|Z_{cal}| = 2.612 > Z_{tab} = 2.58$. So, H_0 is rejected. i.e. H_1 is accepted.

Conclusion: The defective and non-defective pieces produced by machine are not produced in random order.

(b) Binomial Test:

This test used used to test whether the binomial population has two distinct groups of two equal numbers of outcomes or not.

For Sample size ($n \leq 25$)

Test statistics

$$X_0 = \min\{n_1, n_2\}$$

Critical value

Using the binomial distribution for probability $= \sum_{x=0}^{X_0} C(n,x)\left(\frac{1}{2}\right)^n$

Decision:

Accept H_0 at α level of significance if $p > \alpha$ for one tailed test and $2p > \alpha$ for two tailed test, reject otherwise.

For Large Sample size ($n > 25$)

For large sample size, X_0 is normally distributed with mean np and variance npq.

Test statistics

$$Z = \frac{X_0 - \mu}{\sigma} = \frac{X_0 - np}{\sqrt{npq}}$$

Since X_0 is discrete so that continuity correction is made as;

$$Z = \frac{(X_0 \pm 0.5) - np}{\sqrt{npq}}$$, use $+0.5$ if $X_0 < np$ and use -0.5 if $X_0 > np$.

Critical value,

Critical value Z_{tab} is obtained from table according to level of significance and alternative hypothesis.

Decision

Reject H_0 at α level of significance if $Z \geq Z_{tab}$, accept otherwise.

Example 1: The following are defective (D) and non defective (N) electronic items produced in the given order by a certain machine: NNDDNDNNNDDNDDNNDNDNNN.

Test whether defective and non defective are equally produced or not. Use Binomial test at the 0.01 level of significance.

Solution:

Here number of N $(n_1) = 13$

Number of D $(n_2) = 9$.

Problem to test.

Null Hypothesis (H_0): $P = \frac{1}{2}$ i.e, defective and non-defective cables are equally produced.

Alternate Hypothesis (H_1) $= P \neq \frac{1}{2}$ i.e, defective and non-defective cables are not equally produced.

Test Statistics

$$X_0 = \text{Min}(n_1, n_2) = \text{Min}(12, 14) = 12.$$

Critical value

एक ओटा 0, अनि 1 गरेर 9 सम्म राखि सबैको sum गर्ने

$$\text{Now, the probability } (P) = \sum_{X=0}^{9} C(n, x)\left(\frac{1}{2}\right)^n = \sum_{X=0}^{9} C(n, x)\left(\frac{1}{2}\right)^n = 0.262$$

$$2p = 2 \times 0.262 = 0.524$$

Decision: $2P = 0.524 > \alpha = 0.01$.

Accept H_0 at 0.01 level of significance.

Conclusion: Defective and non defective are produced equally.

Example 2:- Out of 50 students willing to express opinion of laptop 30 expressed preferences to brand Dell and 20 for brand Lenovo. Use binomial test to test hypothesis that both brand of laptop are equally popular.

Solution:

Here, number of female students prefer Dell laptop $(n_1) = 30$.

Number of female students prefer Lenovo laptop $(n_2) = 20$.

Problem to test

Null Hypothesis (H_0): $P = \frac{1}{2}$

Alternate Hypothesis (H_1): $P \neq \frac{1}{2}$

Test Statistics:

$$X_0 = \min(n_1, n_2) = \min(20, 30) = 20$$

$$np = 50 \times \frac{1}{2} = 25$$

$$Z = \frac{(x_0 \pm 0.5) - np}{\sqrt{npq}} = \frac{(x_0 + 0.5) - np}{\sqrt{npq}} = \frac{(20+0.5) - 25}{\sqrt{12.5}} = -1.27$$

Level of significance
Let $\alpha = 0.05$.

Critical value
$Z_{\alpha/2} = 1.96$.

Decision
$|Z| = 1.27 < Z_{\alpha/2} = 1.96$.

Accept H_0 at 0.05 level of significance.

Conclusion: Both brand of laptop are equally popular.

© Kolmogorov Smirnov test:

Kolmogorov Smirnov test is used to test goodness of fit. It is alternate to Chi-square test for goodness of fit when sample size is small.

Problem to test:

Null Hypothesis (H_0): Samples come from population with distribution $F_0(x)$.

Alternate Hypothesis (H_1): Samples do not come from population with distribution $F_0(x)$.

Test Statistics:

For finding the test statistics first we need to calculate relative frequency or observed relative frequency (F_o) and expected relative frequency (F_e). Then,

test statistics $D_0 = \text{Max}|F_e - F_o|$

Critical value:

At α level of significance the tabulated value of D is $D_{n,\alpha}$.

Decision:

Reject H_0 at α level of significance if $D_0 \geq D_{n,\alpha}$ accept otherwise.

Example: In a certain computer hardware manufacturing industry six different types of machines are working to cut pieces of wires. The number of wires of unequal length recorded in a day is as follows:

Machine	1	2	3	4	5	6
No. of wire	2	0	4	8	5	11

Do these data provide sufficient evidence that the six machines equally cut the wires of equal length? Apply Kolmogorov Smirnov test at 5% level of significance.

Solution: Problem to test

Null Hypothesis (H_0): Six machines equally cut the wires of unequal length.

Alternate Hypothesis (H_1): Six machines do not equally cut the wires of unequal length.

Test Statistics:

We know, $D_0 = |F_e - F_0|$

For calculation of D_0.

Machine	No. of wire (i.e, frequency)	Observed frequency ($C.f_0$)	observed relative frequency (F_0)	Expected frequency (f_e)	(Cf_e)	F_e	$\|F_e - F_0\|$
1	2	2	2/30	5	5	5/30	3/30
2	0	2	2/30	5	10	10/30	8/30
3	4	6	6/30	5	15	15/30	9/30
4	8	14	14/30	5	20	20/30	6/30
5	5	19	19/30	5	25	25/30	6/30
6	11	30	30/30	5	30	30/30	0

Since Max $|F_e - F_0| = 9/30$

$\therefore D_0 = \frac{9}{30} = 0.3$

Critical value

At 5% level of significance the tabulated value of D is

$D_{30,0.05} = 0.242$

Decision Since $D_0 = 0.30 > D_{30,0.05} = 0.242$. So, H_0 is rejected.

i.e, H_1 is accepted.

Conclusion: Six machines do not equally cut the wires of equal length.

2> Two Independent sample test:

@ Mann-Whitney U-test: [Imp]

Mann-Whitney U-test is a non-parametric test which is used to test the two independent samples whether they come from same population or different population. This test is used as an alternative to the T-test when T-test fails to satisfy the normality assumptions.

For U-test all the observations are combined and ranked as a one group of data from smallest to largest. Incase of same ranks the average rank is assigned. After ranking, the rank value for each sample is summed then U-statistics is calculated as below;

$$U_1 = n_1 n_2 + \frac{n_1(n_1+1)}{2} - \sum R_1$$

i.e, Sum of ranks of 1^{st} sample

$$U_2 = n_1 n_2 + \frac{n_2(n_2+1)}{2} - \sum R_2$$

i.e, Sum of ranks of 2^{nd} sample

where, n_1 = no. of obs. in 1^{st} sample
n_2 = no. of obs. in 2^{nd} sample.

Note: $U_1 + U_2 = n_1 n_2$. (This will help to check that we have ranked right or not).

Now,

$$U_{min} = min\left[U_1, U_2\right].$$

Decision: If $U_{min} > U_{tab}$ then H_0 is accepted otherwise H_0 is rejected. (if $n_1 \leq 10, n_2 \leq 10$).

⇒ But if any sample (n_1 or n_2) is >10 then Z can be calculated as;

$$Z = \frac{U_{min} - \mu_U}{\sigma_U}$$

where, $\mu_U = \frac{n_1 n_2}{2}$

& $\sigma_U = \sqrt{\frac{n_1 n_2 (n_1 + n_2 + 1)}{12}}$

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⇒ If the ranks are same (i.e, tied/repeated) then,

$$\text{corrected } \sigma_U = \sqrt{\frac{n_1 n_2}{n(n-1)}\left[\frac{n^3-n}{12} - \frac{\sum_i t_i^3 - t_i}{12}\right]}$$

where, t_i is no. of times that ith rank is repeated.

Q1. In order to increase their efficiency one group of operators was imparted classroom training and other group was provided on the job training. After the training the time to complete the jobs in minutes was recorded for the both groups are as follows:-

Classroom training		On Job Training	
Operator no.	Time	Operator no.	Time
1	35	1	85
2	39	2	28
3	51	3	42
4	63	4	37
5	48	5	61
6	31	6	54
7	29	7	36
8	41	8	57
9	55		

Test whether both methods of imparting training are equally effective.

Solution:

Problem to test:

Null Hypothesis (H_0): $\mu_1 = \mu_2$

i.e, Both methods of imparting training are equally effective.

Alternate Hypothesis (H_1): $\mu_1 \neq \mu_2$

i.e, Both methods of imparting are not equally effective.

Test Statistics:

We have, $U_1 = n_1 n_2 + \frac{n_1(n_1+1)}{2} - \sum R_1$

$$U_2 = n_1 n_2 + \frac{n_2(n_2+1)}{2} - \sum R_2$$

For finding the value of ΣR_1 and ΣR_2.

Classroom Training		On Job training	
Time	Rank	Time	Rank
35	4	85	17
39	7	28	1
51	11	42	9
63	16	37	6
48	10	61	15
31	3	54	12
29	2	36	5
41	8	57	14
55	13		
	$\Sigma R_1 = 74$		$\Sigma R_2 = 79$

Both values of time for classroom training and on job training are combined as one group and ranked in ascending order

Now,

$$U_1 = 9\times8 + \frac{9\times10}{2} - 74 = 43$$

$$U_2 = 9\times8 + \frac{8\times9}{2} - 79 = 29$$

If $U_1+U_2 \neq n_1n_2$ then our ranking is wrong so correct it and further proceed

Here, $U_1 + U_2 = 43+29 = 72 = n_1 \times n_2 = 9\times8 = 72$

So, $U_{min} = \min[U_1, U_2] = \min[43, 29] = 29$.

<u>Critical value</u>: At 0.05 level of significance the tabulated value of U is $U_{tab} = 15$ (for $n_1 = 9$, $n_2 = 8$).

<u>Decision</u>: Since $U_{min} = 29 > U_{tab} = 15$. So, H_0 is accepted i.e, H_1 is rejected.

<u>Conclusion</u>: Hence both methods of imparting training are equally effective.

Q2. A farmer wishes to determine whether there is a difference in yeilds between two different varieties of wheat I and II. The following data shows the production of wheat per unit area using the two varieties. Can the farmer conclude at significance level 0.01 that a difference exists?

this is $n_1 > 10$ so Z-test

Wheat I	15.9	15.3	16.4	14.9	15.3	16.0	14.6	15.3	14.5	16.6	16.0
Wheat II	16.4	16.8	17.1	16.9	18.0	15.6	18.1	17.2	15.4		

Use Mann Whitney U test.

Solution:

Problem to test:

Null Hypothesis (H_0): $\mu_1 = \mu_2$

i.e, There is no significance difference between the average yeild by two varieties of wheat.

Alternate Hypothesis (H_1): $\mu_1 \neq \mu_2$.

i.e, There is significance difference between the average yeild by two varieties of wheat.

Test Statistics:

For the calculation of U_{min}.

Wheat I	Rank	Wheat II	Rank
15.9	9	16.4	12.5
15.3	5	16.8	15
16.4	12.5	17.1	17
14.9	3	16.9	16
15.3	5	18.0	19
16.0	10.5	15.6	8
14.6	2	18.1	20
15.3	5	17.2	18
14.5	1	15.4	7
16.6	14		
16.0	10.5		
	$\Sigma R_1 = 77.5$		$\Sigma R_2 = 132.5$

Now, $U_1 = n_1 n_2 + \frac{n_1(n_1+1)}{2} - \Sigma R_1 = 11 \times 9 + \frac{11 \times 12}{2} - 77.5 = 87.5$

$$U_2 = n_1 n_2 + \frac{n_2(n_2+1)}{2} - \Sigma R_2 = 11\times 9 + \frac{9\times 10}{2} - 132.5 = 11.5$$

Here $U_1 + U_2 = 87.5 + 11.5 = 99 = n_1 n_2 = 11\times 9 = 99$.

$$U_{min} = \min[27.5, 11.5] = 11.5$$

Z-statistics:

$$Z_{cal} = \frac{U_{min} - \mu_U}{\sigma_U}$$

where, $\mu_U = \frac{n_1 n_2}{2} = \frac{11\times 9}{2} = 49.5$

Corrected $\sigma_U = \sqrt{\frac{n_1 n_2}{n(n-1)}\left[\frac{n^3-n}{12} - \frac{\Sigma t_i^3 - t_i}{12}\right]}$

कुनै value कति चोटि repeat भयो त्यो लेख्ने यस्मा 3 चोटि, 2 चोटि र 2 चोटि repeat छ

$$= \sqrt{\frac{11\times 9}{20(20-1)}\left[\frac{20^3-20}{12} - \left(\frac{3^3-3}{12} + \frac{2^3-2}{12} + \frac{2^3-2}{12}\right)\right]}$$

$$= \sqrt{\frac{99}{380}\left[\frac{7980}{12} - \frac{36}{12}\right]}$$

$$= 13.15$$

Now,

$$Z_{cal} = \frac{11.5 - 49.5}{13.15} = -2.81$$

page no. 309 table value of book

Critical value: The tabulated value of Z at 0.01 level of significance is $Z_{0.01} = 2.575$ (two tailed test).

Decision: Since $Z_{cal} = -2.81 < Z_{tab} = 2.575$. So, H_0 is accepted. i.e, H_1 is rejected.

Conclusion: Hence we can conclude that there is no significance difference between the yeild of two varieties of wheat.

6. Chi-Square Test (χ^2-test): [V.Imp] एउटा Question fix आउछ दोबार.

having qualitative phenomena

Uses:

i) Chi-square test is used to test the independence of attributes

ii) To test the goodness of fit.

For Finding the value of Chi-square we need observed frequency and expected frequency. Here, observed frequencies are obtained by direct observation and expected frequency are generated on the basis of some hypothesis given in question.

Now,

$$\chi^2 = \sum \frac{(O-E)^2}{E}$$

where, O = observed frequency

E = expected frequency.

i) Chi-Square to test goodness of fit:

Problem to test:

Null Hypothesis (H_0): The given distribution fit the given data.

Alternate Hypothesis (H_1): The given distribution does not fit the given data.

Test Statistics:

$$\chi^2_{cal} = \sum \frac{(O-E)^2}{E}$$

Critical value: The tabulated value of χ^2 at α level of significance with $(n-1)$ degree of freedom (d.f) is $\chi^2_{(\alpha, n-1)}$.

Decision: If $\chi^2_{cal} > \chi^2_{tab}$ then H_0 is rejected otherwise H_0 is accepted.

Q1. A dice was rolled for 60 times and observed the following outcomes:

Side	1	2	3	4	5	6	Total
No. of times observed	8	9	13	7	15	8	60

Is the die fair? Test the hypothesis at 5% level of significance.

Solution:

Problem to test:

Null Hypothesis (H_0): The dice is fair. OR $P = \frac{1}{6}$

Alternate Hypothesis (H_1): The dice is not fair. OR $P \neq \frac{1}{6}$.

Test Statistics:

$$\chi^2_{cal} = \sum \frac{(O-E)^2}{E}$$

For the calculation of Expected frequency (E) and calculation of χ^2_{cal}.

ie, $\Sigma O \times$ probability

Observed frequency (O)	Expected frequency (E) where $E = NP$	$O-E$	$(O-E)^2$	$\frac{(O-E)^2}{E}$
8	$60 \times \frac{1}{6} = 10$	-2	4	$4/10$
9	$60 \times \frac{1}{6} = 10$	-1	1	$1/10$
13	" $= 10$	3	9	$9/10$
7	" $= 10$	-3	9	$9/10$
15	" $= 10$	5	25	$25/10$
8	" $= 10$	-2	4	$4/10$
$\Sigma n = 60 = N$	$\Sigma E = 60$			$\Sigma \frac{(O-E)^2}{E} = 5.2$

Critical value: At 0.05 level of significance the tabulated value of χ^2 with $(n-1) = 6-1 = 5$ d.f is $\chi^2_{(0.05, 5)} = 11.070$.

Decision: Since $\chi^2_{cal} = 5.2 < \chi^2_{tab} = 11.070$. So, H_0 is accepted. ie, H_1 is rejected.

Conclusion: Therefore the die is fair.

Q2. A certain chemical plant processes sea water to collect sodium chloride and magnesium. From scientific analysis, sea water is known to contain sodium chloride, magnesium and other elements in the ratio of 62:4:34. A sample of 200 tons of sea water resulted in 130 tons of sodium chloride and 6 tons of magnesium. Are these data consistent with the scientific model at 5% level of significance?

Solution:

Null Hypothesis (H_0): The given data are consistent with the scientific model.

Alternate Hypothesis (H_1): The given data are not consistent with the scientific model.

Test Statistics

we know that, $\chi^2_{cal} = \sum \frac{(O-E)^2}{E}$

i.e, $\sum O \times$ probability of each ratio

Chemical	Observed frequency (O)	Expected frequency (E)	O−E	$(O-E)^2$	$\frac{(O-E)^2}{E}$
Sodium Chloride	130	$200 \times \frac{62}{100} = 124$	6	36	36/124
Magnesium	6	$200 \times \frac{4}{100} = 8$	−2	4	4/8
Remaning	64	$200 \times \frac{34}{100} = 68$	4	16	16/68
	$\sum O = 200$				$\sum \frac{(O-E)^2}{E} = 1.025$

Critical value: The tabulated value of χ^2 at 0.05 level of significance $(3-1)=2$ d.f is $\chi^2_{(0.05,2)} = 5.991$.

Decision: Since $\chi^2_{cal} = 1.025 < \chi^2_{tab} = 5.991$. So, H_0 is rejected. i.e, H_1 is accepted.

Conclusion: The given data are not consistent with the scientific model.

parameter (λ) दिएको छैन find गर्नै पर्छ

Q.N.3. A publishing house got a 500 page book composed for printing. Before printing, the first draft was sent for proof reading. The proof reader detected the number of misprints X on each pages tabulated as follows. Test whether poisson distribution fits the data.

No. of misprint page	0	1	2	3	4	5
No. of pages	221	167	70	30	7	5

Solution:

Null Hypothesis (H_0): Poission distribution fit the given data

Alternate Hypothesis (H_1): Poission distribution does not fit the given data.

Test Statistics:

$$\chi^2_{cal} = \sum \frac{(O-E)^2}{E}$$

For fitting poission, distribution, first we need to find parameter λ.

No. of misprint pages (f)	No. of pages (x)	fx
0	221	0
1	167	167
2	70	140
3	30	90
4	7	28
5	5	25
	N=500	$\sum fx = 400$.

Now, $\text{Mean}(\bar{x}) = \lambda = \frac{\sum fx}{N} = \frac{400}{500} = 0.9$

Here, $P(x) = \frac{e^{-\lambda} \lambda^x}{x!}$

So,

For 0, $P(0) = \frac{e^{-0.9}(0.9)^0}{0!} = e^{-0.9} = 0.4065$

For 1, $P(1) = \frac{e^{-0.9} \cdot (0.9)^1}{1!} = 0.3659$.

For 2, $P(2) = \dfrac{e^{-0.9} \cdot (0.9)^2}{2!} = 0.1646$

For 3 $P(3) = \dfrac{e^{-0.9} \cdot (0.9)^3}{3!} = 0.04439$

For 4 $P(4) = \dfrac{e^{-0.9} (0.9)^4}{4!} = 0.0111$

For 5 $P(5) = \dfrac{e^{-0.9} \cdot (0.9)^5}{5!} = 0.002$

For the calculation of χ^2

round off गरेर fraction part मात्र राखेको

No. of misprint pages	Observed frequency (O)	Probability (P)	E=NP	O−E	$(O-E)^2$	$\frac{(O-E)^2}{E}$
0	221	0.4065	203	18	324	1.596
1	167	0.3659	183	−16	256	1.398
2	70	0.1646	82	−12	144	1.756
3	30	0.04939	25	5	25	1
4	7] 12	0.0111	6] 7	5	25	3.57
5	5	0.002	1			
	$\sum O = N = 500$		$\sum E = 500$			$\sum \frac{(O-E)^2}{E} = 9.323$

E column मा polling गरेकोले corresponding column मा पनि गर्नु पर्छ, Polling एउटा side गरे अर्को side नि गर्नु पर्छ।

6 र 1 लाई polling गरेको means addition गरेर एउटैमा राखेको. 5 भन्दा कम भए कुनै value polling गरेर >5 बनाइन्छ

$\therefore \chi^2_{cal} = 9.323$

Critical value: The tabulated value of χ^2 at 0.05 level of significance with $\underbrace{6-1}_{}-1-1 = 3$ d.f is $\chi^2_{(0.05,3)} = 7.815$

from formula n−1

due to estimation of parameter we subtract 1

due to polling we subtract 1.

If 3 number were combined for polling then we subtract 2 (i.e, 1 less than no.)

Decision: Since $\chi^2_{cal} = 9.3223 > \chi^2_{tab} = 7.815$. So, H_0 is rejected. i.e, H_1 is accepted.

Conclusion: Hence we can conclude that poission distribution does not fit given data.

Q.N.4. In 50 random sample of a manufactured mice, the number of samples containing defective mice is noted below:

No. of defective mice	0	1	2	3	4	5
Frequencies	4	13	17	12	3	1

Can the binomial distribution be a good with $p=0.30$?

Solution:

Null Hypothesis (H_0): Binomial distribution fit the given data.

Alternate Hypothesis (H_1): Binomial distribution does not fit the given data.

Test Statistics:

$$\chi^2_{cal} = \sum \frac{(O-E)^2}{E}$$

For the calculation of probability by using binomial distribution we have, $P(x) = nC_x \, p^x q^{n-x}$.

Here, $n=5$ & $p=0.30 \Rightarrow q = 1-0.30 = 0.70$.

Now,

$$P(0) = 5C_0 (0.3)^0 (0.7)^5 = 0.16307$$

$$P(1) = 5C_1 (0.3)^1 (0.7)^4 = 0.36015$$

$$P(2) = 5C_2 (0.3)^2 (0.7)^3 = 0.3087$$

$$P(3) = 5C_3 (0.3)^3 (0.7)^2 = 0.1323$$

$$P(4) = 5C_4 (0.3)^4 (0.7)^1 = 0.283$$

$$P(5) = 5C_5 (0.3)^5 (0.7)^0 = 0.0024$$

For the calculation of χ^2

No. of defective mice	Observed freq. (O)	Probability (P)	E=NP	O−E	$(O-E)^2$	$\frac{(O-E)^2}{E}$
0	4] 17	0.16307	8] 26	−9	81	$81/26 = 3.11$
1	13	0.36015	18			
2	17	0.3087	16	1	1	$1/16 = 0.0625$
3	12] 16	0.1323	7] 8	8	64	$64/8 = 8$
4	3	0.283	1			
5	1	0.0024	0			
	$\Sigma O = N = 50$		$\Sigma E = 50$			$\Sigma \frac{(O-E)^2}{E} = 11.177$

d.f मा −1

3+1 = 4 मात्र हुन्छ >5 हुँदैन so polling गर्ने 3 digit लिएको . d.f मा −2 हुन्छ

एक side जोडे आर्को side पनि गर्नु पर्छ त्यसैले यो side पनि

$\therefore \chi^2_{cal} = 11.177$

Critical Value: The tabulated value of χ^2 at 0.05 level of significance with $6-1-2-1 = 2$ d.f is $\chi^2_{(0.05,2)} = 5.991$

from formula $n-1$

polling of 3 digit

polling of 2 digit

Decision: Since $\chi^2_{cal} = 11.177 > \chi^2_{tab} = 5.991$. So, H_0 is rejected. i.e, H_1 is accepted.

Conclusion: Hence, Binomial distribution does not fit the given data.

associated/dependent/independent भनेर question आए त्यो यसको हो

ii) Chi-Square test for independence of attributes:-

Problem to test:

H_0: Attributes A and B are independent.

H_1: Attributes A and B are dependent.

Test Statistics:

$$\chi^2 = \frac{N(ad-bc)^2}{(a+c)(b+d)(a+b)(c+d)}$$

(for cell frequency greater than 5 and 2 by 2 table)

But if any frequency is less than 5 in any cell then,

$$\chi^2_{corrected} = \frac{N\left(|ad-bc| - \frac{N}{2}\right)^2}{(a+c)(b+d)(a+b)(c+d)}$$

Q1. The distributions of persons according to sex and blood groups are given below:

	Blood group			
Sex	O	A	B	AB
Male	100	40	45	10
Female	110	35	55	5

Is there any association between sex and blood group?

Solution:

Problem to test

Null Hypothesis (H_0): There is no significant association between sex and blood group (i.e, independent).

Alternate Hypothesis (H_1): There is significant association between sex and blood group. (i.e, dependent).

Test Statistics:

$$\chi^2_{cal} = \sum \frac{(O-E)^2}{E}$$

Here, E is expected frequency $= \frac{\text{Row Total} \times \text{Column Total}}{N \text{ (Obs. freq)}}$

For the calculation of χ^2.

	Observed frequency (O)	Expected frequency (E)	O−E	$(O-E)^2$	$\frac{(O-E)^2}{E}$
Male with O group → MO	100	$\frac{195\times210}{100} = 102$	−2	4	$4/102 = 0.0392$
Male with A group → MA	40	$\frac{195\times75}{40} = 37$	3	9	$9/37 = 0.243$
MB	45	$\frac{195\times100}{45} = 49$	−4	16	$16/49 = 0.326$
MAB	10	$\frac{195\times15}{10} = 7$	3	9	$9/7 = 1.285$
female with O group → FO	110	$\frac{205\times210}{110} = 108$	2	4	$4/108 = 0.037$
FA	35	$\frac{205\times75}{35} = 38$	−3	9	$9/38 = 0.236$
FB	55	$\frac{205\times100}{55} = 51$	4	16	$16/51 = 0.314$
FAB	5	$\frac{205\times15}{5} = 8$	−3	9	$9/8 = 1.125$
	$\sum O = 400$	$\sum E = 400$			$\chi^2_{cal} = \sum\frac{(O-E)^2}{E} = 3.607$

Rough work for calculating Column Total and Row total

Sex \ Blood group	O	A	B	AB	Row Total
Male	100	40	45	10	195
Female	110	35	55	5	205
Column Total	210	75	100	15	400

यो side मा जोरेर देखाउने पनि फरक पर्दैन।

या यो table पहिला बनाएर table अनि मुनिको बनाउने

Critical Value:- The tabulated value of χ^2 at 0.05 level of significance with $(r-1)(c-1)$ d.f $=(2-1)(4-1)=3$ d.f is $\chi^2_{(0.05,3)}=7.815$

no. of rows   no. of columns

Decision:- Since $\chi^2_{cal}=3.60 < \chi^2_{tab}=7.815$. So, H_0 is accepted & H_1 is rejected.

Conclusion: Hence there is no significant association between sex and blood group. (i.e, independent).

Q.2. Test whether the color of son's eyes is associated with that of the father's at 5% level of significance using the data available in the following table.

Father's eye colour	Son's eye colour	
	Not Light	Light
Not Light	230	148
Light	151	471

Solution:

Null Hypothesis (H_0): There is no significant association between colour of son's eyes and colour of father's eyes (ie, independent).

Alternate Hypothesis (H_1): There is significance association between colour of son's eyes and colour of father's eyes (ie, dependent).

Test Statistics:

$$\chi^2_{cal}=\sum\frac{(O-E)^2}{E}$$

Since this is the case of 2 by 2 table and all cell frequencies are greater than 5 so, we use directly formula:

$$\chi^2_{cal}=\frac{N(ad-bc)^2}{(a+b)(c+d)(a+c)(b+d)}$$

Rough

a	b
c	d

$N(ad-bc)^2$

सबै चारै तिर जाइले

$(a+b)(c+d)$
$(a+c)(b+d)$

For the calculation of χ^2 we construct following table:-

Father's eyes colour	Son's eyes colour		Total
	Not light	Light	
Not Light	230 (a)	148 (b)	378 (a+b)
Light	151 (c)	471 (d)	622 (c+d)
Total	381 (a+c)	619 (b+d)	1000 i.e. N=a+b+c+d

$$\therefore \chi^2_{cal} = \frac{1000(230\times 71 - 151\times 148)^2}{378\times 622\times 381\times 619}$$

$$= 13.33$$

Critical value:– The tabulated value of χ^2 at 0.05 level of significance with (2-1) (2-1) d.f = 1.d.f is $\chi^2_{(0.05,1)} = 3.841$.

Decision:– Since $\chi^2_{cal} = 13.33 > \chi^2_{tab} = 3.841$. So, H_0 is rejected, i.e. H_1 is accepted.

Conclusion:– Hence there is significance association between colour of sons eyes and colour of fathers eyes. (i.e, dependent).

Q3. 88 workers of a IT company were interviewed during a sample survey for their smoking habit. Classification of respondents according to their gender and their smoking habit are found as;

Habit	Sex	
	Male	Female
Smoker	40	33
Non-Smoker	3	12

Do the smoking ~~tea~~ habit is associated with gender.

Solution:

Problem to test:

Null Hypothesis (H_0): There is no significant association between smoking habit and gender (i.e, independent).

Alternate Hypothesis (H_1): There is significant association between smoking habit and gender. (i.e, dependent).

Test Statistics:

Since this is the case of 2 by 2 table and one shell frequency is less then 5 then, we have formula,

$$\chi^2_{cal} = \frac{N\left[(ad-bc)-\frac{N}{2}\right]^2}{(a+b)(c+d)(a+c)(b+d)}$$

For the calculation of χ^2.

Habit	Sex		Total
	Male	Female	
Smoker	40 (a)	33 (b)	73 (a+b)
Non-Smoker.	3 (c)	12 (d)	15 (c+d)
Total	43 (a+c)	45 (b+d)	88 (a+b+c+d=N)

cell freq. <5

Now, $$\chi^2_{cal} = \frac{88\left[(40\times2-33\times3)-\frac{88}{2}\right]^2}{73\times15\times43\times45}$$

$= 4.71.$

Critical value: The tabulated value of χ^2 at 0.05 level of significance with $(2-1)(2-1) = 1$ d.f is $\chi^2_{(0.05,1)} = 3.841$.

Decision: Since $\chi^2_{cal} = 4.71 > \chi^2_{tab} = 3.841$. So, H_1 is accepted and H_0 is rejected.

Conclusion: Hence, there is significant association between smoking habit and gender.

यसरि table नदिएर information दिए पहिला Q. को basis मा table बनाउने

Q.N.4: Out of a sample of 120 persons in a village, 76 were administered a new drug for preventing influenza and out of them 24 persons were attacked by influenza. Out of those who were not administered the new drug, 12 persons were not affected by influenza. Is the new drug effective in controlling influenza, test at 5% level of significance.

Solution:-

Given information can be shown in the following table:-

यो side drug र पल्लोमा influenza जुनसुकै side मा लेख्दा पनि फरक पर्दैन

Influenza	Drug		Total
	administered	Not administered	
Attacked	24 (a)	32 (b)	56 (a+b)
Not attacked	52 (c)	12 (d)	64 (c+d)
Total	76 (a+c)	44 (b+d)	120

Problem to test: table बनाएपछि अब पहिलोको जस्तै गरि solve गर्ने

Null Hypothesis (H_0): There is no effect of drug in controlling influenza.

Alternate Hypothesis (H_1): There is effect of drug in controlling influenza.

Test Statistics: This is the case of 2 by 2 table and each cell frequency are greater than 5 so, we can directly use formula;

$$\chi^2_{cal} = \frac{N(ad-bc)^2}{(a+b)(a+c)(b+d)(c+d)}$$

$$= \frac{120(24\times12-52\times32)^2}{56\times76\times44\times64}$$

$$= 18.95.$$

Critical value: χ^2 at 0.05 level of significance with $(2-1)(2-1)=1$ d.f is $\chi^2_{(0.05,1)} = 3.841$.

Decision: Since, $\chi^2_{cal} = 18.95 > \chi^2_{tab} = 3.841$. So, H_0 is rejected i.e, H_1 is accepted.

Conclusion: Hence there is effect of drug in controlling influenza.

(c) Median Test:-

It is used to test the significance of difference between median of two independent sample.

1) If both sample sizes are more than or equal to 10 then test statistics $Z = \frac{a - \mu_a}{\sigma_a}$

where, a is the no. of observations in first sample which is less than or equal to median value of combined sample

Cases:

i) When $n_1 + n_2 = n$ (any even number) then,

$$\mu_a = n_1/2$$

& $\sigma_a = \sqrt{\frac{n_1 n_2}{4(n-1)}}$

ii) When $n_1 + n_2 = n$ (any odd number) then,

$$\mu_a = \frac{n_1(n_1 - 1)}{2n}$$

& $\sigma_a = \sqrt{\frac{n_1 n_2 (n+1)}{4n^2}}$

2) But if $n_1 < 10$ and $n_2 < 10$ then test statistics is:

$$P\text{-value} = \frac{{}^{n_1}C_{U_1} \times {}^{n_2}C_{U_2}}{{}^{n}C_{r}}$$

where, n_1 = no. of obs. in sample 1.

n_2 = no. of obs. in sample 2.

U_1 = No. of obs. in sample 1 which is less than combined mean.

U_2 = No. of obs. in sample 2 which is less than combined mean.

& $r = \frac{n-1}{2}$.

Decision: If P-value $> \alpha$ then, H_0 is accepted. i.e, H_1 is rejected.

If P-value $\leq \alpha$ then, H_0 is rejected i.e, H_1 is accepted.

Q1. The following are the yeilds of 10 plots under two treatments A and B.

Treatment A: 34, 46, 45, 32, 42, 39, 48, 49, 30, 51.

Treatment B: 35, 43, 44, 40, 59, 47, 55, 50, 47, 71.

Use median test to test the effectiveness of two treatment.

Solution:

Problem to test

Null Hypothesis (H_0): There is no significance difference in median yeild of two treatments A and B.

Alternate Hypothesis (H_1): There is significance difference between median yeild of two treatments A and B.

Test Statistics:

We have, $Z = \dfrac{a - \mu_a}{\sigma_a}$ (Since $n_1, n_2 \geq 10$)

For finding value of a first we need to find median of combined series:

Combined both treatment in ascending order

→ 30, 32, 34, 35, 39, 40, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 55, 59, 71.

Now, median = value of $\left(\frac{20+1}{2}\right)^{th}$ item

= value of $(10.5)^{th}$ item

$= \dfrac{45+46}{2}$

$= 45.5$

Now, a = no. of observations in A which are less or equal to median.

$\Rightarrow a = 6$.

Since $n_1 + n_2 = 10 + 10 = 20$ (even)

So, $\mu_a = n_1/2 = 10/2 = 5$

& $\sigma_a = \sqrt{\dfrac{n_1 n_2}{4(n-1)}} = \sqrt{\dfrac{10 \times 10}{4(20-1)}} = 1.147$

$\therefore Z = \dfrac{6 - 5}{1.147} = 0.871$

Critical value: The tabulated value of Z at 0.05 level of significance is $Z_{0.05} = 1.96$.

Decision: Since $Z_{cal} = 0.871 < Z_{tab} = 1.96$. So, H_0 is accepted.
i.e, H_1 is rejected.

Conclusion: Hence, there is no significance difference between median yeild of two treatments A and B.

Q2. A quality controller wishes to determine whether there is a difference in outcome between two different tools of software I and II. The following data shows the outcome of two different tools. Can the controller conclude that a difference exists? Use median test at 5% level of significance.

Software I	24.0	16.7	22.8	19.8	18.9	
Software II	23.2	19.8	18.1	17.6	20.2	17.8

Solution:

Null Hypothesis (H_0): There is no significance difference between the median outcomes of two different software tools.

i.e, $Median_I = Median_{II}$

Alternate Hypothesis (H_1): There is significance difference between the median outcomes of two different software tools.

i.e, $Median_I \neq Median_{II}$

Test Statistics:-

Since n_1 and n_2 are < 10. So, $P\text{-value} = \dfrac{{}^{n_1}C_{U_1} \times {}^{n_2}C_{U_2}}{{}^{n}C_{r}}$

where, U_1 = no. of obs. in sample I which are less than median of combined series.
U_2 = no. of obs. in sample II which are less than median of combined series.
& $r = \frac{n-1}{2}$.

For the calculation of U_1 and U_2:

Combined series:- 16.7, 17.6, 17.8, 18.1, 18.9, 19.8, 20.2, 22.8, 23.2, 24.0

Here, $n = 11$

So, median of $\left(\frac{11+1}{2}\right)^{th}$ item

$=$ value of 6^{th} item

$= 19.8$

Now,

$U_1 = 2$

$U_2 = 3$

So, P-value $= \dfrac{{}^{5}C_2 \times {}^{6}C_3}{{}^{11}C_5}$

$= 0.432$

<u>Decision</u>:- Since P-value $= 0.432 > \alpha = 0.05$. So, H_0 is accepted. i.e, H_1 is rejected.

<u>Conclusion</u>:- Hence, we can conclude that there is no significance difference between the median outcome of two different software tools.

3> Paired Sample Tests:

@ Kruskal Wallis H-test: [V.Imp].

The Mann-Whitney U-test is the test for deciding whether two samples comes from same population or not. But Kruskal Wallis H-test to test for 3 or more than 3 samples. The Test Statistics for this test is given by;

प्रति sample छन् त्यति लेख्ने mostly 3 हुन्छन्

$$H = \frac{12}{N(N+1)} \left\{ \frac{(\Sigma R_1)^2}{n_1} + \frac{(\Sigma R_2)^2}{n_2} + \frac{(\Sigma R_3)^2}{n_3} + \dots \right\} - 3[N+1]$$

where, $n_1, n_2, n_3 \dots$ are the number of elements in each sample.

& $N = n_1 + n_2 + n_3 + \dots$

कुनै एउटा sample ५ भन्दा सानो भएमा

The sampling distribution of H is a Chi-Square distribution with k-1 degree of freedom provided that $n_1, n_2, n_3 \dots \geq 5$.

$\Rightarrow$ If $n_1, n_2, n_3 \dots < 5$ then we use P-value instead of Chi-Square value. P-value is obtained from Krushkal Wallis table for the calculated H.

Decision: If P-value is $> \alpha$ then H_0 is accepted otherwise H_0 is rejected.

Note: If rank is tied (i.e, rank repeat भएको case मा) then test-statistics is given by;

$$\text{Corrected } H = H_c = \frac{H}{\text{A.F (Adjustment Factor)}}$$

where, $$A.F = 1 - \frac{\Sigma (t_i^3 - t_i)}{N^3 - N}$$

कुनै value कति चोटि repeat भयो त्यो number लाई t_i भनिन्छ

Q1. Following are the final examination marks of three group of students who were taught computer by three different methods;

Method I	94	88	91	74	87	97	
Method II	85	82	79	84	61	72	80
Method III	89	67	72	76	69		

Are all three methods equally effective? Use H test at 0.05 level of significance.

Solution:

Problem to test:

Null Hypothesis (H_0): All three methods are equally effective.

Alternate Hypothesis (H_1): All three methods are not equally effective.

Test Statistics

We know,

$$H_{cal} = \frac{12}{N(N+1)}\left\{\frac{(\Sigma R_1)^2}{n_1} + \frac{(\Sigma R_2)^2}{n_2} + \frac{(\Sigma R_3)^2}{n_3} + \cdots\right\} - 3(N+1).$$

For the calculation of ΣR_1, ΣR_2 and ΣR_3.

Method I	Rank	Method II	Rank	Method III	Rank
94	17	85	12	89	15
88	14	82	10	67	2
91	16	79	8	72	4.5 ✓
74	6	84	11	76	7
87	13	61	1	69	3
97	18	72	4.5 ✓		
		80	9		
	$\Sigma R_1 = 84$		$\Sigma R_2 = 55.5$		$\Sigma R_3 = 31.5$

Now,
$$H = \frac{12}{18(18+1)}\left[\frac{(84)^2}{6} + \frac{(55.5)^2}{7} + \frac{(31.5)^2}{5}\right] - 3(18+1)$$
$$= 6.667$$

Since this is case of tied/repeated (repeated ranks are shown by tick mark in above table)

So,
$$\text{Corrected } H = H_c = \frac{H}{\text{Adjustment Factor (A.F)}}$$

एउटा value 2 चोटि repeat भएकोले

$$\text{Here, } A.F = 1 - \frac{\Sigma t_i^3 - t_i}{N^3 - N} = 1 - \frac{2^3 - 2}{18^3 - 8} = 0.998$$

$$\text{Now, } H_c = \frac{H}{A.F} = \frac{6.667}{0.998} = 6.68.$$

Critical value: The tabulated value of χ^2 at 0.05 level of significance with $(3-1)=2$ d.f. is $\chi^2_{(0.05,2)} = 5.991$

Decision: Since $H_c = 6.68 > \chi^2_{tab} = 5.991$. So, H_0 is rejected. i.e. H_1 is accepted.

Conclusion: Hence we can conclude that three methods are not equally effective.

Q.2. For the following scores of 3 groups, apply Kruskal Wallis H test to test the hypothesis that the three groups are not significantly different.

Group	Scores				
A	96	128	83	61	101
B	82	124	132	135	109
C	115	149	166	147	

Solution:

Null Hypothesis (H_0): There is no significance difference between three groups.

Alternate Hypothesis (H_1): There is significance difference between three groups.

Test Statistics:

$$H = \frac{12}{N(N+1)} \left\{ \frac{(\sum R_1)^2}{n_1} + \frac{(\sum R_2)^2}{n_2} + \frac{(\sum R_3)^2}{n_3} \right\} - 3(N+1)$$

For the calculation of $\sum R_1$, $\sum R_2$ and $\sum R_3$.

Score of Group A	Rank	Score of Group B	Rank	Score of Group C	Rank
96	4	82	2	115	7
128	9	124	8	149	13
83	3	132	10	166	14
61	1	135	11	147	12
101	5	109	6		
	$\sum R_1 = 22$		$\sum R_2 = 37$		$\sum R_3 = 46$

Now,

$$H = \frac{12}{14(14+1)}\left[\frac{(22)^2}{5} + \frac{(37)^2}{5} + \frac{(46)^2}{4}\right] - 3(14+1)$$

$$= 6.40.$$

6.40 को nearest value चुन्न है उसको corresponding value निकाला

tabulated value from page no 332

Since $n_3 < 5$

So, P-value for $n = 6.40$ and $n_1 = 5, n_2 = 5$ & $n_3 = 4$ is 0.049

or we can also write 0.01 value the conclusion will be same

Decision: Since P-value $= 0.049 < \alpha = 0.05$.

So, H_0 is rejected. i.e, H_1 is accepted.

Conclusion: Hence, there is significance difference between three groups.

(b) Friedman test:-

It is used to test the significance difference between location of three or more independent populations. In this we rank blocks not as a one whole but each blocks are ranked seperately.

Test Statistics:

$$F_r = \frac{12}{nk(k+1)} \sum_{i=1}^{k} R_i^2 - 3n(k+1).$$

Sum of square of ranks in R_i block

where, k = no. of rows

n = no. of columns

R_i = ranks for each sample (i.e, $R_i = 1, 2, 3, \ldots k$).

rank repeats

If tied occurs then corrected test statistics is;

$$F_r = \frac{\frac{12}{nk(n+1)} \sum_{i=1}^{k} R_i^2 - 3n(k+1)}{1 - \sum \frac{t_i^3 - t_i}{n(k^3 - k)}}$$

where t_i = no. of times i^{th} rank is repeated

Critical value

For $2 \le n \le 9$ and $k = 3$, also $2 \le n \le 5$ and $k = 4$ critical value p is obtained from Friedman probability table.

For $n > 9$ and $k \ge 3$, critical value is $\chi^2_{(\alpha, k-1)}$.

Decision: Accept H_0 at α level of significance if $p > \alpha$ reject otherwise, for $2 \le n \le 9$ and $k = 3$, also for $2 \le n \le 5$ and $k = 4$.

Reject H_0 at α level of significance if $H > \chi^2_{(\alpha, k-1)}$, accept otherwise for $n > 9$ and $k \ge 3$.

Q1. A researcher wants to compare the teaching standard of three English medium schools on the basis of performance of the student's final examination scores. The percentage of passers in I to IV grade in the schools are presented in the following table.

	Grade			
	I	II	III	IV
Alpha	89	98	70	80
Sigma	45	76	40	55
Gamma	20	58	35	67

Test the performances of the schools with respect to pass percentage using Friedman's test.

Solution:

Problem to test:

Null Hypothesis (H_0): There is no significance difference between three schools with respect to pass percentage.

Alternate Hypothesis (H_1): There is significance difference between three schools with respect to pass percentage.

Test Statistics:

We have, $F_r = \frac{12}{nk(k+1)} \sum_{i=1}^{k} R_i^2 - 3n(k+1)$ where, n = no. of columns, k = no. of rows

For the calculation of F_r

	Grade									
	I	Rank	II	Rank	III	Rank	IV	Rank	R_i	R_i^2
Alpha	89	3	98	3	70	3	80	3	12	144
Sigma	45	2	76	2	40	2	55	1	7	49
Gamma	20	1	58	1	35	1	67	2	5	25
										$\sum R_i^2 = 218$

Now, $F_r = \frac{12 \times 218}{4 \times 3(3+1)} - 3 \times 4(3+1)$

$= 6.5$

Critical value: Since $n=4, k=3$ and $F_r = 6.5$, So from table, p-value $= 0.042$

Decision: Since P-value $= 0.042 < \alpha = 0.05$. So, H_0 is rejected. i.e, H_1 is accepted.

Conclusion: Hence there is significance difference between three schools with respect to percentage.

Q.N.2. The score of 3 matched groups under the six conditions are given below:-

Group	Condition					
	I	II	III	IV	V	VI
A	9	5	2	5	6	7
B	6	4	3	4	6	5
C	5	1	3	3	6	5

Apply the Friedman two way ANOVA test to identify if there is significantly difference in variation between matched groups. Use 5% level of significance.

Solution:-

Null Hypothesis (H_0):- There is no significance difference between the three matched groups.

Alternate Hypothesis (H_1):- There is significance difference between the three matched groups.

Test Statistics:-

We have,

$$F_r = \frac{\frac{12}{nk(k+1)} \sum_i R_i^2 - 3n(k+1)}{1 - \sum \frac{(t_i^3 - t_i)}{n(k^3-k)}}$$

(Since this is the case of tied ranks).

Now, for the calculation of F_r

Group	Condition												R_i	R_i^2
	I	Rank	II	Rank	III	Rank	IV	Rank	V	Rank	VI	Rank		
A	9	3	5	3	2	1	5	3	6	2	7	3	15	255
B	6	2	4	2	3	2.5	4	2	6	2	5	1.5	12	144
C	5	1	1	1	3	2.5	3	1	6	2	5	1.5	9	81
														$\sum_i R_i^2 = 450$

Here,

$$\sum \frac{(t_i^3 - t_i)}{n(k^3-k)} = \frac{2^3-2}{6(3^3-3)} + \frac{2^3-2}{6(3^3-3)} + \frac{3^3-3}{6(3^3-3)}$$

$$= 0.251$$

$$\therefore F_r = \frac{\frac{12 \times 450}{6 \times 3(3+1)} - 3 \times 6(3+1)}{1 - 0.251}$$

$$= 4$$

Critical value:- Since $n=6$, $k=3$ and $F_r=4$. So, from table p-value $= 0.184$

Decision, Since p-value $= 0.184 > \alpha = 0.05$. So, H_0 is accepted.

© Wilcoxon Matched pair signed rank test:

It is non-parametric test which is used to compare the two populations for which observations are paired.

Problem to test:

Null Hypothesis (H_0):- There is no significance difference between two population.

Alternate Hypothesis (H_1):- There is significance difference between two population.

Test Statistics:-

ⓐ For $n \leq 25$

Test statistics $T = \min(+S, -S)$. For the calculation of $+S$ and $-S$, first we need to take the difference between the two each pairs of observations then we rank these differences then;

+ve sign -ve sign

$+S$ = Sum of all the positive ranks

& $-S$ = Sum of all the negative ranks.

Critical Value → The tabulated value of T at α level of significance with n degree of freedom $= T_{\alpha,n}$.

Decision → If $T \leq T_{\alpha,n}$ then H_0 is rejected otherwise H_0 is accepted.

ⓑ For $n > 25$

We first calculate test statistics as we did for $n \leq 25$ then we use Z statistics as;

$$Z_{cal} = \frac{T - \mu_T}{\sigma_T} \quad \text{where, } \mu_T = \frac{n(n+1)}{4}$$

$$\sigma_T^2 = \frac{n(n+1)(2n+1)}{24}$$

Critical value → Critical value or $Z_{tabulated}$ is obtained from table according to level of significance.

Decision → If $Z > Z_{tabulated}$ then reject H_0, otherwise accept H_0.

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Q1. The weight (kg) of 5 people before they stopped smoking are as follows:-

Before	66	80	69	52	75
After	71.	82	68	56	73.

Use wilcoxon matched pairs signed rank test for paired observations to test the hypothesis at 0.05 level of significance that giving up smoking has no effect on a person's weight against the alternative hypothesis that one's weight increases if he or she quits smoking.

Solution:

Problem to test:-

Null Hypothesis (H_0):- There is no significance difference in the weight before smoking and after quit smoking.

Alternate Hypothesis (H_1):- There is significance increase in the weight after quit smoking.

Test Statistics:-

We have, $T = \min(+S, -S)$

For the calculation of $+S$ and $-S$.

Before X	After Y	$d = Y - X$	Rank
66	71	5	5
80	82	2	2.5
69	68	−1	−1
52	56	4	4
75	73	−2	−2.5

Now, $+S = 5 + 2.5 + 4 = 11.5$

$-S = 1 + 2.5 = 3.5$

So, $T = \min(11.5, 3.5) = 3.5$.

Critical value → The tabulated value of T at 0.05 level of significance with $n = 5$ d.f is $T_{0.05,5} = 1$.

Decision → Since $T = 3.5 > T_{0.05,5} = 1$. So, H_0 is accepted i.e, H_1 is rejected.

Conclusion → Hence there is no significance difference in the weight before smoking and after quit smoking.

Q2. The following data gives the additional hours of sleep gained by 31 patients in an experiment to test the effect of a drug. Do these data give evidence that the drug produces additional hours of sleep?

Sleep hours gained by patients are; 0.5, 0.7, 0.1, -0.2, 1.2, 1.5, -2, 4, 0.1, 3.4, 3.7, -1.1, 0.8, -0.8, 1.3, 2.7, -3.4, 0.1, 0.6, 2.3, 0.1, 2.7, -0.9, 3.1, 2.0, 1.2, 1.2, -1.8, 1.0.

Solution:

Additional hours sleep (d_i)	Rank
0.5	6
0.7	8
0.1	2.5
-0.2	-5
1.2	15
1.5	18
-2	-21.5
4	31
0.1	2.5
3.4	28
3.7	30
-1.1	-13
0.8	9.5
-0.8	-9.5
1.3	17
2.7	24.5
-3.4	-28
-1.9	-20
3.4	28
0.1	2.5
0.6	7
2.3	23
0.1	2.5
2.7	24.5
-0.9	-11
3.1	26
2.0	21.5
1.2	15
1.2	15
-1.8	-19
1.0	12

Here, $n=31$

Sum of $-$signed ranks $(-S) = 5+21.5+13+9.5+28+20+11+19 = 127$

Sum of $+$signed ranks $(+S) = \frac{n(n+1)}{2} - (-S) = \frac{31(1+31)}{2} - 127$

$= 369.$

$T = \min(+S, -S)$
$= \min(369, 127)$
$= 127.$

Problem to test:

H_0: drug does not produce additional hours of sleep.

H_1: drug produces additional hours of sleep.

additional hours shows one-tailed test

Test Statistic:-

$T = 127$

$\mu_T = \frac{n(n+1)}{4} = \frac{31(31+1)}{4} = 248$

$\sigma_T^2 = \frac{n(n+1)(2n+1)}{24} = \frac{31(31+1)(62+1)}{24} = 2604$

$\Rightarrow \sigma_T = \sqrt{2604} = 51.029$

Hence, $Z = \frac{T - \mu_T}{\sigma_T} = \frac{127-248}{51.029} = -2.37$

Critical value:- Let $\alpha = 5\%$ be the level of significance then critical value is $Z_{0.05} = 1.645$ (Since one-tailed test).

Decision:- $|Z| = 2.37 > Z_{0.05} = 1.645$. So, H_0 is rejected. i.e, H_1 is accepted.

Conclusion:- Hence, drug produces additional hours of sleep.

(d) Cochran Q test:-

Problem to test:

Null Hypothesis (H_0):- There is no significance difference between the treatments.

Alternate Hypothesis (H_1):- There is significance difference between the treatment effect.

Test Statistics:-

$$Q = \frac{(k-1)\left\{k\sum_{i=1}^{k} R_i^2 - \left(\sum_{i=1}^{k} R_i\right)^2\right\}}{k\sum_{j=1}^{n} C_j - \sum_{j=1}^{n} C_j^2}$$

where, k = no. of treatments with condition $k>2$.
n = set of objects.
$\sum R_i$ = Sum of all positives row wise.
$\sum C_j$ = Sum of all positives column wise.

Critical value: The tabulated value of χ^2 at α level of significance with $(k-1)$ d.f is $\chi^2_{\alpha, k-1}$.

Decision: If $Q > \chi^2_{\alpha, k-1}$, H_0 is rejected, otherwise H_0 is accepted.

Q1. Four objective questions are given to 5 students and the results of correct answer (1) and wrong answers (0) are arranged in the following table.

Objective Questions	Students				
	1	2	3	4	5
Q_1	1	0	0	1	1
Q_2	0	1	1	0	1
Q_3	1	1	1	0	0
Q_4	0	0	1	0	1

Apply Cochran q test for testing the hypothesis that there is no significant difference between four objective questions with respect to correct answers.

Solution:-

Null Hypothesis (H_0):- There is no significance difference between four objective questions with respect to their correct answer.

Alternate Hypothesis (H_1):- There is significance difference between four objective questions with respect to their correct answer.

Test Statistics:-

We have,

$$Q = \frac{(k-1)\left\{k\sum_{i=1}^{k} R_i^2 - \left(\sum_{i=1}^{k} R_i\right)^2\right\}}{k\sum_{j=1}^{n} C_j - \sum_{j=1}^{n} C_j^2}$$

For the calculation of $\sum R_i$, $\sum R_i^2$, $\sum C_j$ and $\sum C_j^2$.

Objective Questions	Students 1	2	3	4	5	R_i	R_i^2
Q_1	1	0	0	1	1	3	9
Q_2	0	1	1	0	1	3	9
Q_3	1	1	1	0	0	3	9
Q_4	0	0	1	0	1	2	4
C_j	2	2	3	1	3	$\sum R_i = 11$ $\sum C_j = 11$	$\sum R_i^2 = 31$
C_j^2	4	4	9	1	9	$\sum C_j^2 = 27$	

Now, $Q = \dfrac{(4-1)\{4\times 31 - (11)^2\}}{4\times 11 - 27} = \dfrac{3(124-121)}{44-27} = 0.529$

Critical value: The tabulated value of χ^2 at 0.05 level of significance with $k-1 = 4-1 = 3$ d.f is $\chi^2_{0.05,3} = 7.815$.

Decision:- Since $Q = 0.529 < \chi^2_{0.05,3} = 7.815$. So, H_0 is accepted. i.e, H_1 is rejected.

Conclusion:- Hence, there is no significance difference between four objective questions with respect to their correct answer.